Introduction: General overview of the methodology

The proposed methodology consists in a two steps approach. Each of these steps, described hereafter, are independent.

The step 1 consists in the generation of a reduced sample of scenarios through a given methodology (several options being possible and described in section 1).

The step 2 consists in adjusting the scenarios produced in the step 1. These adjustments aim to ensure that the overall sample of scenario has acceptable (1) martingale properties (2) market consistent properties. These adjustments are described in section 2.

It should be noted that this two steps approach allows to use any kind of method to generate the scenarios regardless of whether they are produced by a risk neutral model or by a real world model as the adjustment step ensures their martingale properties.

1. Step 1 : Methodological options for the production of the sample of scenarios

<u>Method 1: use of pure stochastic trajectories</u>

The most immediate option for generating the PHRSS scenarios consists in using a stochastic model (that can be more or less complex) to generate scenarios.

For this purpose, it is possible to use a simple model such as a basic Gaussian stochastic process to simulate evolutions of the risk factors considered. More complex models could also be used (e.g. Hull and White, G2++ or LMM model for interest rates).

However, as the PHRSS is intended to be a materiality assessment in the context of a proportionality measure rather than a real stochastic valuation of the TP, a simple model seems preferable to ensure robustness, transparency, and simplicity.

It is therefore proposed for this option to use the following modelling of the risk factors:

- The interest rates are modelled under a Gaussian dynamic centered on forward rates (parallel shift):

$$\tilde{r}(t,m) = r^f(t,m) + \sigma_{IR} \sum_{k=1}^{t} \varepsilon_k^{IR}$$

With $r^f(t,m)$ the forward rate seen at time 0 for period t related to maturity m and $\varepsilon_k^{IR} \approx N(0,1)$.

- As for the equity-like indexes (equities total return, real estate total return), they are modelled with a Black and Scholes model:

$$S^{EQ}(t) = S^{EQ}(t-1) \times \frac{1}{P(t-1,1)} \times e^{-0.5\sigma_{EQ}{}^2 + \sigma_{EQ}\varepsilon_t^{EQ}};$$

$$S^{RE}(t) = S^{RE}(t-1) \times \frac{1}{P(t-1,1)} \times e^{-0.5\sigma_{RE}{}^2 + \sigma_{RE}\varepsilon_t^{RE}}.$$

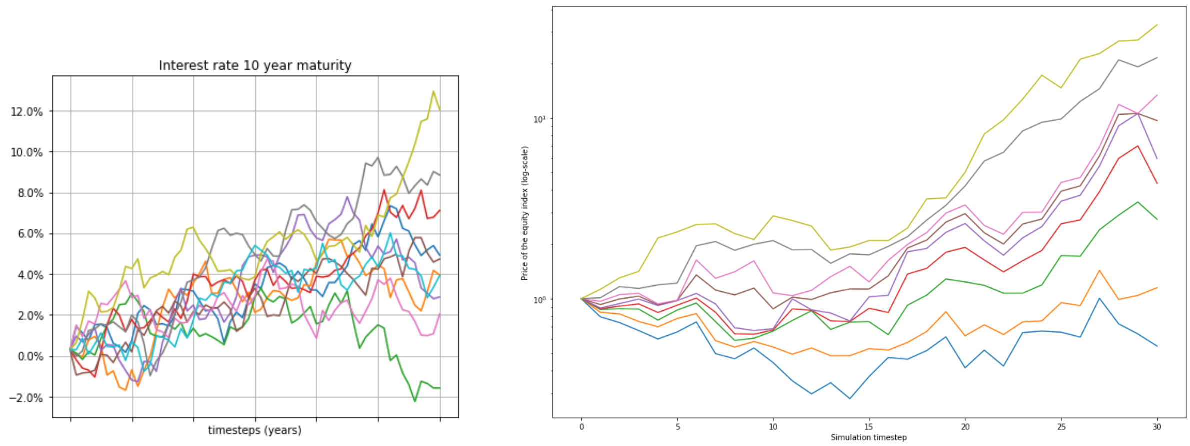
With $\varepsilon_t^{EQ} \approx N(0,1)$, $\varepsilon_t^{RE} \approx N(0,1)$.

- The innovations of the different stochastic risk factors (ε_k^{IR},ε_t^{EQ},ε_t^{RE}) are simulated independently (no dependence structure embedded).

Pros: This simple methodology can be easily implemented and allows having « real ESG » looking scenarios.

Cons: However, as for a full ESG, the trajectories are erratic and very dependent on the random number generator seed used to produce the scenarios. As the number of scenarios is intended to be very limited (approx. 10 scenarios), the methodology is very sensitive to a sampling error and could lead to an instability of the results through years. Scenarios can also be difficult to interpret.

Figure 1 : illustration of the method « pure stochastic trajectories »



Method 2: use of percentiles level lines

In order to solve the issue of the instability of the results and to reduce the dependency of on the random number generator seed, the use of percentile scenarios has been considered.

The methodology consists in generating an important number of scenarios (e.g. 1 000, 10 000 scenarios) with a model such as the one described for the method 1.

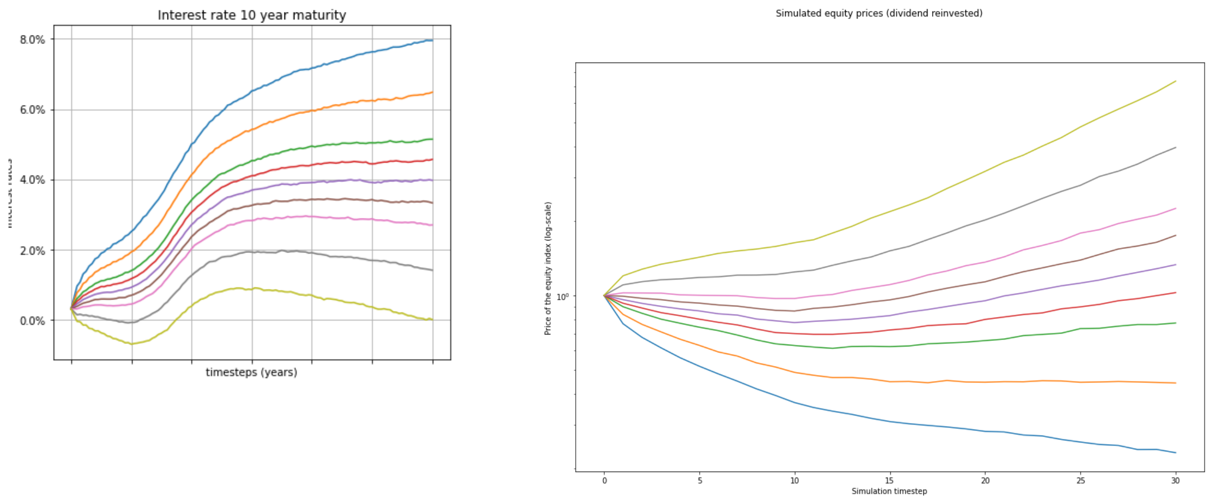
Then, several **percentiles** are defined (e.g. 10%, 20%, 50%, 70%, 90%, …), and the percentile scenarios at each time-step are obtained by selecting the defined percentiles of each risk factor's evolution over the time-step $\left(\left(q_\alpha(RF_i(t))\right)_{i,t}\right)_{\alpha=10\%,\dots,90\%,\dots}$.

The scenarios are continuous increase or decrease in the value of the risk factors, which could be an issue for the equity-like indexes. Indeed, for a given year an increase in 10 % of equities is rather common due to the high volatility of these assets. However, a continuous increase of 10 % each year on a 30 years projection is very unlikely. As the percentile lines methodology consists in taking the percentile of the risk factors (here, the change in market value) independently for each year of simulation, the equity like indexes simulated might be extreme. To cope with this issue, the percentiles have been defined for equity-like indexes as the percentiles of the values of the indexes rather than the percentiles of the capital change.

Pros: This method results in much smoother and stable scenarios than the pure stochastic trajectories. The trajectories are relatively easy to interpret (strong increase in the IR, moderate increase, …).

Cons: The scenarios are continuous increase or decrease in the value of the risk factors: there is therefore no "internal volatility" in one given scenario, which might be an issue for some liabilities.

Figure 2 : illustration of the method « percentile level lines »



<u>Method 3: use of ranked scenarios with conditional expectations and nearest neighbours</u>

The two previous methodologies show significant drawbacks as:

- The raw stochastic scenarios do not provide stable results and in sensitive to sampling error ;
- The use of percentile level lines based on the risk factors treats independently each timestep, resulting in extreme scenarios for equity like indexes ;
- The use of percentile level lines introduce smoothing in the scenarios and therefore reduces the volatility.

Restoring the time coherence of the scenarios while maintaining a certain smoothing of the trajectories can be obtained with an alternative method: the use of ranked scenarios with conditional expectation combined with nearest neighbour research.

In this methodology, the scenarios are not ranked independently for each timestep and risk factor. A reference portfolio is built with a certain proportion w_{bonds} of bonds (assuming an average duration D), equity (w_{equity}), and property ($w_{property}$).

The weights $w_{bonds,equity,property}$ could for instance be based on the EIOPA reference portfolio.

Similarly to the previous method, the input is based on a high number of scenarios produced with Method 1. A reference horizon is then defined, and the value of the portfolio is calculated at this horizon. Then, the scenarios are ranked according to the value of the portfolio for this given time horizon.

Then, several percentiles are defined (e.g. 10%, 20%, 50%, 70%, 90%, …). Simply selecting the scenario which exactly corresponds to each of these quantiles q_α would lead to the same drawbacks as the Method 1. A dependency on the random number seed would indeed remain.

To cope with this issue, it is proposed in this alternative methodology to use conditional expectation, i.e. to define the scenarios as the average scenario that would lead at time T to a value P_α of the reference portfolio that corresponds to the q_α percentile. In practice, this average scenario is based on a window whose size is adjustable. A large window will lead to very smooth scenarios, while a window of size = 1 will allow to pick a single scenario.

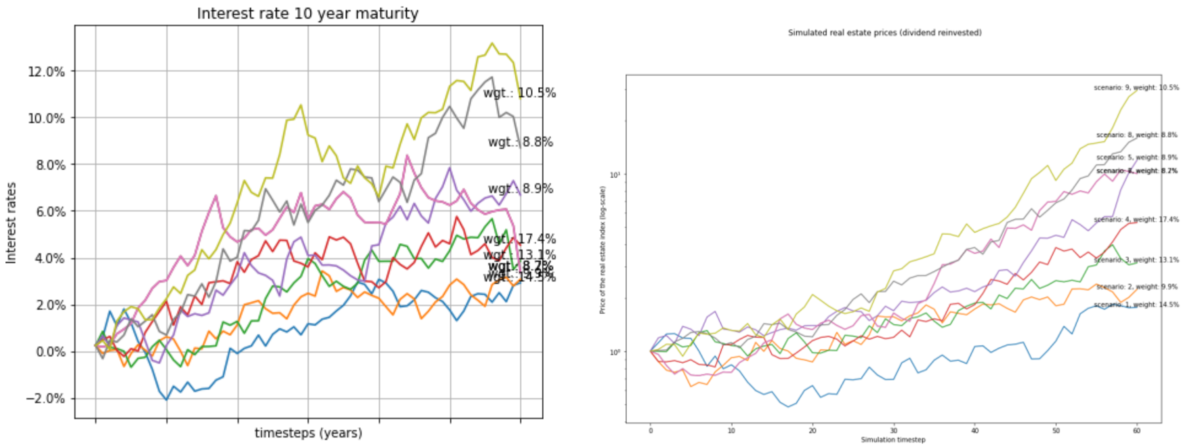
However, at this stage, the scenarios produced by this method can suffer from a strong smoothing effect that reduces the inner volatility of the scenarios. To cope with that issue, a nearest neighbour research is introduced to find in the original large sample of pure stochastic trajectories the scenario that minimizes the distance with the average scenario obtained for a given quantile of the value of the portfolio. The final scenario which is picked by the methodology is therefore directly extracted from the pure stochastic trajectories, but a reduced sampling error and a certain coherence of the trajectory.

This ensures both (i) more explainable scenarios than pure stochastic trajectories, (ii) limitation of the dependency on the random number generator seed (iii) to keep the internal time coherence of the scenarios and the inner volatility.

Pros: This method results in more reliable and representative scenarios than the pure stochastic trajectories as well as reduced sampling error. The trajectories are relatively easy to interpret (strong increase in the IR, moderate increase …). Compared to percentile line as well as ranked scenarios with conditional expectation but without nearest neighbour research, the methodology allows to maintain the volatility through avoiding smoothing effect.

Cons: this method introduces a dependence on a reference portfolio which might not fit all undertakings. The method is still dependent on the random number generator seed.

Figure 3 : illustration of the method « ranked scenarios with conditional expectation » and nearest neighbours



2. Step 2 : Adjustments to be made to the scenarios

The step 1 allows to obtain a sample of scenarios. However, at this stage, these scenarios have no reason for being either risk-neutral nor market consistent.

Several adjustments are therefore proposed to fulfil as much as possible these requirements.

<u>Adjustment A: moment matching</u>

The adjustment A consist in using moment matching techniques to adjust risk factors simulations in order to ensure convergence towards martingale tests targets.

These adjustments are computed step by step (deflators, ZC prices, equity and real estate) on risk factors. These moment matching technique allows to obtain following martingale tests targets:

$$E\big(D(t)\big) = P(0,t)$$

$$E(D(t)P(t,T)) = P(0,T)$$

$$E(D(t)S(t)) = S(0)$$

$$E(D(t)RE(t)) = RE(0)$$

Example on equity risk

The equity martingale test target is defined as (1): $E(D(t)S(t)) = S(0)$

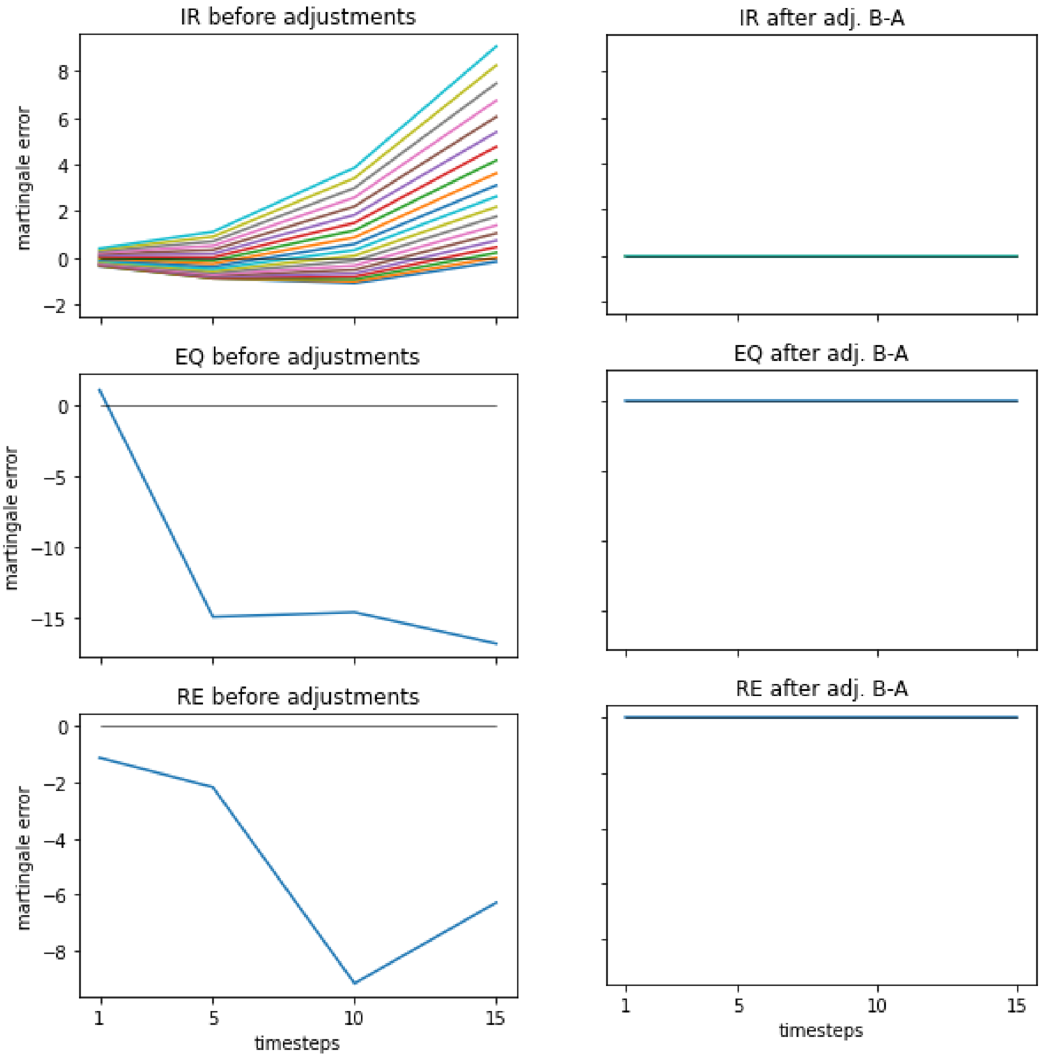
Note $S^{adj}(t)$ the adjusted index defined by the dynamics: $S^{adj}(t) = S^{adj}(\mathrm{t}-1) \times \frac{S^{init}(t)}{S^{init}(t-1)} \times AdjFactor_t$

The formula below allows to estimate the **adjustment factor** so that the equity martingale test (1) is met by S^{adj}:

$$AdjFactor_t = \frac{S(0)}{E\left(D^{adj}(t) \times S^{adj}(\mathrm{t}-1) \times \frac{S^{init}(t)}{S^{init}(t-1)}\right)}$$

Figure 4 : illustration of moment matching adjustment

Martingale test before adjustment are shown on the left, and martingale test after adjustment are shown on the right showing that the test is passed as expected. These results are shown for Interest Rates, Equity, and Real Estate.



<u>Adjustment B: (re)weighting</u>

While the adjustment A allows to force the scenarios martingale properties, it is also necessary to ensure correct market consistent properties. In particular, the volatility of the scenarios included in the PHRSS should not be too low, in order to ensure that the Time Value of Options and Guarantees materiality is correctly estimated.

While a full ESG usually produces scenarios whose probability is uniform (e.g. 1 % probability for each of 100 scenarios), fine tuning the probability of each scenario so that the overall sample has the expected properties can help to ensure the quality of the PHRSS.

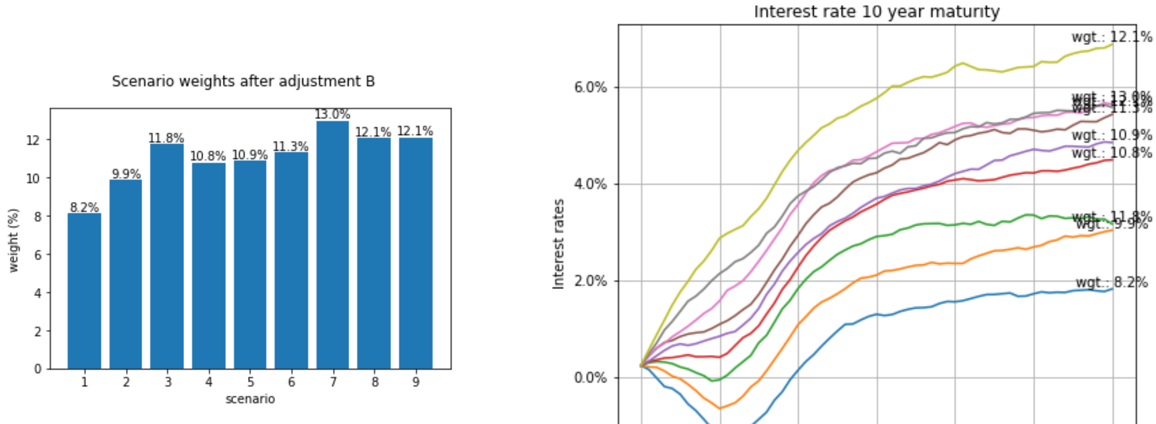
An optimisation algorithm is therefore used to minimize a combination of market consistency error and martingale error, while adding a penalty to ensure that all scenarios are used. Indeed, from a theoretical perspective, the criteria described before can be matched with only two scenarios. However, for the robustness of the assessment of the materiality of the TVOG, it is preferable to have as many scenarios as possible. The variety of the options and guarantees embedded in the liabilities indeed is indeed wider than the financial derivatives used to assess the market consistent criteria. The volatility assumptions used for the market consistency assessment are described in section 3.

The optimal weights $p^{\star}$ are therefore defined as follows:

$$(p_1^*, \dots, p_N^*) = \underset{(p_1,\dots,p_N)}{\operatorname{ArgMin}} \left\{ w_1 \sum_{c \in C} \left(\sum_{s=1}^{N} p_s CF_s^c - MarketPrice_c \right)^2 + w_2 \sum_t \left(E\left(\widehat{D(t)}\right) - P(0,t) \right)^2 \right.$$
$$+ w_3 \sum_t \left(E(\widehat{D(t)S}(t)) - S(0) \right)^2 + w_4 \sum_t \left(E(\widehat{D(t)RE}(t)) - RE(0) \right)^2$$
$$\left. + w_5 \sum_s \frac{1}{p_s + \delta} \right\}.$$

With $\delta \ll 1$ and $p_1, \dots, p_n$ constrained so that they are positive and their sum equal to 1.

Figure 5 : illustration of the weights of the scenarios



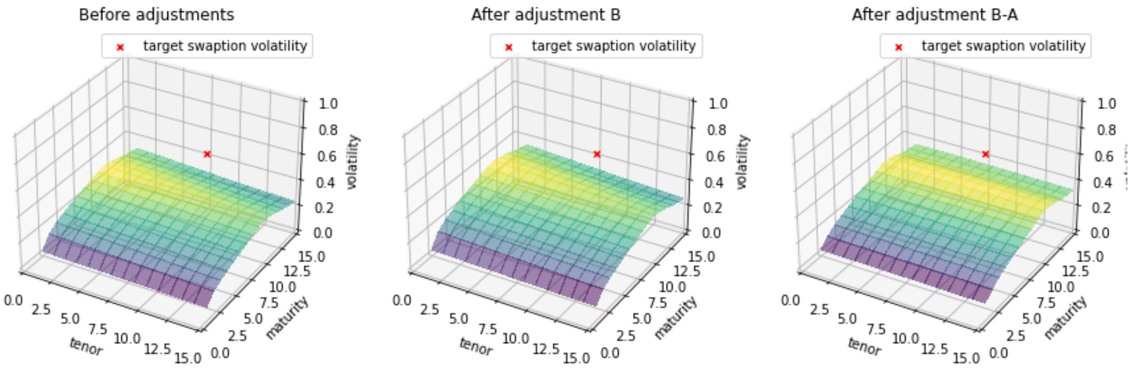
	0	1	2	3	4	5	6	7	8
Weight (%)	8.2	9.9	11.8	10.8	10.9	11.3	13.0	12.1	12.1

Combination of adjustments

The adjustments A and B are then intended to be combined to match as much as possible the targets in terms of martingale properties and market consistency.

In this case, even if the volatility must reflected as much as possible, it is essential that the martingale test is passed. To ensure this, an adjustment « A » must be performed at the end of the process, after the weights of the scenarios are calculated. This adjustment might slightly change the simulated volatility.

Figure 6 : illustration of the impact of adjustments on the swaption implied volatilities surface



3. Market data used to calibrate the model

Market consistent valuation as expected by the article 22§3 of the Delegated Regulation usually requires the use of deep, liquid, and transparent market data. Three kinds of market data are usually used to calibrate an ESGs:

- The EIOPA risk free rate, which is accessible on the EIOPA website and updated every month
- Implied volatilities or derivatives prices so that the scenario volatility reflects the expectations of the markets at the time of valuation
- Where implied volatilities are not available due to the absence of DLT data, historical volatilities are often used.

To calibrate the PHRSS, two options are possible:

Option 1: use of market data. This would however require EIOPA to buy market data to external providers with the associated cost. Besides, replacement hypotheses would need to be used for risk factors and currencies that do not offer DLT market data.

Option 2: use of real world hypotheses based on the standard formula stresses.

As the PHRSS is intended to provide a materiality assessment of the TVOG, it might not be necessary to perfectly match the criterions of a fully economic valuation of the balance sheet. In practice, the stresses of the standard formula can be inverted to obtain real-world "implied volatilities" to calibrate the models.

While increasing transparency and providing a simple proportionality solution, this option would also have the merit to (1) show clearly that the PHRSS is not intended to replace a real stochastic valuation for undertakings with material options and guarantees (2) avoid any market data licencing fees (3) ensure a certain stability of the PHRSS assessment across time.

The formulas that follow provide the estimation of volatilities needed for the calibration of the PHRSS for Interest Rates, Equity and Real Estate. Similar approaches could be developed for other risk factors such as inflation.

IR volatility

For IR we consider an absolute shock equal to 1% (level of upward shock observed in last closing exercises). The volatility parameter is calculated by solving following equation:

$$q_{99.5\%}(\sigma_{IR}.\varepsilon^{IR}) = 1\%$$

We obtain:

$$\sigma_{IR} = \frac{1\%}{q_{99.5\%}(\varepsilon^{IR})}$$

$$\sigma_{IR} = 0.39\%$$

Note: this volatility depends on the level of interest rates

Equity (EQ) volatility
To derive EQ volatility we consider a S2 shock equal to 39%. By neglecting IR drift, we get the equation below:

$$e^{-0.5\sigma_{EQ}{}^2+\sigma_{EQ} \cdot q_{0.5\%}(N(0,1))} = 1 - 39\%$$

EQ volatility parameter is determined as a solution of this equation. We obtain: $\sigma_{EQ} = 19\%$.

Real Estate (RE) volatility
To derive RE volatility we consider a S2 shock equal to 25%. By neglecting IR drift, we get the equation below:

$$e^{-0.5\sigma_{RE}{}^2+\sigma_{RE} \cdot q_{0.5\%}(N(0,1))} = 1 - 25\%$$

RE volatility parameter is determined as a solution of this equation. We obtain: $\sigma_{RE} = 11\%$

Note: the volatilities obtained for EQ and RE are actually close to market-implied volatilities used by undertakings in their ESGs.

Option to be used for the information request:

At this stage, and in order to avoid market data licencing issues, it is proposed to use the option 2 (real world volatilities).

4. Example of the use of the PHRSS

Let's consider a very simple policy with the following features:

- Single premium paid upfront = 100 €
- Minimum guaranteed rate : i = 0.2 %
- Term : T = 10 years

Define MV_t as the market value of the assets held by the undertaking at time t. The policy offers a profit-sharing mechanism with a 80 % profit sharing rate. At the term T of the policy, the contract offers the maximum between the single premium capitalized with the MGR and the fraction of the gain on assets made by the insurer.

At maturity, the benefits of the policy and the cash flows arising from the contract can be calculated as follows:

$$CF_T = \max\left(100\text{ €} \times (1+\mathrm{i})^T; 80\% \times (MV_T - 100\text{ €})\right)$$

The assets are invested on the following asset mix: 75 % bonds with 10 years maturity, 5 % cash, 10 % equity, and 10 % real estate.

The Best Estimate of the policy is calculated as the discounted weighted average of CF_T.

$$BE = \sum w_k D(k,T) \times CF(k,T)$$

Where $CF(k,T)$ is the cash flow of the policy in the scenario k at time T, $D(k,t)$ de deflator, and w_k the weight of the scenario.

	Deterministic with central equivalent scenario	**Method 1: pure stochastic trajectories**	**Method 2: Percentile line scenarios**	**Method 3: Ranked Scenarios with conditional expectation**
Best Estimate	99.6	102,3	102,3	102,3
Value of Inforce	0.4	-2,3	-2,3	-2,3
TVOG	0	2,6	2,6	2,6

The following table provides the results for a different random number generator initial seed.

	Deterministic with central equivalent scenario	**Method 1: pure stochastic trajectories**	**Method 2: Percentile line scenarios**	**Method 3: Ranked Scenarios with conditional expectation**
Best Estimate	99.6	101,7	102,3	102,4
Value of Inforce	0.4	-1,7	-2,3	-2,4
TVOG	0	2,1	2,6	2,8

5. Description of the scenarios of the information request

The table below describes the different scenario sets that are used in the first information request. Although a larger number of scenario sets would be needed in order to test all possible combinations of options, it is proposed to restrict the number of scenario sets so as to limit the information request burden while still exploring options deemed material. Indeed, the choice of the simulation seed for method #2 is not anticipated to be a material option, while the materiality of the percentile list is not anticipated to be different for method #3 and for method #2. These considerations lead to proposing the following list of scenario sets.

	Step 1 (scenarios)	**Step 2 (adjustments)**	**Market Data**	**Specificity**
Scenario set 1	Method #1	B + A	Real world S2 SF volatilities	Seed s_1
Scenario set 2	Method #1			Seed s_2
Scenario set 3	Method #2			Percentile list A
Scenario set 4	Method #2			Percentile list B
Scenario set 5	Method #3			Seed s_1
Scenario set 6	Method #3			Seed s_2