QIS3

Calibration of the underwriting risk, market risk and MCR

April 2007
# Table of Contents

**TABLE OF CONTENTS**

**SECTION 1 – CALIBRATION OF THE UNDERWRITING RISK MODULES**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INTRODUCTION</strong></td>
<td>4</td>
</tr>
<tr>
<td><strong>Purpose</strong></td>
<td>4</td>
</tr>
<tr>
<td><strong>Overview</strong></td>
<td>4</td>
</tr>
<tr>
<td><strong>LIFE UNDERWRITING RISK</strong></td>
<td>5</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>5</td>
</tr>
<tr>
<td><strong>Correlations between u/w risk components</strong></td>
<td>5</td>
</tr>
<tr>
<td><strong>Choice of shocks for biometric risks</strong></td>
<td>6</td>
</tr>
<tr>
<td><strong>Mortality risk</strong></td>
<td>6</td>
</tr>
<tr>
<td><strong>Longevity risk</strong></td>
<td>6</td>
</tr>
<tr>
<td><strong>Disability and sickness</strong></td>
<td>7</td>
</tr>
<tr>
<td><strong>Choice of scenarios for lapse, expense and revision risk</strong></td>
<td>8</td>
</tr>
<tr>
<td><strong>Lapse risk</strong></td>
<td>8</td>
</tr>
<tr>
<td><strong>Expense risk</strong></td>
<td>8</td>
</tr>
<tr>
<td><strong>Revision risk</strong></td>
<td>9</td>
</tr>
<tr>
<td><strong>Factors for mortality, disability and lapse catastrophe risk</strong></td>
<td>9</td>
</tr>
<tr>
<td><strong>Mortality and disability catastrophe risk</strong></td>
<td>9</td>
</tr>
<tr>
<td><strong>Mortality catastrophe risk</strong></td>
<td>10</td>
</tr>
<tr>
<td><strong>Disability catastrophe risk</strong></td>
<td>10</td>
</tr>
<tr>
<td><strong>Lapse catastrophe risk</strong></td>
<td>11</td>
</tr>
<tr>
<td><strong>NON-LIFE UNDERWRITING RISK</strong></td>
<td>11</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>11</td>
</tr>
<tr>
<td><strong>Overall calculation of premium and reserve risk</strong></td>
<td>12</td>
</tr>
<tr>
<td><strong>Market-wide factors for premium and reserve risk</strong></td>
<td>13</td>
</tr>
<tr>
<td><strong>Premium risk</strong></td>
<td>13</td>
</tr>
<tr>
<td><strong>Reserve risk</strong></td>
<td>15</td>
</tr>
<tr>
<td><strong>Credibility constant for premium risk</strong></td>
<td>15</td>
</tr>
<tr>
<td><strong>Correlations between different LOBs</strong></td>
<td>18</td>
</tr>
<tr>
<td><strong>Aggregation to an overall sigma</strong></td>
<td>18</td>
</tr>
<tr>
<td><strong>Rationale for aggregation formula</strong></td>
<td>18</td>
</tr>
<tr>
<td><strong>Setting of the correlation coefficients</strong></td>
<td>20</td>
</tr>
<tr>
<td><strong>CAT risk</strong></td>
<td>22</td>
</tr>
<tr>
<td><strong>HEALTH UNDERWRITING RISK</strong></td>
<td>23</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>23</td>
</tr>
<tr>
<td><strong>Expense risk</strong></td>
<td>23</td>
</tr>
<tr>
<td><strong>Results</strong></td>
<td>24</td>
</tr>
<tr>
<td><strong>Claim/mortality/cancellation risk</strong></td>
<td>25</td>
</tr>
<tr>
<td><strong>Epidemic/accumulation risk</strong></td>
<td>26</td>
</tr>
</tbody>
</table>

**SECTION 2 – CALIBRATION OF THE MARKET RISK**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INTRODUCTION</strong></td>
<td>28</td>
</tr>
<tr>
<td><strong>Purpose</strong></td>
<td>28</td>
</tr>
<tr>
<td><strong>Scope</strong></td>
<td>28</td>
</tr>
<tr>
<td><strong>General assumptions</strong></td>
<td>28</td>
</tr>
<tr>
<td><strong>Overview</strong></td>
<td>29</td>
</tr>
</tbody>
</table>
Section 1

Calibration of the underwriting risk modules

Introduction

Purpose

1.1 This section deals with the calibration of the underwriting risk modules (life underwriting risk, non-life underwriting risk and health underwriting risk) within the SCR standard formula. It does not only contain the actual results, but also describes the rationale and assumptions underlying the calibration.

1.2 The calibration builds upon the advice CEIOPS previously submitted to the Commission and takes into account the amended Framework for Consultation \(^1\). The paper takes specifically the following SCR features into consideration:

- Risk measure: Value at Risk (VaR);
- Confidence level: 99.5%;
- Time horizon: 1 year.

Overview

1.3 Section 2 describes the calibration of the life underwriting risk sub-modules. This includes the size of the volatility and trend shocks for biometric risks, the scenarios for expense risk and lapse risk and also the factors for disability and lapse catastrophe risk.

1.4 Section 3 lays out the determination of the risk factors for non-life underwriting risk. This comprises the setting of the correlations between different LOBs, the determination of the market-wide factors for premium and reserve risk, the setting of the credibility constant for the premium risk approach and also the choice of the transregional scenarios for CAT risk.

1.5 Finally, section 4 presents an overview of the calibration for health underwriting risk, including the factors and parameters for expense and claims risk and the risk factor for epidemic health risk.

1.6 The results of the calibration for non-life underwriting risk are summarised in the annex.

**Life underwriting risk**

*Introduction*

1.7 The life underwriting risk module is comprised of the following components

- Mortality risk
- Longevity risk
- Disability risk
- Lapse risk
- Expense risk
- Revision risk
- Catastrophe risk

1.8 In QIS 2 for mortality, longevity and disability risk two risk components had been tested respectively, one for trend risk and one for volatility risk. However, compared to the trend risk, the volatility risk proved to be considerably lower. Thus for QIS 3 it has been decided to reduce the complexity of the design of the underwriting risk module by maintaining the trend risk components only, assuming the volatility risk components to be implicitly covered by the trend and catastrophe risk components.

1.9 For mortality, longevity, disability, lapse and expense risks, CEIOPS has proposed in CP20\(^2\) to apply a scenario based approach.

1.10 Conversely, for the catastrophe risk component, CEIOPS has proposed a factor based approach.

1.11 The components for mortality, longevity, disability, lapse, expense and catastrophe risks are then aggregated together through the application of a correlation matrix.

*Correlations between u/w risk components*

1.12 The assumed correlations between the different components have been assessed judgmentally, in the context of the adverse scenarios that are being contemplated. We have taken as a starting point the matrix on which we consulted in CP20, and which was also applied in QIS2.

However, the assumed correlations between expenses and mortality, expenses and longevity, and between lapses and longevity have been reduced slightly to reflect comments received during QIS2 and subsequently.

The assumed correlation of 0.5 between mortality and disability has now been taken as the same figure as adopted in QIS2 for the correlation between sickness and mortality. This reflects the likelihood of a reasonably strong link between increased disability or sickness and subsequent earlier deaths.

Questions have been raised about the assumed zero correlation between mortality and longevity. The cohorts of policyholders exposed to mortality and longevity risk are generally different. A zero correlation is therefore seen as being a prudent assumption that allows for the potentially different risk drivers for these particular cohorts of policyholder. For example, the current trends towards greater obesity could cause an increase in mortality rates for the affected generation, at the same time as the older generation are still experiencing improved longevity.

Choice of shocks for biometric risks

Mortality risk

For mortality risk, we had regard to information derived from a study published in 2004 by Watson Wyatt about the 99.5% assumptions over a 12 months time horizon that firms were proposing to make for their ICAS submissions in the UK. This indicated a range of between 10 and 35%, with an average of around 23%. However, it is thought that this assumption may cover both trend and volatility risk, as well as possibly cat risk.

More recent ICAS submissions are believed to have included a fairly low level of mortality trend shock. This may reflect the likelihood that the probability distribution for mortality is skewed, with a current trend towards improving mortality.

It is also relevant to note that many firms may not allow explicitly for future improvements when assessing the best estimate mortality rates for insured lives.

Accordingly, in the light of this information, it is suggested that the 20% mortality shock in QIS2 should now be reduced to 10% for the purpose of QIS3.

Longevity risk

For longevity risk, we had regard to information derived from a study published in 2004 by Watson Wyatt about the 99.5% assumption over a 12 months time horizon, that firms were proposing to make for their ICAS submissions in the UK, for the reduction in mortality rates, when
expressed as a single uniform permanent decrease in mortality. This indicated a range of between 5% and 35%, with an average of around 18% as the decrease in mortality that was assumed.

1.21 More recent ICAS submissions in the UK are believed to have shown though an assumed decrease of around 25% in mortality rates, to cover longevity risk, and this is understood to be consistent with some external expert advice that has been received.

1.22 This is also consistent with the observed experience in recent years, with an accelerating rate of improvement in longevity for retired persons. See for example the paper on 'Longevity in the 21st century' by Willets et al in BAJ Volume 10 part 5. This shows for example in Table 6.11b of this paper the quantum jump in mortality between the various standard tables that were utilised in the UK for annuitant mortality over the last 25 years. Each of these standard mortality tables included an allowance for future improvements in longevity based on the best estimates made at that time (e.g. the table PA(90) took the average experience for all firms between 1979 and 1982 and added an allowance for future improvements based on then current thinking). However, as can be seen, each of these standard tables has underestimated the actual rates of improvement that have occurred.

1.23 These improvements are believed to be attributable to a combination of factors, such as significant medical advances, particularly in the treatment of heart disease and cancer, a reduction in the number of smokers, and better living conditions. There are a wide range of views among academics about the potential for further significant improvements in longevity. However, the above suggested 25% reduction in mortality rates would be equivalent to around another 3 years expected life for a man aged 65, and would be consistent with the quantum shifts in longevity trends seen in recent years.

Disability and sickness

1.24 For disability risk, we had regard to information derived from a study published in 2004 by Watson Wyatt about the 99.5% assumption over a 12 months time horizon that firms were proposing to make for their ICAS submissions in the UK, for the increase in sickness and disability rates. This indicated a wide range of between 10% and 60%, with an average of around 40%, as the level of increase that was assumed in the number of new sickness and disability claims.

1.25 It is likely that much of this potential variation would be attributable to short-term factors such as epidemics, and also to the effect of the economic cycle which can increase the number of longer-term claims. For example, significant variations in the numbers of new longer-term sickness claims between different 4-year periods can be observed from the CMI reports (e.g. CMI No17) published by the UK actuarial profession. However, there could also be more permanent changes as a result of a resurgence of diseases such as tuberculosis, or that are
attributable to a lack of relevance or credibility of the data on which the best estimates are based.

1.26 Since comments from firms claimed the 40% average increase in disability rates as being slightly high, it is proposed to adopt a factor of 35% for the increase in sickness and disability rates over the next 12 months, reducing to a 25% increase over best estimate thereafter (the latter figure representing an allowance for both a permanent increase in sickness rates over the assumed best estimate, and the effect on claims of the economic cycle).

Choice of scenarios for lapse, expense and revision risk

Lapse risk

1.27 For lapse risk, we had regard to information derived from a published study in 2004 by Watson Wyatt about the 99.5% assumption over a 12 months time horizon that firms were proposing to make for their ICAS submissions in the UK, for the changes in lapse rates, either upwards or downwards. This indicated a range of between 35% and 60% in the rate of assumed lapses, with an average of around 50% as the change in the rate of lapses that was assumed.

1.28 However, we are also conscious that a 50% increase in the rate of lapses may underestimate the potential increase, if lapse rates are currently quite low. Consequently, as proposed previously by CEIOPS, we are applying a minimum increase of 3% per annum in the assumed lapse rate.

Expense risk

1.29 For expense risk, we had regard to information derived from a published study in 2004 by Watson Wyatt about the 99.5% assumption over a 12 months time horizon that firms were proposing to make for their ICAS submissions in the UK, for the potential increase in the level of expenses. This indicated a range of between 5% and 50% in the rate of assumed lapses, with an average of around 26% as the increase in the level of expenses that was assumed.

1.30 More recent ICAS submissions in the UK are believed to have shown though an assumed increase of around 10% in the level of expenses in the following year, together with an increase of between 1% and 2 % per annum in the rate of future expense inflation.

1.31 Increases of 10% or more in expense levels have certainly been observed in the accounts of undertakings from one year to the next. An increase of between 1 and 1.5% in the rate of inflation would also be consistent with the postulated movement in nominal rates of interest that are proposed for the interest rate risk component of the market risk module. Given that real rates of interest are likely to be fairly stable, this suggests that a 1% per annum increase in the rate of expected future inflation would be a consistent financial assumption.
Accordingly, it is proposed that undertakings should assess the impact of a scenario of a 10% increase in the level of expenses in the following year, together with a 1% per annum increase in the assumed rate of inflation.

However, in case of some policies the undertakings are entitled to adjust the expense loadings or charges. Making allowance for this opportunity for those policies it is assumed that 75% of the additional expenses can be recovered from year 2 onwards through increasing the charges payable by policyholders.

Revision risk

For revision risk, the 3% increase scenario was calibrated using historical data for pensions in payment for the workers’ compensation line of business in Portugal.

We fitted a binomial compound distribution to the historical data, assuming a binomial distribution for the frequency process and a lognormal distribution to model the severity of revision. The aggregate loss distribution was derived using Monte Carlo simulation for different portfolio sizes. All pensions were assumed to be independent and their annual amount was assumed to be constant. Different assumptions were considered for pensions homologated and pensions not yet defined, the latter with higher frequency and severity volatilities.

The 3% scenario corresponds to the 99.5% quantile of the aggregate loss distribution for an average sized portfolio comprising pensions at different legal stages in ‘average’ proportions.

It is recognised that further study is needed to assess the adequacy of the scenario, namely by increasing the number of historical years, the number of firms providing data and, probably, differentiating the scenarios per type of pension, e.g. pensions not yet homologated are more prone to revision risk. Also, the analysis will need to be extended to cover other lines of business and markets.

Factors for mortality, disability and lapse catastrophe risk

Mortality and disability catastrophe risk

Mortality and disability catastrophe risk reflects the potential effect of epidemics and other hazards such as large fires, earthquakes, war, and terrorism. The epidemic risk is thought to outweigh all these other potential risks at present and consequently is the only catastrophe considered further here.

The particular risk at the forefront of thinking by the WHO at present is that of an avian flu pandemic, thought here are of course other potential diseases such as SARS or ebola that need to be borne in mind as well.
Mortality catastrophe risk

1.40 It is known that there are recurring serious flu epidemics every 20-30 years. The most intense of these epidemics in living memory was the 1918 epidemic. This is believed to have been a significant mutation of earlier flu viruses that caused serious illness and resulted in a mortality rate of as much as 1-2% of the population in some countries, focused mainly on those aged between 20 and 40, but with lower rates of mortality in other age groups. Over Europe as a whole, the additional death rate was around 5 per mille. It is not entirely clear why the death rate varied between countries or why the 20-40 age group was particularly badly affected. One theory though is that many of the deaths resulted from an overreaction of the immune system which is at its strongest level for individuals in this age group, and which caused the lungs of infected people to be overwhelmed by a form of pneumonia.

1.41 The current H5N1 flu strain is of considerable concern to the WHO. While at present, it is fairly difficult to transmit to humans, there has been around a 50% or higher death rate from known cases of people infected people by this virus. It is also believed that it may only be a matter of time before the virus mutates or combines with a human flu virus to become contagious between humans. It could then spread very rapidly within the global population, though the prognosis for people infected with the mutated virus is as yet unknown.

1.42 There are a number of possible mitigants such as vaccines and antiviral drugs. However, it would take some time to develop an effective and widely available specific vaccine for this virus and this might well not be available to prevent a widespread outbreak. It is also known that the current H5N1 virus is developing some resistance to the most commonly available antiviral drug.

1.43 If we assume though as a starting point that the 1918 epidemic represents a 1 in 200 year event, then this would suggest an additional number of deaths of around 5 per mille. However, it may be reasonable to allow some reduction for the medical advances that have taken place since then, albeit that there is still considerable uncertainty about the potential effectiveness of these mitigating measures in combating a mutated H5N1 (or any other) virus.

1.44 Accordingly, it is proposed that we assume a capital component equal to 1.5 per mille of capital at risk for mortality catastrophe risk.

Disability catastrophe risk

1.45 For sickness and disability catastrophe risk, a major epidemic is also likely to be the main risk factor. However, we are already assuming an increase of 35% in the level of new claims to calculate the capital component for sickness and disability risk, and a serious flu epidemic may be more likely to cause death within a week or two than a lengthy
sickness claim. Accordingly, it is proposed to assume an additional level of claims of 1.5 per mille.

_Lapse catastrophe risk_

1.46 A further 'catastrophe' risk to consider is that of a sudden adverse policyholder reaction in the event of either a loss of reputation of a firm, or some other operational difficulty, resulting in a sizeable number of surrenders or lapses of unit-linked policies. This risk may not be fully reflected in the operational risk component. The most effective means of covering this risk would be to hold capital to cover the difference between the surrender value of the policies and the technical provision held. This would also achieve some equivalence with the banking and investment sectors, where firms are not allowed to anticipate on the balance sheet the expected profits from future management charges. This could though be offset by the lapse risk component in respect of unit-linked policies and possibly by part of the operational risk component.

1.47 Accordingly, it is proposed to include a capital component for 75% of the difference between the surrender value of the policies and the technical provision held. This parameter will be reviewed further in the light of the QIS3 results.

_Non-life underwriting risk_

_Introduction_

1.48 Non-life underwriting risk is the specific insurance risk arising from non-life insurance contracts. It relates to the uncertainty about the results of the insurer’s underwriting. This includes uncertainty about:

- the amount and timing of the eventual claim settlements in relation to existing liabilities;
- the volume of business to be written and the premium rates at which it will be written; and
- the premium rates which would be necessary to cover the liabilities created by the business written.

1.49 The non-life underwriting risk module is comprised of the following components

- Premium and reserve risk
- Catastrophe risk

1.50 For QIS3, CEIOPS has extended the approach under QIS2 by deriving a capital charge for the combined premium and reserve risk in a single calculation, based on separate analyses of premium and reserve risk at the level of individual lines of business. This charge is then aggregated
with the CAT risk charge to an overall non-life underwriting SCR, assuming a correlation of zero between premium/reserve risk and catastrophe risk.

**Overall calculation of premium and reserve risk**

1.50 Analogously to the computation of the risk capital charges for premium and reserve risk under QIS2, in QIS3 the capital charge $NL_{pr}$ for the combined risk is calculated as

$$\rho(\sigma) \cdot V$$

where

- $V$ = volume measure
- $\sigma$ = standard deviation of the underlying risk driver
- $\rho(\sigma)$ = a function of the standard deviation

1.51 The function $\rho(\sigma)$ is set as follows:

$$\rho(\sigma) = \frac{\exp(N_{0.995} \cdot \sqrt{\log(\sigma^2 + 1)}) - 1}{\sqrt{\sigma^2 + 1}},$$

where $N_{0.995}$ is the 99.5% quantile of the standard normal distribution.

1.52 Assuming a log-normal distribution of the underlying risk, this ensures that the overall risk capital charge for premium and reserve risk is consistent with the VaR 99.5% standard.

1.53 The following diagram illustrates the values of $\rho(\sigma)$, showing that, for small and medium-sized standard deviations, $\rho(\sigma)$ is roughly equal to $3 \cdot \sigma$:
1.54 The volume measure $V$ and the standard deviation $\sigma$ of the combined ratio for the overall non-life insurance portfolio are determined in two steps as follows:

- in a **first step**, for each individual line of business (LOB) standard deviations and volume measures for both premium risk and reserve risk are determined;

- in a **second step**, the standard deviations and volume measures for the premium risk and the reserve risk in the individual LOB’s are aggregated to derive an overall volume measure $V$ and an overall standard deviation $\sigma$.

1.55 In the following, the calibration of the parameters/assumptions needed to perform these two steps is set out.

**Market-wide factors for premium and reserve risk**

1.56 In the premium and reserve risk sub-module, for each individual LOB market-wide estimates of the standard deviation for premium risk and reserve risk are specified.

**Premium risk**

1.57 For premium risk, the calibration of these market-wide factors was carried out by analysing undertaking’s specific estimations of the volatility for premium risk (derived from historic loss ratios) using data from the German insurance market. Specifically, the following approach was chosen:

1.58 Ideally, the market-wide estimate $\sigma_M$ can be chosen such that it is near to the undertaking’s specific estimate $\sigma_{\text{ind}}$, i.e.

$$\sigma_M \approx \sigma_{\text{ind}}, \text{ so that } V \cdot \sigma_M \approx V \cdot \sigma_{\text{ind}}$$

for each individual insurer and for each relevant volume measure $V$.

1.59 Therefore, the market-wide estimate $\sigma_M$ was determined using least squares optimisation, i.e. it was chosen such that the sum of the squares of the residuals is minimised:

$$VS = \sum_{\text{ind}} (V \cdot \sigma_M - V \cdot \sigma_{\text{ind}})^2$$

where the volume measure $V$ was determined as the average gross premium income for the individual insurer in the LOB, over the period that was used to derive the insurer’s individual estimate of the standard deviation.

1.60 The result of performing the linear regression analysis are described in the following table:
<table>
<thead>
<tr>
<th>LOB</th>
<th>No. of firms</th>
<th>No. of loss ratios&lt;sup&gt;3&lt;/sup&gt;</th>
<th>Statistical sigma</th>
<th>Chosen sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accident and health</td>
<td>112</td>
<td>1.609</td>
<td>6%</td>
<td>5%&lt;sup&gt;4&lt;/sup&gt;</td>
</tr>
<tr>
<td>Motor, third party liability</td>
<td>96</td>
<td>1.364</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Motor, other classes</td>
<td>98</td>
<td>1.394</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Marine, aviation and transport</td>
<td>58</td>
<td>815</td>
<td>13%</td>
<td>12.5%</td>
</tr>
<tr>
<td>Fire and other damage of property</td>
<td>127</td>
<td>1.821</td>
<td>9%</td>
<td>10%</td>
</tr>
<tr>
<td>Third-party liability</td>
<td>99</td>
<td>1.421</td>
<td>6%</td>
<td>10%</td>
</tr>
<tr>
<td>Legal expenses</td>
<td>30</td>
<td>430</td>
<td>5%</td>
<td>5%</td>
</tr>
</tbody>
</table>

1.61 For the case of motor third class liability, the results of performing the linear regression analysis are illustrated in the following diagram:

![Motor, third party liability](image)

In this diagram, the values for some large insurers were left out to ensure anonymity of the data.

1.62 To allow a more refined calibration, the “accident and health” LOB was split into three sub-lines:

- Accident and health - worker’s compensation business
- Accident and health – health

<sup>3</sup> For each individual insurer, loss ratios in the time period between 1988 and 2002 (where available) were used to derive an insurer-specific estimate of the standard deviation

<sup>4</sup> This factor was chosen for LOB3 (accident and health – other)
• Accident and health – other

1.63 The calibration of the premium risk factor for worker’s compensation business was carried out using statistical data from the Portuguese insurance market, following a similar approach as described above.

1.64 The calibration of the premium risk factor for health business was derived from a statistical analysis on basis of data from the French insurance market, also taking into account the results of a study conducted by Swiss Re.\textsuperscript{5}

1.65 The calibration of the premium risk factors for the remaining LOBs (credit and suretyship, assistance, miscellaneous non-life insurance and the three non-proportional reinsurance classes) was chosen judgementally, taking into account the feedback received from QIS2.

Reserve risk

1.66 The calibration of the reserve risk factor for the following LOBs:

• Motor, third party liability, and

• Third party liability

was carried out using statistical data from the UK insurance market, following a similar approach as was used for the determination of the premium risk factors.

1.67 For health business, as in the case of premium risk, the calibration of the reserve risk factor was derived from a statistical analysis on basis of data from the French insurance market.

1.68 For the remaining LOBs, the reserve risk factors were assessed judgementally, using as a starting point the base reserve risk factors applied in QIS2\textsuperscript{6}. Taking into consideration results from ICAS submissions in the UK and feedback from the QIS2 exercise, the QIS2 base reserve factors were adjusted downwards in some cases (assistance and miscellaneous non-life insurance).

Credibility constant for premium risk

1.69 The standard deviation for premium risk in the individual LOB is derived as a credibility mix of an undertaking-specific estimate and the market-wide estimate as follows:

\[
\sigma_{(prem,lob)} = \sqrt{c_{lob} \cdot \sigma^2_{(U,prem,lob)} + (1 - c_{lob}) \cdot \sigma^2_{(M,prem,lob)}},
\]

\textsuperscript{5} Swiss Re sigma No 4/2006 “Solvency II: an integrated risk approach for European insurers”

\textsuperscript{6} Note that the overall reserve risk factor in QIS2 was determined as a product of the base risk factor and a size factor, ranging between 1 and 2.44.
where

\[ \sigma_{(\text{prem,lob})} = \text{Resulting estimate of the standard deviation for premium risk} \]
\[ c_{\text{lob}} = \text{Credibility factor for LOB} \]
\[ \sigma_{(U,\text{prem,lob})} = \text{Undertaking-specific estimate of the standard deviation for premium risk} \]
\[ \sigma_{(M,\text{prem,lob})} = \text{Market-wide estimate of the standard deviation for premium risk (calibrated as described above)} \]

1.70 The credibility factor \( c_{\text{lob}} \) is defined as:

\[
c_{\text{lob}} = \begin{cases} 
\frac{n_{\text{lob}}}{n_{\text{lob}} + k_{\text{lob}}} & \text{if } n_{\text{lob}} \geq 7 \\
0 & \text{otherwise}
\end{cases}
\]

where

\[ k_{\text{lob}} = \text{credibility constant depending on the individual LOB} \]
\[ n_{\text{lob}} = \text{number of historic loss ratios available (at most 15) for insurer i in the LOB} \]

1.71 The calibration of the credibility factor was derived by an application of the classical Bülmann-Straub credibility model. For this, the following set of assumptions on the loss ratios

\[ LR_{\text{lob},i}^1, LR_{\text{lob},i}^2, \ldots, LR_{\text{lob},i}^{n_{\text{lob},i}} \]

of individual insurer i were used:

(a) Conditionally, for fixed structure parameter \( \Theta_i \),

\[ LR_{\text{lob},i} = (LR_{\text{lob},i}^1, \ldots, LR_{\text{lob},i}^{n_{\text{lob},i}}) \]

are independent and there exist functions \( \mu(\Theta_i) \) and \( \sigma(\Theta_i) \), such that:

i. \( E(LR_{\text{lob},i}^j | \Theta_i) = \mu(\Theta_i) \)

ii. \( V(LR_{\text{lob},i}^j | \Theta_i) = \frac{\sigma^2(\Theta_i)}{P_{\text{lob},ij}} \)
iii. \( \mu_4(LR^j_{lob,i} | \Theta_i) = \frac{3}{P^2_{lob,ij}} \bullet \sigma^4(\Theta_i) \)

where

\[
E(LR^j_{lob,i} | \Theta_i) = \text{The expected value of } LR^j_{lob,i} \text{ given } \Theta_i
\]

\[
V(LR^j_{lob,i} | \Theta_i) = \text{The variance of } LR^j_{lob,i} \text{ given } \Theta_i
\]

\[
\mu_4(LR^j_{lob,i} | \Theta_i) = \text{The fourth central moment of } LR^j_{lob,i} \text{ given } \Theta_i
\]

\[
P_{lob,ij} = \text{earned gross premium of the insurer i in the LOB and in historic year j}
\]

\[
n_{lob,i} = \text{number of historic loss ratios for insurer i}
\]

(b) The pairs \((\Theta_1, LR^1_{lob,1}), (\Theta_2, LR^2_{lob,2}), …, (\Theta_N, LR^N_{lob,N})\) are independent and the risk parameters \(\Theta_1, \Theta_2, ..., \Theta_N\) are independent and identically distributed.

1.72 Under these assumptions, it can be shown\(^7\) that the credibility factor \(c_{lob}\) is given by:

\[
c_{lob} = \frac{n_{lob,i} - 1}{n_{lob,i} - 1 + 2 \bullet \frac{E(\sigma^4(\Theta_i))}{V(\sigma^2(\Theta_i))}} = \frac{V(\sigma^2(\Theta_i))}{V(\Sigma_i)^2},
\]

where

\[
\Sigma_i = \sqrt{\frac{1}{(n_{lob,i} - 1)} \bullet \sum_j P_{lob,ij} \bullet (LR^j_{lob,i} - \mu_{lob,i})^2}
\]

1.73 Using unbiased estimators for the structural parameters

\[
\xi = V(\Sigma_i^2)
\]

and

\[
\varphi = V(\sigma^2(\Theta_i))
\]

constructed from the statistical data\(^8\), estimates of the credibility factor \(c_{lob}\) in the individual LOBs were determined. From this, values for the

---

\(^7\) see e.q. (21) in Centeno, Lourdes: The Bühlmann-Straub Model with the premium calculated according to the variance principle, Insurance: Mathematics and Economics 8 (1989) 3-10.

\(^8\) see section 4 in the paper of Centeno (1989)
credibility constants \( k_{\text{lob}} \) were derived. Depending on the individual LOB, these ranged between 3 and 5.

1.74 In the light of this analysis and for reasons of simplification, the credibility constant \( k_{\text{lob}} \) was set as 4.0 for each LOB.

Correlations between different LOBs

Aggregation to an overall sigma

1.75 To derive the overall charge for premium and reserve risk, the results of the analysis on the level of individual LOBs are combined to derive an overall standard deviation as follows:

\[
\sigma = \frac{1}{\sqrt{V^2}} \cdot \left( \sum_{r,c} CorrLob_{r,c} \cdot V_r \cdot V_c \cdot \sigma_r \cdot \sigma_c \right),
\]

where

\( \sigma \) = standard deviation for the overall portfolio

\( V \) = overall volume measure

\( r,c \) = All indices of the form (prem,lob) or (res,lob)

\( CorrLob_{r,c} \) = the cells of the correlation matrix \( CorrLob \)

\( V_r, V_c \) = Volume measures for the individual lines of business

\( \sigma_r, \sigma_c \) = standard deviations for premium and reserve risk in the individual line of business

Rationale for aggregation formula

1.76 The rationale for the aggregation formula is as follows:

1.77 For an individual insurer, the overall risk variable for premium and reserve risk is described by the sum of the risk variables for the individual LOBs, where for each LOB a distinction is made between premium and reserve risk. Hence:

\[
X = \sum_{\text{lob}} \left( X_{(\text{prem},\text{lob})} + X_{(\text{res},\text{lob})} \right)
\]

where

\( X \) = risk variable for premium and reserve risk for the overall portfolio

\( X_{(\text{prem},\text{lob})} \) = risk variable for premium risk in the individual LOB
\[ X_{(\text{res},\text{lob})} \] = risk variable for the systematic part of reserve risk in the individual LOB

1.78 From these assumptions, it follows that the variance of the overall risk is given by:

\[
\sigma^2(X) = \sigma^2\left( \sum_{\text{lob}} X_{(\text{prem}, \text{lob})} + X_{(\text{res}, \text{lob})} \right)
\]

\[ = \sum_{r,c} \text{Corr}(X_r, X_c) \cdot \sigma(X_r) \cdot \sigma(X_c) \]

where

\[ \sigma(X) \] = the standard deviation of the overall risk \( X \)

\[ r,c \] = All indices of the form \((\text{prem}, \text{lob})\) or \((\text{res}, \text{lob})\)

\[ \text{Corr}(X_r, X_c) \] = the correlation between the risk variables \( X_r \) and \( X_c \)

\[ \sigma(X_r), \sigma(X_c) \] = the standard deviations of the risk variables \( X_r \) and \( X_c \)

1.79 The overall standard deviation that needs to be derived is defined relative to the overall volume measure \( V \), so that

\[ \sigma(X) = \sigma \cdot V \]

1.80 Likewise, the standard deviations for premium and reserve risk that are determined on the level of individual LOBs are defined relative to the volume measures for premium and reserve risk, so that:

\[ \sigma(\text{X}_{(\text{prem}, \text{lob})}) = \sigma_{(\text{prem}, \text{lob})} \cdot V_{(\text{prem}, \text{lob})}; \text{ and} \]

\[ \sigma(\text{X}_{(\text{res}, \text{lob})}) = \sigma_{(\text{res}, \text{lob})} \cdot V_{(\text{res}, \text{lob})} \]

for each LOB, where

\[ \sigma_{(\text{res}, \text{lob})} \] = the standard deviation of reserve risk in the individual LOB, relative to the volume measure

\[ \sigma_{(\text{prem}, \text{lob})} \] = the standard deviation of premium risk in the individual LOB, relative to the volume measure

1.81 Hence \( \sigma \) is described by the following formula:
Comparing this with the aggregation formula used in the specifications (see above), we see that the two coincide when we interpret the entries of the correlation matrix \( CorrLob \) as approximations of the correlations between the relevant risk variables, i.e. when we determine \( CorrLob \), such that

\[
CorrLob_{r,c} \approx Corr(X_r, X_c)
\]

for each pair of indices \( r,c \) of the form \((\text{prem},\text{lob})\) or \((\text{res},\text{lob})\).

**Setting of the correlation coefficients**

Within the correlation matrix \( CorrLob \), coefficients have to be set between the following pairs of indices:

- between \((\text{prem},\text{lob}1)\) and \((\text{prem},\text{lob}2)\);
- between \((\text{prem},\text{lob}1)\) and \((\text{res},\text{lob}2)\); and
- between \((\text{res},\text{lob}1)\) and \((\text{res},\text{lob}2)\)

for each pair \((\text{lob}1,\text{lob}2)\) of individual LOBs.

In view of the insufficiency of currently available data, the setting of these correlation coefficients will necessarily include a certain degree of judgement. This is also true because, when selecting correlation coefficients, allowance should be made for non-linear tail correlation, which is not captured under a “pure” linear correlation approach.\(^9\) To allow for this, the correlations used should be higher than simple analysis of relevant data would indicate.

For reasons of simplification, \( CorrLob \) was determined such that:

- for any two LOBs \( \text{lob}1 \) and \( \text{lob}2 \), the correlations within premium and reserve risk coincide, i.e. the correlation coefficients between \((\text{prem},\text{lob}1)\) and \((\text{prem},\text{lob}2)\) and between \((\text{res},\text{lob}1)\) and \((\text{res},\text{lob}2)\) are the same;
- for each individual \( \text{lob} \), the correlation between premium and reserve risk is set as 50%.\(^{10}\), and

---

\(^9\) For example, two risk variables \( X \) and \( Y \) may have zero linear correlation, but may nonetheless be dependent “in the tail” (i.e. in the occurrence of adverse events). In fact, such a situation is not uncommon for variables related to insurance risk. In such cases, the correlation matrix used in the standard formula to aggregate the risk capital charges for the two risks should be set to capture such tail dependence, i.e. the related correlation coefficient should be set higher than zero.

\(^{10}\) Note that a similar assumption was made in QIS2 with respect to the dependence between premium and reserve risk.
- the correlations between \((\text{prem}, \text{lob}_1)\) and \((\text{res}, \text{lob}_2)\) for different LOBs \(\text{lob}_1\) and \(\text{lob}_2\) are determined as 50% of the correlation between \((\text{prem}, \text{lob}_1)\) and \((\text{prem}, \text{lob}_2)\).

Therefore, the matrix \(\text{CorrLob}\) was specified as:

\[
\text{CorrLob} = \begin{pmatrix}
\text{CorrLob}_{\text{prem}} & \alpha \cdot \text{CorrLob}_{\text{prem}} \\
\alpha \cdot \text{CorrLob}_{\text{prem}} & \text{CorrLob}_{\text{prem}}
\end{pmatrix}
\]

where

- \(\text{CorrLob}\) = the correlation matrix for premium and reserve risk, arranged in such a way that the first (respectively, the last) 15 rows and columns refer to the indices of the form \((\text{prem}, \text{lob})\) (respectively, to the indices of the form \((\text{res}, \text{lob})\))

- \(\text{CorrLob}_{\text{prem}}\) = the correlation matrix for premium risk

- \(\alpha\) = Factor representing on overall assumption between premium and reserve risk (set as 50%)

As a starting point for the determination of the correlation matrix for premium risk, an analysis of the statistical correlations between individual LOBs (on the level of individual insurers) in the German insurance market was carried out.

For a given pair \((\text{lob}_1, \text{lob}_2)\) of LOBs \(\text{lob}_1\) and \(\text{lob}_2\), the analysis used historic loss ratios of individual insurers in \(\text{lob}_1\) and \(\text{lob}_2\) during the time period 1988 to 2002.

For each individual insurer (where at least 10 historic loss ratios in each of \(\text{lob}_1\) and \(\text{lob}_2\) were available), an insurer-specific estimate of the correlation between \(\text{lob}_1\) and \(\text{lob}_2\) was derived. An overall estimate of the correlation between \(\text{lob}_1\) and \(\text{lob}_2\) was then determined as the average of these insurer-specific correlations.

Moreover, to visualise the dependency between the loss ratios in \(\text{lob}_1\) and \(\text{lob}_2\), diagrams were produced showing the standardised residuals of the loss ratios of the individual insurers (with respect to loss ratios in \(\text{lob}_1\) in the \(x\)-coordinate, and with respect to loss ratios in \(\text{lob}_2\) in the \(y\)-coordinate).

For example, in the case of the dependency between:

- \(\text{lob}_1\): Motor, third party liability; and
- \(\text{lob}_2\): Third party liability,
an average overall correlation of 28% (using the data of 89 firms and 1.269 loss ratios) was derived, together with the following plot of standardised residuals:

![Residuals: Motor, third party liability and Third-party liability](image)

1.92 It is interesting to note that the plot clearly shows a non-linear dependency between lob1 and lob2, with an accumulation of residuals in the lower left corner (i.e. the dependency increases under the occurrence of adverse events).

1.93 A final choice of the correlation coefficients of the matrix CorrLob\textsubscript{prem} was derived by taking into account results from internal models of selected insurers, as well as the general considerations regarding the selection of correlation coefficients as laid out in para. 1.84.

**CAT risk**

1.94 Regional CAT scenarios were specified judgementally by local supervisors having regard to the main natural catastrophes to which their insurers were exposed.

1.95 The European windstorm catastrophe requires insurers to apply judgement to estimate a 1 in 200 year event, unless their business is concentrated in one regional area where a regional scenario may be regarded as an application of the European windstorm scenario.

1.96 The man-made scenario requires the insurer to take the most severe of those on the list or to apply judgement to select a 1 in 200 year event that is more severe. The scenarios on the list were selected as being scenarios that it was reasonable to expect most insurers to be able to meet rather than as being specifically 1 in 200 year events. For this reason the most severe was to be selected rather than combining them in some other way.

1.97 Since the scenarios were for the most part independent, they are to be combined assuming zero correlation.
1.98 The threshold was chosen so as to ensure that insurers did not have to estimate the effect of more than a few scenarios, while at the same time not ignoring scenarios that might be material. It should be noted that adding a second scenario equal to 25% of the first only increases the capital charge for non-life catastrophe risk by 3.1% ($\sqrt{1+.25^2}$).

**Health underwriting risk**

*Introduction*

1.99 The health underwriting risk module is concerned with underwriting risk in health insurance that is practised on a similar technical basis to that of life assurance. Health underwriting risk is split into three components:

- expense risk;
- claim/mortality/cancellation risk; and
- epidemic/accumulation risk.

*Expense risk*

1.100 The capital charge for health expense risk is determined as

$$Health_{exp} = \lambda_{exp} \cdot \sigma_{h \, exp} \cdot P_{ay}$$

where

- $\lambda_{exp} = $ expense risk factor which is set to deliver a health expense risk charge consistent with a VaR 99.5% standard
- $\sigma_{h \, exp} = $ the standard deviation of the expense result in relation to the gross premium over the previous ten-year period
- $P_{ay} = $ gross premium earned for the accounting year

1.101 To determine an appropriate value for the factor $\lambda_{exp}$, an analysis of the empirical distribution of the health expense results (for the German market) was carried out. An application of standard statistical testing tools (Kolmogorow-Smirnow- and Shapiro-Wilk-Test) yielded the result that it would be appropriate to assume that the expense result follows a normal distribution.

1.102 Since the calibration follows a VaR 99.5% standard, this led to setting the factor $\lambda_{exp}$, as 2.58.

1.103 The standard deviation of the expense risk result is generally calculated on the basis of undertaking’s specific data and therefore
does not require an explicit calibration. However, in cases where the undertaking’s specific data is insufficient to determine $\sigma_{\text{hexp}}$, the standard deviation is estimated as a convex combination of the undertaking’s specific estimate and a market-wide parameter $f_{\text{exp}}$.

1.104 The calibration of the parameters $f_{\text{exp}}$ was carried out on the bases of data of 43 health insurance undertakings in the German market. Ideally, these parameter can be chosen such that the market-wide estimate is near to the undertaking’s specific estimate, so that

$$f_{\text{exp}} \approx \sigma_{\text{hexp}}, \text{ i.e. }$$

$$P \cdot f_{\text{exp}} \approx P \cdot \sigma_{\text{hexp}}$$

for each individual insurer and for each relevant volume measure $P$.

1.105 Therefore, the parameter $f_{\text{exp}}$ was determined using least squares optimisation, i.e. it was chosen such that the sum of the squares of the residuals is minimised:

$$VS = \sum_{\text{ind}} (P \cdot f_{\text{exp}} - P \cdot \sigma_{\text{hexp}})^2$$

Results

1.106 The result of performing a linear regression analysis as described in para. 1.105 is shown below:

In this diagram, the values for some large insurers were left out to guarantee the anonymity of the data.

1.107 On the basis of this analysis, the factor $f_{\text{exp}}$ was set to 2%.
Claim/mortality/cancellation risk

1.108 Similarly to the case of expense risk, the capital charge for health claim / mortality / cancellation risk is determined as

\[ Health_{cl} = \lambda_{cl} \cdot \sigma_{h_{cl}} \cdot P_{ay} \]

where

\[ \lambda_{cl} = \text{health}_{cl} \text{ risk factor which is set to deliver a health claim / mortality / cancellation risk charge consistent with a VaR 99.5\% standard} \]

\[ \sigma_{h_{cl}} = \text{the standard deviation of the claim / mortality / cancellation result in relation to the gross premium over the previous ten-year period} \]

\[ P_{ay} = \text{gross premium earned for the accounting year} \]

1.109 To determine an appropriate value for the the factor \( \lambda_{cl} \), an analysis of the empirical distribution of the health_{cl} results (for the German market) was carried out. As in the case of expense risk, an application of standard statistical testing tools yielded the result that it would be appropriate to assume that the health_{cl} result follows a normal distribution. Hence the factor \( \lambda_{cl} \) was set as 2.58.

1.110 As for expense risk, in cases where the undertaking’s specific data is insufficient to determine \( \sigma_{h_{cl}} \), the standard deviation is estimated as a convex combination of the undertaking’s specific estimate and a market-wide factor \( f_{cl} \).

1.111 The calibration of the factor \( f_{cl} \) was carried out using the same linear regression approach as for expense risk. The result of performing a linear regression analysis is shown below:
In this diagram, the values for some large insurers were left out to guarantee the anonymity of the data.

1.112 On the basis of this analysis, the factor $f_{cl}$ was set to 3%.

**Epidemic/accumulation risk**

1.113 The capital charge for epidemic/accumulation risk is determined as

$$Health_{ac} = \lambda_{ac} \cdot P_{ay} \cdot \frac{claims_{ay}}{MP_{ay}}$$

where

- $\lambda_{ac}$ = Health$_{ac}$ risk factor which is set to deliver a health epidemic/accumulation risk charge consistent with a VaR 99.5% standard
- $P_{ay}$ = gross premium earned for the accounting year
- $MP_{ay}$ = total gross premium earned for the accounting year in the health insurance market
- $claims_{ay}$ = claims expenditure for the accounting year in the health insurance market

1.114 In accordance with international guidelines issued by the World Health Organisation (WHO) to respond to threats and occurrences of pandemic influenza, the German state and German Länder worked out a national influenza pandemic plan under the leadership of the Robert
Koch Institute (RKI).\textsuperscript{11} As part of this plan, it was estimated for Germany that, under an assumed influenza infection rate of 50% and within a time period of eight weeks, 25% of the population would seek medical consultation and 0.75% of the population would require clinical treatment, not regarding additional therapeutical and prophylactical measures.

The likelihood for the occurrence of such a scenario was considered to lie within the 99.5%-Quantile used for the SCR, and therefore this scenario was taken into account to determine $\lambda_{ac}$. On the basis of this analysis, the factor $\lambda_{ac}$ was set to 6.5%.

\textsuperscript{11}The Robert Koch Institute (cf. \url{www.rki.de}) is the central German federal institution responsible for disease control and prevention and is therefore the central federal reference institution for both applied and response-orientated research as well as for the Public Health Sector. The pandemic plan issued by the RKI is available under: \url{www.rki.de/cln_011/nn_965184/DE/Content/InfAZ/I/Influenza/influenzapandemieplan_I,templateId=raw,property=publicationFile.pdf/influenzapandemieplan_I}
Section 2

Calibration of the market risk

Introduction

Purpose

2.1 This section deals with the calibration of the market risk module within the SCR standard formula. The FSC is commissioned to provide support to the Pillar I group on the calibration of market risk within the SCR standard formula. The paper contains analysis of the magnitude of market shocks that would be consistent with the 99.5%/1yr standard for the SCR\textsuperscript{12}. The results will be reflected in the calibration tested for QIS3.

Scope

2.2 The section not only contains the actual results, but also describes the process followed, e.g. what steps were taken, which assumptions were made etc. It explores possible extensions/amendments to the initial calibration work, for example, treatment of fat tails behaviour, other currencies/countries, different data periods/data sources, other model assumptions etc. The work includes analysis of the impact of using different levels of granularity (global, regional, and country level).

2.3 This section focuses on the analysis of four market risk shocks. For QIS3, market risk is divided into the following sub-risks: interest rate, equity, property, spread, concentration, and currency risk. This paper studies interest rate, equity, property, and currency risk. Analyses of spread risk, concentration risk and other non-insurance risks, such as operational risk and counterparty default risk, are described in separate papers.

2.4 This analysis also investigates the dependency structure between the, in general, most significant market sub-risks, i.e. the correlation coefficient between interest rate and equity risk.

2.5 Work to date has largely been based on data from the euro area. In this paper the calibration will be extended to non-euro EU countries.

General assumptions

2.6 The calibration of the market risk module within the SCR standard formula is based on an economic approach, i.e. maximum use of historical observed changes in market rates and market prices.

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\textsuperscript{12} Although, the analysis is based on long-dated time series, it reflects developments to date. Future market movements might necessitate amendments.
2.7 The analysis of the magnitude of market shocks builds upon the advice CEIOPS previously submitted to the Commission and it takes into account the amended Framework for Consultation\(^{13}\). The paper takes specifically the following SCR features into consideration:

- Risk measure: Value at Risk (VaR);
- Confidence level: 99.5%;
- Time horizon: 1 year.

**Overview**

2.8 Subsection 2 elaborates on the magnitude of market shocks for interest rate risk. The analysis compares the used EMU term structure with term structures of several non-euro EU countries. Compared to the QIS2 calibration work, a larger number of maturity buckets is considered. Furthermore, the paper suggests that the stress factors for interest rate risk should be higher.

2.9 Subsection 3 extends the analysis of equity risk of global development markets indices. The paper introduces possible methods for correcting negative fat tails and considers other time periods. Furthermore, instead of applying global indices, the paper analyses the impact of using different levels of granularity (regional level). The paper concludes that global equity shocks, consistent with the 99.5 confidence level, approximately correspond to a range between 32% and 35%. For QIS3, CEIOPS decided to test a 32% stress factor which is consistent with the treatment in the banking sector.

2.10 Subsection 4 presents a proposal for property risk. The paper extends the analysis to other EU countries. Furthermore, it elaborates on the de-smoothing mechanism applied. Similar to the QIS2 calibration work, a 20% stress factor for property risk is proposed.

2.11 Subsection 5 broadens the analysis of currency risk. Firstly, the analysis is based on longer data series. Secondly, the analysis does not only consider the euro as local currency. Thirdly, the paper investigates the impact of using different currency baskets. Compared to the QIS2 calibration work, the paper suggests changing the stress factor for currency risk from 25% to 20%.

2.12 Finally, subsection 6 analyses the dependency structure between interest rate and equity risk. A ‘top down’ approach for the calibration of the correlation coefficient between interest rate and equity risk is used. This approach is chosen in order to ensure that the overall $\text{SCR}_{\text{mixt}}$ risk charge, given the calibration of the individual shocks to a $\text{VaR} 99.5\%$ standard, is again consistent with a $\text{VaR} 99.5\%$ standard. For QIS3 this paper proposes to use a 0 correlation coefficient between interest rate and equity risk.

\(^{13}\) European Commission (2006) - Amended Framework for Consultation on Solvency II MARKT/2515/06.
Interest rate risk

Introduction

2.13 The QIS2 technical specification document provides a table with stress factors for interest rate risk (see §5.46). Both the up stress, $s_{up}(n)$, and the down stress, $s_{down}(n)$, are constant over five maturity buckets:

<table>
<thead>
<tr>
<th>Maturity n (years)</th>
<th>1-3</th>
<th>4-6</th>
<th>7-12</th>
<th>13-18</th>
<th>18+</th>
</tr>
</thead>
<tbody>
<tr>
<td>relative change $s_{up}(n)$</td>
<td>0.75</td>
<td>0.5</td>
<td>0.4</td>
<td>0.35</td>
<td>0.3</td>
</tr>
<tr>
<td>relative change $s_{down}(n)$</td>
<td>-0.4</td>
<td>-0.35</td>
<td>-0.3</td>
<td>-0.25</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

2.14 The altered term structures can be derived by multiplying the current term structure by the relevant stress factor. For example, the n-year spot rate in 12 months time in the up stress situation corresponds with:

$$ R_{12}(n) = R_0(n) \times (1 + s_{up}(n)) $$

(a)

where, $R_0(n)$ is the current n-year spot rate based on the term structure data supplied, and $s_{up}(n)$ follows from the table.

Numerical example

2.15 For a 4% 10-year interest rate, the upward stressed interest rate corresponds to 5.6% \([4\% \times (1+0.4)]\), while the downward stressed interest rate equals to 2.8% \([4\% \times (1-0.3)]\).

Data series

2.16 The stress factors for the changes in interest rates are calibrated on two data sources:

- German zero rates, maturities 1y up till 10y, available from 1972, monthly data, (source: Bundesbank).


Modelling approach

2.17 The observed data showed that in general higher interest rates were associated with higher absolute changes in interest rates. The log-
normal model exhibits this property and the calibration of the log-normal model appeared more robust than the normal model\textsuperscript{14}.

2.18 The log-normal model treats proportionate changes in interest rates as a log-normal process, so it has been assumed that the distribution of the n-year spot rate in 12 months is given by:

\[ R_{12}(n) = R_0(n) \times e^X \]  \hspace{1cm} (b)

where X is distributed \( N(\mu_n, \sigma_n^2) \).

2.19 For \( y \) sufficiently close to zero, \( \ln(1+y) \) is approximately \( y \), hence, formula (b) can be rearranged to:

\[ X = \ln \left( \frac{R_{12}(n)}{R_0(n)} \right) = \ln \left( 1 + \left( \frac{R_{12}(n) - R_0(n)}{R_0(n)} \right) \right) \approx \left( \frac{R_{12}(n) - R_0(n)}{R_0(n)} \right) \]  \hspace{1cm} (c)

showing that the log-normal model assumes that the absolute change in interest rates, \([R_{12}(n)-R_0(n)]\), linearly depends on the level of interest rate, \( R_0(n) \) [Campbell, Lo and MacKinlay (1997)\textsuperscript{15}].

Further amendments/assumptions

2.20 The annualised standard deviations are given for the following maturities.

<table>
<thead>
<tr>
<th>Time to maturity</th>
<th>1-year</th>
<th>2-year</th>
<th>3-year</th>
<th>4-year</th>
<th>5-year</th>
<th>6-year</th>
<th>7-year</th>
<th>8-year</th>
<th>9-year</th>
<th>10-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stand. Dev. (annual.)</td>
<td>0.27</td>
<td>0.23</td>
<td>0.21</td>
<td>0.20</td>
<td>0.19</td>
<td>0.17</td>
<td>0.17</td>
<td>0.16</td>
<td>0.15</td>
<td>0.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time to maturity</th>
<th>1-year</th>
<th>2-year</th>
<th>3-year</th>
<th>4-year</th>
<th>5-year</th>
<th>6-year</th>
<th>7-year</th>
<th>8-year</th>
<th>9-year</th>
<th>10-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stand. Dev. (annual.)</td>
<td>0.20</td>
<td>0.21</td>
<td>0.17</td>
<td>0.14</td>
<td>0.14</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
</tr>
</tbody>
</table>

*Long-end stress factors*

2.21 For interest rates having maturities longer than 10 years (often described as the long end of a term structure), no long-dated data series were available. To determine stress factors for these long end interest rates that are consistent with the short end stress factors, the factors were fitted on information from both data sources\textsuperscript{16}.

Results

\textsuperscript{14} Two mean reversion models (Black-Karasinski and Cox-Ingersoll-Ross) were also considered. However, based on the observed data, the mean reversion assumption did not hold and, additionally, the resulting shocks were highly dependent on the exact model chosen.


\textsuperscript{16} For the long end interest rates, the annualised standard deviations are determined by using a constant volatility ratio (yearly data/daily data).
2.22 The following table provides the resulting stress factors for interest rate risk.

<table>
<thead>
<tr>
<th>Maturity n (years)</th>
<th>$S_{up}(n)$</th>
<th>$S_{down}(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.94</td>
<td>-0.51</td>
</tr>
<tr>
<td>2</td>
<td>0.77</td>
<td>-0.47</td>
</tr>
<tr>
<td>3</td>
<td>0.69</td>
<td>-0.44</td>
</tr>
<tr>
<td>4</td>
<td>0.62</td>
<td>-0.42</td>
</tr>
<tr>
<td>5</td>
<td>0.56</td>
<td>-0.40</td>
</tr>
<tr>
<td>6</td>
<td>0.52</td>
<td>-0.38</td>
</tr>
<tr>
<td>7</td>
<td>0.49</td>
<td>-0.37</td>
</tr>
<tr>
<td>8</td>
<td>0.46</td>
<td>-0.35</td>
</tr>
<tr>
<td>9</td>
<td>0.44</td>
<td>-0.34</td>
</tr>
<tr>
<td>10</td>
<td>0.42</td>
<td>-0.34</td>
</tr>
<tr>
<td>11</td>
<td>0.42</td>
<td>-0.34</td>
</tr>
<tr>
<td>12</td>
<td>0.42</td>
<td>-0.34</td>
</tr>
<tr>
<td>13</td>
<td>0.42</td>
<td>-0.34</td>
</tr>
<tr>
<td>14</td>
<td>0.42</td>
<td>-0.34</td>
</tr>
<tr>
<td>15</td>
<td>0.42</td>
<td>-0.34</td>
</tr>
<tr>
<td>16</td>
<td>0.41</td>
<td>-0.33</td>
</tr>
<tr>
<td>17</td>
<td>0.40</td>
<td>-0.33</td>
</tr>
<tr>
<td>18</td>
<td>0.39</td>
<td>-0.32</td>
</tr>
<tr>
<td>19</td>
<td>0.38</td>
<td>-0.31</td>
</tr>
<tr>
<td>20</td>
<td>0.37</td>
<td>-0.31</td>
</tr>
<tr>
<td>21</td>
<td>0.37</td>
<td>-0.31</td>
</tr>
<tr>
<td>22</td>
<td>0.37</td>
<td>-0.31</td>
</tr>
<tr>
<td>23</td>
<td>0.36</td>
<td>-0.31</td>
</tr>
<tr>
<td>24</td>
<td>0.36</td>
<td>-0.31</td>
</tr>
<tr>
<td>25</td>
<td>0.36</td>
<td>-0.31</td>
</tr>
<tr>
<td>26</td>
<td>0.36</td>
<td>-0.31</td>
</tr>
<tr>
<td>27</td>
<td>0.36</td>
<td>-0.31</td>
</tr>
<tr>
<td>28</td>
<td>0.36</td>
<td>-0.31</td>
</tr>
<tr>
<td>29</td>
<td>0.36</td>
<td>-0.31</td>
</tr>
<tr>
<td>30</td>
<td>0.36</td>
<td>-0.31</td>
</tr>
</tbody>
</table>

Other EU countries

2.23 The analysis has been extended to interest rates of the following non-euro EU countries: Denmark, Sweden and the UK. The comparison was based on:

- relevant zero swap rates (source: Datastream);
- maturities 2y-5y-10y-20y-30y;
- overlapping time periods starting from 1997;
data on a daily basis.

2.24 The following table shows for each country and maturity the corresponding standard deviation:

**Comparison of standard deviations**

<table>
<thead>
<tr>
<th>Country</th>
<th>2-year</th>
<th>5-year</th>
<th>10-year</th>
<th>20-year</th>
<th>30-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMU</td>
<td>0.21</td>
<td>0.17</td>
<td>0.14</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>DEK</td>
<td>0.19</td>
<td>0.16</td>
<td>0.14</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>SEK</td>
<td>0.18</td>
<td>0.17</td>
<td>0.14</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>UK</td>
<td>0.15</td>
<td>0.14</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Swap rates daily data from 1997-2006 (data source: Datastream)

2.25 For all three added countries, the standard deviations were more or less consistent with the EMU results, with the exception of the short end of the UK term structure that showed lower results. However, for all countries the short end of the term structure exhibits higher standard deviations than the long end.

**Possible future work**

2.26 Explore further on:

- Correction for the bias in the time series (since it starts at high interest rates in 1973 and ends with low interest rates in 2005).
- Fat tails correction: log-lin, extreme value etc.

**Equity risk**

**Introduction**

2.27 The QIS2 technical specification document provides a 40% stress factor for equity risk (see §5.53-§5.54). Equity risk arises from the level or volatility of market prices for equities. The stress factor for equity risk can be calibrated at different levels of granularity (i.e. global or region level). For reasons of simplicity, the level of granularity used for QIS2 was set at the global level. For QIS3, this hypothesis is refined, and equity risk is divided into two parts. The first part contains equity invested in indices of development markets, and the second component comprises the remaining elements, e.g. emerging markets, non-listed equities and alternative investments.

**Data series**

2.28 The stress factor for equity risk was calibrated on the following data sources:

**Information on the MSCI indices**

2.29 MSCI Developed Markets index covers 23 developed market country indices. In order to calculate total returns at an aggregated level, country weights were calculated on the basis of their market capitalisation. The index does not include emerging markets and private equity investments\(^{17}\).

2.30 The total returns of the MSCI Developed Markets index are calculated on the assumption that dividends are re-invested in the index on the day the security is quoted ex-dividend.

**Modelling approach**

2.31 As a first step, the global returns are assumed to follow a normal distribution. However, this assumption might need some modification. The observed returns are negatively skewed and exhibit a negative fat tail.

2.32 There are multiple options to deal with thickness of negative tails. For example, there are the so-called extreme value theory and log-linear estimation methods. The log-linear estimation method extrapolates the tail in the historical probability distribution using log-linear regression for the historically worst out-comes\(^{18}\). Extreme value theory makes use of the assumption that the distribution of the tail converges to a limit distribution. The Gumbel distribution is a special case of the Generalized Extreme Value distribution and is especially suitable for light tailed distributions.

**Results**

<table>
<thead>
<tr>
<th>Provider:</th>
<th>MSCI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region:</td>
<td>Global</td>
</tr>
<tr>
<td>Time period:</td>
<td>1970-2006</td>
</tr>
<tr>
<td>Frequency:</td>
<td>quarterly</td>
</tr>
<tr>
<td>Inflation-adjustment:</td>
<td>nominal returns(^{19})</td>
</tr>
<tr>
<td>Currency:</td>
<td>hedged(^{20})</td>
</tr>
<tr>
<td>Arithmetical mean:</td>
<td>11.5%</td>
</tr>
<tr>
<td>Geometric mean(^{21}):</td>
<td>10.1%</td>
</tr>
</tbody>
</table>

\(^{17}\) To the extent feasible, future analyses could expand to both sub-categories.
\(^{18}\) See for example the calibration of the equity shock within the recently introduced traffic light system in Sweden.
\(^{19}\) Real (or inflation-adjusted) equity returns are important when effects on purchasing power are directly included. However, the current SCR specification for market risk is in a nominal context, e.g. interest rate risk is calibrated on nominal interest rate term structures. Hence, nominal equity returns are used in preference to real equity returns.
\(^{20}\) This analysis uses hedged returns, so the equity returns exclude movements in exchange rates. Since equity positions are included in the currency risk model, the use of un-hedged returns for equity risk would lead to double counting.
\(^{21}\) In practice, and based on the reinvesting assumption, the analyses of financial time series often use the geometric mean instead of the arithmetic mean. The geometric mean of a set of positive
The geometric mean equals to 10.1% and the standard deviation corresponds to 16.9%. Based on the assumption of normally distributed equity returns, the 99.5 confidence level corresponds to a shock of 33.4%. After fat tail correction, the 99.5 confidence level corresponds to a shock of approximately 35%, depending on the exact correction method chosen.

**Other time periods**

Institutional investors consider the MSCI Developed Markets index as one of the main benchmarks for worldwide equity investments. Unfortunately, the MSCI data is only available from 1970. Consequently, important pre-WWII market declines are not included in the MSCI world index. On the other hand, due to the globalisation of financial markets, the dependency between the markets of the developed countries has increased. This effect is clearly observed in the tables below. The tables show the correlations between five major equity markets for three different time periods. These correlations are calculated on the basis of Dimson total return indices (source: Ibbotson Associates), dated from 1900 and on a yearly basis\(^{22}\).

---

**Arithmetic mean:** \[ \bar{x} = \frac{\sum x}{n} \]

**Geometric mean:** \[ \left( \prod (1 + R_t) \right)^{\frac{1}{n}} - 1 \]

The Dimson indices lead to approximately equal results: (1) geometric mean equals to 11.2%, (2) the standard deviation corresponds to 16.5% (3) based on normality the 99.5 confidence level corresponds to a shock of 31.2% (4) after fat tail correction, the 99.5 confidence level corresponds to a shock of approximately 35%, depending on the exact correction method chosen.
<table>
<thead>
<tr>
<th></th>
<th>FRA</th>
<th>GER</th>
<th>JPN</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>1900-2005</td>
<td>1</td>
<td>0.31</td>
<td>0.16</td>
<td>0.38</td>
<td>0.24</td>
</tr>
<tr>
<td>1970-2005</td>
<td>0.75</td>
<td>0.51</td>
<td>0.46</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>1985-2005</td>
<td>0.82</td>
<td>0.68</td>
<td>0.77</td>
<td>0.67</td>
<td></td>
</tr>
</tbody>
</table>

(Data source: Ibbotson)

**Level of granularity**

2.35 The table below shows the outcome of applying the analyses at different levels of granularity.

<table>
<thead>
<tr>
<th>Region/country:</th>
<th>Global</th>
<th>Europe</th>
<th>Global ex-Europe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency:</td>
<td>quarterly</td>
<td>quarterly</td>
<td>quarterly</td>
</tr>
<tr>
<td>VaR 99.5% Normal</td>
<td>-33%</td>
<td>-38%</td>
<td>-34%</td>
</tr>
<tr>
<td>VaR 99.5% Log-lin</td>
<td>-34%</td>
<td>-38%</td>
<td>-34%</td>
</tr>
<tr>
<td>VaR 99.5% Gumbel</td>
<td>-35%</td>
<td>-36%</td>
<td>-35%</td>
</tr>
</tbody>
</table>

2.36 In line with expectations, the more diversified the index the lower the shock. In this context, investing purely in European indices will lead on average to a 3% increase, while global ex-Europe investments show slightly lower increases compared to the global index.

**Possible future work**

2.37 explore further on:

- Fat tails corrections: An option is, to fit a Generalized Pareto Distribution (GPD) to the tail of the equity return distribution. There are Maximum Likelihood (ML) estimators for the GPD and the GPD can be determined with regression models.
**Property risk**

*QIS2 calibration results*

2.38 The QIS2 technical specification document provides a 20% stress factor for property risk (see §5.59-§5.60). Property risk arises from the level and volatility of market prices of property. For reasons of simplicity, QIS2 offered no distinction between direct and indirect real estate or between different types of real estate investment (offices, retail, residential etc.).

*Data series*

2.39 The stress factor for property risk is calibrated on the following data sources:

- Dutch direct real estate data, 1977-2005, yearly basis, (source: ROZ-IPD);
- French direct real estate data, 1998-2005, yearly basis, (source: IPD);
- German direct real estate data, 1996-2005, yearly basis, (source: IPD);
- Swedish direct real estate data, 1997-2005, yearly basis, (source: IPD);
- UK direct real estate data, 1971-2005, yearly basis, (source: IPD);

*Information on the index*

2.40 The IPD indices are based on annualised total returns (capital growth + income return) of direct investments in real estate.

2.41 The total returns are based on valuation data (i.e. surveyors’ estimates of property values), rather than actual market prices. The returns calculated from this data are often referred to as a “smoothed” property returns, because valuation prices are usually smoothed over time.

2.42 The French, German, and Swedish IPD data lack long-dated information of their property markets. Consequently, the corresponding analyses do not include a full property cycle.

*Modelling approach*

2.43 For reasons of simplicity, it is assumed that the property returns are normally distributed. Choosing a more sophisticated model might give a better fit, however, currently no sufficient data exists to model the negative tail of the distribution very precisely. The purpose here is to
use a simple and transparent model to produce reasonable estimators for the lower percentiles.

**Investment market size**

2.44 In the IPD Pan-European property index\(^{23}\), the individual country returns are grossed-up according to the estimated value of the investment market in each country. The table below shows the end 2005 market size for each of the five countries mentioned earlier.

<table>
<thead>
<tr>
<th>Investment market size</th>
<th>Market size</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRA</td>
<td>138714</td>
<td>14%</td>
</tr>
<tr>
<td>GER</td>
<td>259375</td>
<td>26%</td>
</tr>
<tr>
<td>NL</td>
<td>68032</td>
<td>7%</td>
</tr>
<tr>
<td>SEK</td>
<td>58212</td>
<td>6%</td>
</tr>
<tr>
<td>UK</td>
<td>462469</td>
<td>47%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>986802</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

*In million euro end 2005 (source: IPD)*

**Smoothing mechanism**

2.45 Since transaction prices are important, an “unsmoothed” property returns can be derived from the observable smoothed data. [Fisher and Geltner (2000)]

2.46 The following simple de-smoothing mechanism is applied:

\[
\hat{R}_t = \omega \times R_t + (1 - \omega) \times \hat{R}_{t-1}
\]  

(a)

, where

\( \hat{R}_t \): smoothed property return at time \( t \);

\( R_t \): unsmoothed property return at time \( t \);

\( \omega \): weight.

2.47 In other words, it is assumed that the IPD data is smoothed as a weighted average of last year’s smoothed and this year’s actual return. The weight, \( \omega \), can be found using the existing auto-covariance in the observed data\(^{24}\).

**Results**

2.48 The estimated \( \omega \) for each individual country is shown in the table below.

---

\(^{23}\) See for more information the technical note on the IPD Pan-European property index version 2006.

\(^{24}\) \( \omega \) is estimated from the slope coefficient of the regression of the smoothed returns on their values lagged one year, with the estimation undertaken on a rolling basis.
<table>
<thead>
<tr>
<th>Country</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRA</td>
<td>0,4388</td>
</tr>
<tr>
<td>GER</td>
<td>0,1046</td>
</tr>
<tr>
<td>NL</td>
<td>0,4696</td>
</tr>
<tr>
<td>SEK</td>
<td>0,5523</td>
</tr>
<tr>
<td>UK</td>
<td>0,6522</td>
</tr>
</tbody>
</table>

2.49 Compared to the other countries, the German returns showed a relatively high level of auto-covariance, leading to a relatively low value for $\omega$.

2.50 Based on the specific de-smoothing mechanism, the standard deviations of the “unsmoothed” property returns can be determined.

<table>
<thead>
<tr>
<th>Weight</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Adjust. factor Unsm./Smooth</th>
<th>99.5% shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Smoothed Unsmoothed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>14%</td>
<td>10,5% 3,4% 7,6%</td>
<td>222% -8,92%</td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>26%</td>
<td>3,6% 1,7% 9,3%</td>
<td>541% -20,36%</td>
<td></td>
</tr>
<tr>
<td>NL</td>
<td>7%</td>
<td>9,4% 5,1% 8,4%</td>
<td>163% -12,20%</td>
<td></td>
</tr>
<tr>
<td>SEK</td>
<td>6%</td>
<td>9,9% 7,2% 11,4%</td>
<td>158% -19,40%</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>47%</td>
<td>12,4% 10,3% 16,0%</td>
<td>156% -28,87%</td>
<td></td>
</tr>
</tbody>
</table>

2.51 After de-smoothing, the standard deviations, although to a lesser extent, still significantly differ between the five countries. The level of granularity to be chosen is highlighted as a Pillar I WG issue.

Return truncation

2.52 Since the historical series considerably vary in length, co-variances are estimated by using the shortest common subset of returns, thereby discarding some information contained in the longer series\textsuperscript{25}. Therefore, instead of using a market-weighted basket of the five countries, the 99.5% shock was conservatively rounded.

2.53 On the basis of these analyses this paper suggests the following stress factor for property risk.

<table>
<thead>
<tr>
<th>Property</th>
<th>Stress scenario</th>
<th>20%</th>
</tr>
</thead>
</table>

Possible future work

2.54 explore further on:

fat tails correction: log-lin, extreme value etc. [no sufficient data available]

illiquidity correction;

return truncation correction.

**Currency risk**

*Introduction*

2.55 The QIS2 technical specification document provides a 25% stress factor for currency risk (see §5.65 - §5.66). Currency risk arises from the level or volatility of currency exchange rates. The stress factor for currency risk relates to a simultaneous change (rise or fall) in value of all other currencies against the local currency in which the undertaking prepares its local regulatory accounts.

2.56 Compared to the QIS2 calibration work, this section extends in multiple areas. Firstly, the calibration is based on much longer data series of several exchange rates. Secondly, work to date has been based on exchange rates versus the euro. In this section the analysis will be extended to some other relevant currencies, such as GBP. Finally, this section will test the impact of using different currency baskets. On the basis of these analyses this paper suggests that the stress factor for currency risk should be 20% in preference to 25%.

*Data series*

2.57 The calibration of the stress factor for currency risk is based on the exchange rates of a basket of seven currencies versus the euro. The following data series were used:

Exchange rates versus the (synthetic) euro, 1958-2006, monthly basis, (source: Datastream).

- US dollar (USD)
- British pound (GBP)
- Argentine peso (ARP)
- Japanese yen (JPY)
- Swedish krone (SEK)
- Swiss franc (CHF)
- Australian dollar (AUD)

*Modelling approach*
For reasons of simplicity, it is assumed that the relative changes in exchange rates are normally distributed. The graph below shows the lower percentiles of the observed distribution of the monthly relative changes in the dollar-euro exchange rate. Looking at the negative tail of the distribution, the normality assumption seems to be acceptable. Of course, choosing a more sophisticated model might give a better fit. However, the purpose here is to use a simple and transparent model to produce reasonable estimators for the lower percentiles.

Currency basket

The relative weights of the currencies within the basket were based on an estimation of currency positions hold by Dutch financial institutions.

- US dollar: 35%
- British pound: 24%
- Argentine peso: 13%
- Japanese yen: 8%
- Swedish krone: 7%
- Swiss franc: 7%
- Australian dollar: 6%

The Argentine peso is used as a proxy for all currency exposure to emerging markets.²⁶

Further amendments/assumptions

Time scaling

²⁶ Please notice that the analysed period includes the fixed pegged USD time period (1992-2001).
Given the monthly frequency of the data, and the fact that the SCR is based on a one-year time horizon, the holding period applied in the calculation of VaR needed to be scaled up to a one year risk evaluation. This adjustment assumes that the monthly distributions are statistically independent.

Results for the euro

The table below presents the (annualised) standard deviations of the seven exchange rates versus the euro and the currency basket assumed. The table also shows the corresponding 99.5% shocks on the basis of the normality assumption.

<table>
<thead>
<tr>
<th>Standard deviations (annualised) and 99.5% currency shocks versus the euro</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>8.69%</td>
</tr>
<tr>
<td>-22.12%</td>
</tr>
</tbody>
</table>

Exchange rates versus synthetic euro, monthly basis, 1958 -2006, (source: Datastream)

For this specific currency basket, the 99.5 confidence level corresponds to a shock around 17%. When we exclude the Bretton-Woods period, the 99.5% shock equals 20%.

Other local currency

Instead of looking at exchange rates to the euro, the same analysis can be applied for the British pound.

<table>
<thead>
<tr>
<th>Standard deviations (annualised) and 99.5% currency shocks versus GBP</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>9.03%</td>
</tr>
<tr>
<td>-23.25%</td>
</tr>
</tbody>
</table>

Exchange rates versus GBP, monthly basis, 1958 -2006, (source: Datastream)

Assuming the same currency basket, the standard deviation for the British pound is higher and the corresponding 99.5% shock equals to 21%.

Other currency baskets

An interesting sensitivity analysis of the currency basket assumption would be, to change the current weights of the three major country weights. The table below depicts the impact on the resulting 99.5% shocks for the euro.
Given its highest standard deviation, and specific dependency with the other currencies, the ARP-EUR exchange rate has the most impact on the total outcome. The decision to include non- or 20%- ARP exposure leads approximately to a 10% difference at total level.

The same analysis was applied for the British pound and the table below shows the outcome. All results fall within the 15%-25% range.

### Sensitivity analysis of the GBP currency basket

<table>
<thead>
<tr>
<th>Currency</th>
<th>Current</th>
<th>EUR</th>
<th>ARG</th>
<th>USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
<td>35%</td>
<td>23%</td>
<td>46%</td>
<td>50%</td>
</tr>
<tr>
<td>JPY</td>
<td>8%</td>
<td>7%</td>
<td>11%</td>
<td>7%</td>
</tr>
<tr>
<td>SEK</td>
<td>7%</td>
<td>6%</td>
<td>8%</td>
<td>6%</td>
</tr>
<tr>
<td>CHF</td>
<td>7%</td>
<td>6%</td>
<td>8%</td>
<td>5%</td>
</tr>
<tr>
<td>GBP</td>
<td>24%</td>
<td>22%</td>
<td>28%</td>
<td>18%</td>
</tr>
<tr>
<td>AUD</td>
<td>6%</td>
<td>6%</td>
<td>7%</td>
<td>5%</td>
</tr>
<tr>
<td>ARP</td>
<td>13%</td>
<td>10%</td>
<td>10%</td>
<td>9%</td>
</tr>
</tbody>
</table>

| Standard deviation | 8.19% | 7.27% | 9.38% | 9.87% |
| 99.5% shock        | -21.10% | -18.72% | -24.17% | -25.41% |

Exchange rates versus GBP, monthly basis, 1958 -2006, (source: Datastream)

On the basis of these analyses this paper suggests the following stress factor for currency risk.

| Stress scenario | 20% |

Possible future work

2.70 explore further on:
fat tails correction: log-lin, extreme value etc.

different model assumptions.

**Correlation matrix**

**Introduction**

2.71 The QIS2 technical specification document provides a correlation matrix for interest rate, equity, property and currency risk (see §5.17). The correlation coefficient between interest rate and equity risk corresponded to a VaR 99.5% standard.

<table>
<thead>
<tr>
<th>CorrMkt</th>
<th>Mkt(_{\text{int}})</th>
<th>Mkt(_{\text{eq}})</th>
<th>Mkt(_{\text{prop}})</th>
<th>Mkt(_{\text{fx}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mkt(_{\text{int}})</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mkt(_{\text{eq}})</td>
<td>0.75</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mkt(_{\text{prop}})</td>
<td>0.75</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mkt(_{\text{fx}})</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
</tr>
</tbody>
</table>

2.72 CEIOPS stated in CP20 that it recognises that on market risk the QIS2 approach for the market risk module did not give due recognition for diversification effects and that some of the correlation assumptions between interest rate, equity, property and currency risk would need to be revised downwards (see CP20 §5.124).

2.73 This paper focuses on the correlation coefficient between interest rate and equity risk. For most insurers, interest rate and equity risk will dominate their market risk module.

2.74 This analysis uses a ‘top down’ approach for the calibration of the correlation coefficient between interest rate and equity risk. This approach is chosen in order to ensure that the overall SCR\(_{\text{mkt}}\) risk charge, given the calibration of the individual shocks to a VaR 99.5% standard, is again consistent with a VaR 99.5% standard.

**Data series**

2.75 The correlation coefficient between interest rate and equity risk was calibrated on the following data sources:

- German zero rates, different maturities available from 1972, yearly data, (source: Bundesbank).
Modelling approach

2.76 In order to carry out the 'top down' approach some simplified model assumptions were made. For example, the analysis assumes a constant asset mix (30% equity and 70% bonds\(^{27}\)) and works with a constant positive 10 year duration gap between the duration of liabilities and the duration of bond investments and liabilities. These assumptions will be subject to a sensitivity analysis.

2.77 The historical returns of this model portfolio are assumed to follow a normal distribution. Figure 1 compares the empirical data with this normality assumption. Figure 1 shows that the normality assumption seems appropriate for the balanced portfolio.

Results

2.78 Based on the normality assumption, the 99.5% confidence level corresponds to an 18% overall shock for market risk.

Total return model portfolio

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetical mean:</td>
<td>7,6%</td>
</tr>
<tr>
<td>Geometrical mean:</td>
<td>7,1%</td>
</tr>
<tr>
<td>Sigma:</td>
<td>9,8%</td>
</tr>
<tr>
<td>99.5 perc: N() &amp; Mu(_{geo})</td>
<td>18,0%</td>
</tr>
</tbody>
</table>

\(^{27}\) See Financial Conditions and Financial Stability in the European Insurance and Occupational Pension Fund Sector 2005-2006, CEIOPS-FS-14/06S, December 2006, Figure 10B, p. 11.
2.79 Given this 18% overall shock for the model portfolio, the corresponding correlation coefficient between interest rate and equity risk can be derived. The calibration work for the individual shocks for interest rate and equity risk, as described in section 2 and 3, is used to determine the corresponding correlation coefficient of the model portfolio. The analysis assumes for equity risk a 35% shock, and applies for interest rate risk the stress factors from the table stated in para 2.22.

<table>
<thead>
<tr>
<th>SCRmkt (rho=0,25)</th>
<th>16,8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCRmkt (rho=0,5)</td>
<td>18,4%</td>
</tr>
<tr>
<td>SCRmkt (rho=0,75)</td>
<td>19,8%</td>
</tr>
</tbody>
</table>

2.80 The exact correlation coefficient corresponding to the overall 99.5% shock equals 0.44.

2.81 The table below shows the split of the resulting SCR\textsubscript{mkt} of the model portfolio into the interest rate and equity risk parts\textsuperscript{28}.

<table>
<thead>
<tr>
<th>weight (implied) shock</th>
<th>(SCR_{mkt}^i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds 70% 15%</td>
<td>10,7%</td>
</tr>
<tr>
<td>Equity 30% 35%</td>
<td>10,5%</td>
</tr>
<tr>
<td>Portfolio 100%</td>
<td>18,0%</td>
</tr>
</tbody>
</table>

2.82 Sensitivity analysis

The sensitivity analysis shows that the result is extremely sensitive to the assumed percentage invested in equity and used duration gap. The subsequent table reveals that the correlation coefficient between interest rate and equity risk ranges between 0.75 and 0.10. Given this great uncertainty, this paper proposes for QIS3 purposes to use a rounded number of 0 for the correlation coefficient between interest rate and equity risk. This also allows symmetry between the assumptions about the direction of movements in the value of equities during an interest rate increase scenario and an interest rate decrease scenario. In addition, it may be noted that the main interest rate risk for insurers is likely to be a fall in interest rates. In a falling interest rate scenario, it is arguable that the direction of movement in equities is just as likely to be upwards as downwards.

\textsuperscript{28} The 15% implied shock for interest rate risk is based on the 34% downwards stress factor from the table stated in §2.22, a positive 10 year duration gap between the duration of liabilities and the duration of the bond investments, and a risk-free 10 year rate of 4.5%.
### Table 6.1. The derived correlation coefficient between interest rate and equity risk for different % in equity and duration gap assumptions

<table>
<thead>
<tr>
<th>Duration gap</th>
<th>25%</th>
<th>30%</th>
<th>35%</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.39</td>
<td>0.23</td>
<td>0.10</td>
</tr>
<tr>
<td>10</td>
<td>0.56</td>
<td>0.44</td>
<td>0.36</td>
</tr>
<tr>
<td>15</td>
<td>0.75</td>
<td>0.60</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Possible future work

Possible further work will include:

- Analysis on property and currency risk.
Section 3

Calibration of the MCR

Market risk

3.1 The calibration of the MCR market risk module generally adopts the methodology and the results of the calibration of the SCR market risk module, outlined elsewhere in the Annex. However the reduced granularity of the MCR also meant that, in certain cases, broad average assumptions had to be used.

ALTERNATIVE 1

3.2 The calibration has been benchmarked on a member state market, taking into account about 400 undertakings, using the results of QIS1 and QIS2; this aimed at adjusting balance sheets in order to reflect changes in the valuation of liabilities and assets.

Equity component

3.3 The MCR equity risk charge was calibrated using normally distributed returns with a 10% return and a 16.9% volatility. This lead to a 11.7% capital charge, rounded to 12%.

Property component

3.4 Regarding property risk, it seemed that the QIS2 methodology overestimated return rates, due to estimations based on the 1998-2005 period (for the reference market); such estimations excluded the crisis that took place in the beginning of the 1990s on the property reference market. Those estimates would have lead to a negative capital charge on a 90% VaR basis. For that reason, a 7% return and 12% volatility were chosen.

Fixed-income component

3.5 The fixed income charge covers the interest rate risk and its initial calibration follows the calibration of the QIS 2 interest rate risk module. The liability side was taken into account, not on a company by company basis but on a market-wide basis: a mean rate of 4% was chosen, combined with a 2 year duration for non-life business, and a 7 year duration for life business. A 20% shock was applied and lead to the coefficients in the specifications: 2,7% for non-life business and 5,4% for life business.

ALTERNATIVE 2

Equity and property components
3.6 For the equity component and the property component, the calibrations are the same as in Alternative 1, see above.

_Credit spread component_

3.7 The risk weight applied to FI* was derived from the SCR spread risk module, assuming a 5-year duration and A-rated bonds, and the factor was adjusted for the 90% VaR level shock and rounded.

_Interest rate risk component_

3.8 The modelling of the upward and downward interest rate shocks as a function of maturity follows the approach used to calibrate the SCR standard formula interest rate risk submodule; the calibration of the MCR took into account the shocks corresponding to the 90% VaR level. The average values for medium maturities (the former middle maturity bucket) of 7-12 years were chosen, leading to $s_{up} = 0.18$ and $s_{down} = -0.20$. Reference to the shorter durations and their higher shock factors was deliberately avoided, because of the concern that this would close the gap between the MCR and the SCR.

3.9 Below is the table of the 90%-level stress factors for interest rate risk, from which the above parameters have been derived.

<table>
<thead>
<tr>
<th>maturity n (years)</th>
<th>$s_{up}(n)$</th>
<th>$s_{down}(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.37</td>
<td>-0.31</td>
</tr>
<tr>
<td>2</td>
<td>0.31</td>
<td>-0.28</td>
</tr>
<tr>
<td>3</td>
<td>0.28</td>
<td>-0.26</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>-0.25</td>
</tr>
<tr>
<td>5</td>
<td>0.23</td>
<td>-0.23</td>
</tr>
<tr>
<td>6</td>
<td>0.21</td>
<td>-0.22</td>
</tr>
<tr>
<td>7</td>
<td>0.20</td>
<td>-0.21</td>
</tr>
<tr>
<td>8</td>
<td>0.19</td>
<td>-0.21</td>
</tr>
<tr>
<td>9</td>
<td>0.18</td>
<td>-0.20</td>
</tr>
<tr>
<td>10</td>
<td>0.17</td>
<td>-0.20</td>
</tr>
<tr>
<td>11</td>
<td>0.17</td>
<td>-0.20</td>
</tr>
<tr>
<td>12</td>
<td>0.17</td>
<td>-0.20</td>
</tr>
<tr>
<td>13</td>
<td>0.17</td>
<td>-0.20</td>
</tr>
<tr>
<td>14</td>
<td>0.17</td>
<td>-0.20</td>
</tr>
<tr>
<td>15</td>
<td>0.17</td>
<td>-0.20</td>
</tr>
<tr>
<td>16</td>
<td>0.17</td>
<td>-0.19</td>
</tr>
<tr>
<td>17</td>
<td>0.17</td>
<td>-0.19</td>
</tr>
<tr>
<td>18</td>
<td>0.16</td>
<td>-0.19</td>
</tr>
<tr>
<td>19</td>
<td>0.16</td>
<td>-0.18</td>
</tr>
<tr>
<td>20</td>
<td>0.16</td>
<td>-0.18</td>
</tr>
<tr>
<td>21</td>
<td>0.15</td>
<td>-0.18</td>
</tr>
<tr>
<td>22</td>
<td>0.15</td>
<td>-0.18</td>
</tr>
<tr>
<td>23</td>
<td>0.15</td>
<td>-0.18</td>
</tr>
<tr>
<td>24</td>
<td>0.15</td>
<td>-0.18</td>
</tr>
<tr>
<td>25</td>
<td>0.15</td>
<td>-0.18</td>
</tr>
<tr>
<td>26</td>
<td>0.15</td>
<td>-0.18</td>
</tr>
<tr>
<td>27</td>
<td>0.15</td>
<td>-0.18</td>
</tr>
<tr>
<td>28</td>
<td>0.15</td>
<td>-0.18</td>
</tr>
</tbody>
</table>
Non-life underwriting risk

3.10 No independent model was built to calibrate \( \text{MCR}_{nl} \); the present calibration aims to approximate the 90%-level SCR-equivalent (\( \text{SCR}_{90} \)) in a simpler algebraic structure. The model used to determine the \( \text{SCR}_{90} \) charge was the same as in the SCR standard formula non-life premium and reserve risk submodule, yet with a treatment of premium risk relying solely on market-wide volatility factors.

3.11 The factors for each line of business and for both premium and reserve risk were derived as

\[
\alpha_{\text{lab}} = \rho_{90}(\sigma_{\text{prem,lob}}), \quad \beta_{\text{lab}} = \rho_{90}(\sigma_{\text{res,lob}})
\]

3.12 where \( \rho_{90} \) is the 90% VaR equivalent of the risk measure function \( \rho \) used in the standard formula \( \text{NL}_{\text{prem}} \) sub-module; and where \( \sigma_{\text{prem,lob}} \) and \( \sigma_{\text{res,lob}} \) are the volatility factors used in the SCR for each line of business.

3.13 The 0.65 floor applied to the concentration/diversification factors was selected to minimise the sum of the squares of the residuals

\[
\sum \left( \frac{\text{MCR}_{nl}}{\text{SCR}_{90}} - 1 \right)^2
\]

for a sample of insurer data. A real life sample of 80 insurers from 2 countries was used; however, since the data matching the QIS3 segmentation were not always available, the calculation involved some rough estimates. Because of this, a simulated (randomly generated) sample was also set up to supplement the calibration.

3.15 The two approaches lead to very similar results. In terms of MCR-to-SCR ratios, the calibration exercise indicated that, under the present choice of parameters, the ratio of the \( \text{SCR}_{90} \) charge to the (non-personalised) SCR non-life premium and reserve risk charge generally falls close to 47%. For the samples used in the calibration exercise, the ratio of the MCR non-life underwriting risk charge to the corresponding standard formula charge (with no personalisation applied) generally fell between 35% and 55%.

3.16 The diagrams below illustrate the relationship between \( \text{MCR}_{nl} \) and \( \text{SCR}_{90} \). Actual QIS3 results may differ from this, because of the entity-specific volatilities for premium risk in the SCR, and also because the MCR inputs do not project a premium growth for the forthcoming year.
MCR vs SCR_90 (real-life data)

MCR vs SCR_90 (simulated data)
Life underwriting risk

3.17 The mortality and longevity components are calculated on the same technical basis as in the factor-based SCR\textsubscript{life} proxies, with a calibration of 90\% VaR instead of 99.5\% VaR.

3.18 The definition of the unit linked charge is an initial one meant for QIS3 purposes only. Parallel to the review of the standard formula operational risk charge, CEIOPS will revise this component after QIS3.

3.19 The present MCR life underwriting risk formula does not take into account disability and morbidity risks. The inclusion of a risk charge reflecting these risks will be considered after QIS3.

Health underwriting risk

3.20 This subsection describes the technical basis of the calibration of the MCR health underwriting risk charge.

3.21 The calibration is based on market data regarding the following variables:

- \( I_k \): Number of sampled risks of the \( k \)-th insurer,
- \( A_k \): Overall number of risks of the \( k \)-th insurer,
- \( \text{SumClaim}_k \): Sum of sampled claims of the \( k \)-th insurer, i.e., \( \sum_{i \in I_k} x_{i,k} \),
- \( Q\text{SumClaim}_k \): Sum of sampled squares of claims of the \( k \)-th insurer, i.e., \( \sum_{i \in I_k} x_{i,k}^2 \),
- \( s_k \): estimated standard deviation of the random variable supposed to describe the claim per person and accounting year of the \( k \)-th insurer.

3.22 The purpose of the calibration is to determine the coefficient \( \rho \) in the MCR health formula

\[
\text{MCR}_{\text{health}}^\rho = c \cdot \frac{\rho}{\sqrt{A_k}} \cdot \text{SumClaim}_k,
\]

in such a way that the MCR capital charge, together with technical provisions, provides a 90\% confidence level that the available capital for a risk (unbiased randomly chosen from all risks on the market) will stay above the technical provision reserved for that risk.

3.23 The factor \( \rho \) is formally determined from the equation
\[
\sum A_k \cdot \text{Prob}\left( X < c \cdot \rho \frac{\text{SumClaim}_k}{I_k \cdot s_k} \right) = \frac{\sum A_k \cdot \Phi\left( c \cdot \rho \frac{\text{SumClaim}_k}{I_k \cdot s_k} \right)}{\sum A_k} = 0.9
\]

where \( X \) denotes an \( \text{N}(0,1) \) distributed random variable and \( \Phi \) denotes the cumulative distribution function of \( \text{N}(0,1) \). The equality is solved for \( \rho \) by a Newton or fixed-point procedure.

3.24 Applying the above methodology to data obtained from a member state market, estimates of \( \rho \) fell between 2.1 and 7.6. For QIS3, a calibration of \( \rho = 5 \) was chosen.