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CEIOPS' Advice for Level 2 Implementing Measures on Solvency II:

SCR Standard Formula

Calibration of Non-life Underwriting Risk

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1 Executive summary

1.1 This paper provides the proposed calibration of the non-life underwriting risk module (premium and reserve sub-modules) in accordance with the requirements of Article 104 of the Level 1 text[1]. This is the revised version of CP71, following stakeholder feedback from consultation and further collection of data.

1.2 The Commission has requested CEIOPS to base its recommendations on evidence from as wide a range of Member States and types of undertakings within the EEA as possible. CEIOPS members have provided a wider range of data than was available for the QIS3, QIS4 and previous CP71 analysis exercises. However this data was mainly gross of reinsurance, with a more limited coverage of net of reinsurance data. Consequently CEIOPS decided to perform the main analysis using exclusively gross of reinsurance data, and has produced separate recommendations on how to obtain appropriate net factors to use in the SCR standard formula.

Data

1.3 The data used for this exercise comes from fifteen Member States. This represents a significant improvement compared to previous calibration exercises undertaken by CEIOPS. Only six Member States provided data for the previous CP71 analysis, and only three for QIS3 and QIS4.

1.4 The data was judgementally filtered to remove, to the best possible extent as best as possible:

- Distortions due to mergers and acquisitions
- Typographic mistakes
- Apparent inconsistencies between different years and between opening reserve and closing reserve for the same company
- Catastrophe losses
- As well as other features which were considered to be incorrect based on expert judgement.

1.5 Data available for some lines of business was still limited despite collecting further data. The analysis produced for these lines of business is thus naturally not as robust as that for lines of business with more data.

[1] Article 104 of the Level 1 text states that each of the risk modules referred to in paragraph 1 shall be calibrated using a Value at Risk measure with a 99.5% confidence level, over a one year period

Assumptions

1.6 CEIOPS has performed the analysis in line with the requirements underlying the design of the standard formula, such as:

- Provide an estimate for a set of factors which are pan-European
- Allowance has been made for an average level of geographical diversification, as implied by the data.
- No allowance for underwriting cycle
- No allowance for expected profits and losses
- No allowance for a size factor ie diversification by volume. This has the implication that the proposed calibration may overestimate for large portfolios and underestimate for small portfolios.

1.7 In addition:

- No explicit allowance has been made for inflation in the calibration process. Implicitly therefore it assumed that the inflationary experience in the period 1999 to 2008 was representative of the inflation that might occur in the future. The period analysed was a relatively benign period with low inflation in the countries supplying data and without unexpected inflation shocks which might be expected to increase the factors.
- When assessing the capital requirements under the standard formula approach, the impact on the net asset value (difference between asset market value and insurance liabilities) is assessed under the assumption that the risk margin does not change after the stress.

1.8 CEIOPS would like to highlight that any changes made to the assumptions underlying the design of the standard formula would require a recalibration of the proposed factors.

Methodologies

1.9 A range of methodologies was used to test different sets of assumptions and goodness of fit. The methods used were based on sound statistical analysis and (some) were based on published actuarial papers. Some of the methods are directly comparable to the methods used under QIS 4.

1.10 A variety of methods was used to estimate the factors across all undertakings and Member States for each line of business. However results vary across methods because each method uses different underlying assumptions. For example:

- Some methods will place more weight on volatilities estimated for larger companies which tend to have lower standard deviations thus producing a lower overall result.
- Other methods will give an equal weight to each undertaking and as a result will tend to produce a higher overall result.

Results

1.11 The final gross technical fitted result across all methods was derived by taking an average of the methods that best fit the data. CEIOPS would like to highlight that the selection was not conservatively selected, but rather based on the goodness of fit results and the adequacy of the method. Furthermore by taking an average, CEIOPS is ensuring that the factors are not biased towards factors most appropriate for larger portfolios (and hence lower). The analysis shows that for most lines of business the factors should be higher for smaller and medium portfolios.

1.12 In line with industry comments, CEIOPS has recommended an adjustment factor for Premium Risk that is undertaking specific, and so it is not possible to provide a net premium factor. For reserve risk, CEIOPS used the net data available from Member States to estimate an adjustment to the gross estimate.

1.13 To get a further insight and consider other information available, CEIOPS supplemented the above analysis with additional exercises provided by CEIOPS or the industry (see section 4.5). These additional exercises also suggest that factors proposed for QIS4 may not be appropriate at least for some lines of business.

1.14 Having considered the results from the technical analysis along with these other analyses and wider considerations, CEIOPS recommends that the factors for the premium and reserve risk sub modules should be as follows:

LOB	Net premium factor[2]	Net reserve factor
Motor vehicle liability	11.5%*(NCR_i/GCR_i)	9.5%
Motor Other	8.5%*(NCR_i/GCR_i)	12.5%
MAT	23%*(NCR_i/GCR_i)	17.5%

[2] CEIOPS has recommended an adjustment factor for Premium Risk that is undertaking specific, and so it is not possible to provide a net premium factor. NCR and GCR stand for net combined ratio and gross combined ratio respectively

Fire and Other damage	15%*(NCR_i/GCR_i)	12%
Third party liability	17%*(NCR_i/GCR_i)	16%
Credit and suretyship	28%*(NCR_i/GCR_i)	25%
Legal expenses	8%*(NCR_i/GCR_i)	9%
Assistance	5%*(NCR_i/GCR_i)	12.5%
Miscellaneous	15.5%*(NCR_i/GCR_i)	20%
NPL Property	20%*(NCR_i/GCR_i)	25.5%
NPL Casualty	18.5%*(NCR_i/GCR_i)	25%
NPL MAT	16.5%*(NCR_i/GCR_i)	25%

Observations

1.15 Throughout this document, CEIOPS has endeavoured to show transparency in the process it has followed as far as possible.

1.16 Finally CEIOPS recognises that as the Standard Formula is intended to be pan-European, it is not possible to select a factor that fits all portfolio specificities and works perfectly for all undertakings operating in the EEA. The Solvency 2 framework provides a wide range of approaches for an undertaking to determine its SCR. Undertakings that consider that some or all of the standard parameters within the Standard Formula do not appropriately reflect their risk profile, may wish to consider using undertaking specific parameters or applying for the approval of a (partial) internal model.

2. Introduction

2.1. In its letter of 19 July 2007, the European Commission requested CEIOPS to provide final, fully consulted advice on Level 2 implementing measures by October 2009 and recommended CEIOPS to develop Level 3 guidance on certain areas to foster supervisory convergence. On 12 June 2009 the European Commission sent a letter with further guidance regarding the Solvency II project, including the list of implementing measures and timetable until implementation.[3]

2.2. This paper aims at providing advice with regard the calibration of the non-life underwriting risk module as requested in Article 109(b) of the Solvency II Level 1 text. [4]

2.3. This paper covers advice in respect of:

a. The calibration of premium and reserve risk sub module of the non-life underwriting risk module. In particular a description of the data, analysis, assumptions and methodology used to calibrate the standard deviations required for the calculation of the risk sub module.

b. The calibration of the factor required under the "Factor Method" for the catastrophe risk sub-module

2.4. This advice does not include details of the calibration of:

c. The catastrophe risk Standardised Scenarios required under the non-life catastrophe risk sub-module. This advice shall be provided for June 2010.

d. The calibration of the non-SLT classes (sickness, accident and Workers' compensation) under the standard formula SCR Standard Formula Health Underwriting module. For details please refer to revised CP72.

2.5. This advice should be read in conjunction with CEIOPS final advice – CEIOPS-DOC-41-09 : SCR Standard Formula - Article 111 Non-Life Underwriting Risk.

2.6. The term undertaking relates to both insurance and reinsurance undertakings and "LoB" refers to line of business unless otherwise explicitly mentioned.

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[3] See http://www.ceiops.eu/content/view/5/5/

[4] Text adopted by the European Parliament on 22 April 2009, see http://www.europarl.europa.eu/sides/getDoc.do?pubRef=-//EP//NONSGML+TA+20090422+SIT-03+DOC+WORD+V0//EN&language=EN

3. Extract from the Level 1 text

Legal basis for implementing measure

3.1. According to the guiding principles referred to in the Commission's letter, the legal basis for the advice presented in this paper is primarily found in Article 109 of the Level 1 text, which states:

Article 109 – Implementing measures

In order to ensure that the same treatment is applied to all insurance and reinsurance undertakings calculating the Solvency Capital Requirement on the basis of the standard formula, or to take account of market developments, the Commission shall adopt implementing measures laying down the following:

(a) a standard formula in accordance with the provisions of Articles 101 and 103 to 108;

(b) any sub-modules necessary or covering more precisely the risks which fall under the respective risk modules referred to in Article 104 as well as any subsequent updates;

(c) the methods, assumptions and standard parameters to be used, when calculating each of the risk modules or sub-modules of the Basic Solvency Capital Requirement laid down in Articles 104, 105 and 105bis, as well as the adjustment mechanism referred to in Article 105ter;

[…]
(k) the simplified calculations provided for specific sub-modules and risk modules, as well as the criteria that insurance and reinsurance undertakings, including captive insurance and reinsurance undertakings, shall be required to meet in order to be entitled to use each of these simplifications, as set out in Article 108;

[…]

Other relevant Articles for providing background to the advice

Article 101 Calculation of the Solvency Capital Requirement

1. *The Solvency Capital Requirement shall be calculated in accordance with paragraphs 2 to 5:*

2 *The Solvency Capital Requirement shall be calculated on the presumption that the undertaking will carry on its business as a going concern.*

3. *The Solvency Capital Requirement shall be calibrated so as to ensure that all quantifiable risks to which an insurance or reinsurance undertaking is exposed are taken into account. It shall cover existing business, as well as* *the new business expected to be written over the next twelve months. With respect to existing business, it shall cover unexpected losses only.*

It shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period.

4. The Solvency Capital Requirement shall cover at least the following risks:

(a) non-life underwriting risk;

[…]

5 When calculating the Solvency Capital Requirement, insurance and reinsurance undertakings shall take account of the effect of risk mitigation techniques, provided that credit risk and other risks arising from the use of such techniques are properly reflected in the Solvency Capital Requirement.

Article 104 - Design of the Basic Solvency Capital Requirement

1. *The Basic Solvency Capital Requirement shall comprise individual risk modules, which are aggregated in accordance with point 1 of Annex IV.*

It shall consist of at least the following risk modules:

(a) non-life underwriting risk;

(b) life underwriting risk;

(c) health underwriting risk;

(d) market risk,

(e) counterparty default risk.

[…]

6. With regard to risks arising from catastrophes, geographical specifications may, where appropriate, be used for the calculation of the life, non-life and health underwriting risk modules.

Article 105 - Calculation of the Solvency Capital Requirement

The non-life underwriting risk module shall reflect the risk arising from non-life insurance obligations, in relation to the perils covered and the processes used in the conduct of business.

It shall take account of the uncertainty in the results of insurance and reinsurance undertakings related to the existing insurance and reinsurance obligations as well as to the new business expected to be written over the next twelve months.

It shall be calculated, in accordance with point 2 of Annex IV, as a combination of the capital requirements for at least the following sub-modules:

(a) the risk of loss, or of adverse change in the value of insurance liabilities, resulting from fluctuations in the timing, frequency and severity of insured events, and in the timing and amount of claim settlements (non-life premium and reserve risk);

(b) the risk of loss, or of adverse change in the value of insurance liabilities, resulting from significant uncertainty of pricing and provisioning assumptions related to extreme or exceptional events (non-life catastrophe risk).

[…]

4. Premium and reserve risk calibration

4.1. CEIOPS points out that the calibration in this advice is being considered to be in line with 99.5% VaR and a one year time horizon. QIS5 will give an indication of the overall impact of the proposed calibrations, not limited to the SCR but including technical provisions and own funds.

4.2. CEIOPS' advice on non-life underwriting risk (CEIOPS-DOC-41-09), provides advice in respect of the design of the non life underwriting risk module, in particular the methods, assumptions and standard parameters to be used when calculating this risk module.

4.3. Overall, the premium and reserve risk capital charge is determined as follows:

$$NL_{pr} = \rho(\sigma) \bullet V$$

where

V	=	Volume measure
σ	=	combined net standard deviation, resulting from the combination of the reserve and premium risk standard deviations
$\rho(\sigma)$	=	A function of the standard deviation

4.4. The overall volume measure V is determined as follows:

$$V = \sum_{Lob} V_{lob}$$

where, for each individual line of business LoB, V_{lob} is the volume measure for premium and reserve risk:

$$V_{lob} = V_{(prem,lob)} + V_{(res,lob)}$$

4.5. The function $\rho(\sigma)$ is specified as follows:

$$\rho(\sigma) = \frac{\exp(N_{0.995} \bullet \sqrt{\log(\sigma^2 + 1)})}{\sqrt{\sigma^2 + 1}} - 1$$

where

$N_{0.995}$	=	99.5% quantile of the standard normal distribution

4.6. The function $\rho(\sigma)$ is set such that, assuming a lognormal distribution of the underlying risk, a risk capital charge consistent with the VaR 99.5% standard is produced. Roughly, $\rho(\sigma) \approx 3 \bullet \sigma$.

4.7. The overall net standard deviation σ is determined as follows:

$$\sigma = \sqrt{\frac{1}{V^2} \cdot \sum_{rxc} CorrLob_{r,c} \cdot \sigma_r \cdot \sigma_c \cdot V_r \cdot V_c}$$

where

r,c	=	All indices of the form (lob)
$CorrLob^{rxc}$	=	the cells of the correlation matrix *CorrLob*
V_r, V_c	=	Volume measures for the individual lines of business, as defined above

4.8. In order to estimate the capital charge for the Non life premium and reserve risk submodule, CEIOPS needs to provide calibrated factors for the following inputs:

- Net standard deviation for premium risk σ(prem,LoB)
- Net standard deviation for reserve risk σ(res,LoB)
- correlation factors between LoB (CorrLob)

4.9. The corresponding LoBs shall be:

LoB number	
1	Motor, vehicle liability
2	Motor, other classes
3	Marine, aviation, transport (MAT)
4	Fire and other property damage
5	Third-party liability
6	Credit and suretyship
7	Legal expenses
8	Assistance
9	Miscellaneous
10	Non-proportional reinsurance – property
11	Non-proportional reinsurance – casualty
12	Non-proportional reinsurance – MAT

4.1 General Observations

QIS 3 and QIS 4 calibration

4.10. During the CP71 consultation, stakeholders emphasized that the parameters provided by CEIOPS deviated significantly from previous exercises and that QIS 4 was a better benchmark.

4.11. CEIOPS would like take this opportunity to provide some background in respect of QIS 4 and QIS 3 as well as to highlight the main differences between the current and previous analyses.

4.12. CEIOPS provided the first non-life calibration paper as part of QIS 3 (CEIOPS- FS-14/07). The calibration was carried out with German data for premium risk, some UK and German data for reserve risk and French data for the health segments. The exercise was carried out on a best efforts basis with the very limited data set available at the time and working under the assumption that the application of the above approach would be suitable for premium and reserve risk. The document presented a simple approach regarding fitting underwriting risk.

4.13. CEIOPS also provided a calibration for the QIS 4 exercise which was presented in the QIS 4 Technical Specifications which made some adjustments to the results of the QIS 3 calibration.

4.14. CEIOPS has worked on the basis that it is able to refine calibrations as and when data becomes available. For example the following note was attached to TS.XIII.B.25 in the QIS4 Technical Specifications (MARKT/2505/08):

"Please note that the proposed calibration for the "reserve risk" standard deviations is tentative and has been developed for QIS4 purposes only. It is recommended that further work should be carried out in order to refine this calibration by dedicating a specific workstream to this issue."

4.15. During June to September 2009 CEIOPS decided to carry out a full calibration exercise using data which was representative of EEA, fully laying out assumptions, applying a range of methods and carrying out goodness of fit tests. CP 71 was the result of this work.

4.16. During CP71 and the current revised version, it was acknowledged that there were various issues in respect of previous calibrations:

Data Applicability for the whole of the EEA

4.17. The previous calibrations were performed using data from an unrepresentatively small set of member states within the EEA.

4.18. Whilst the introduction of more data leads to heterogeneity calibration problems, the resultant parameters should be more appropriate for more undertakings within the EEA.

4.19. CEIOPS have included Method 1 in CP 71 (for both premium risk and reserve risk) as this is the closest of all the methods presented to the approach used in the earlier calibrations. This has been adjusted to allow for some of the issues identified, but clearly still has some of the same limitations. As can also be seen in CP 71, this method also tends to give the lowest calibrations, as expected from the issues identified.

Relationship between volatility and volume measure

4.20. CP 71 identifies a clear relationship between the level of volatility of the undertaking and its associated volume measure. Namely that, in general, the larger the undertaking's volume the smaller the associated undertaking standard deviation.

4.21. The approach used in historic calibrations to derive a single factor from the company specific estimates of volatility placed a significant weight (the volume measure squared) upon the volatilities from the larger firms, with the smallest volatilities. This has the effect of materially understating the resultant fitted volatility in relation to the underlying firms.

Fitting Algorithm

4.22. The previous calibrations used a single fitting approach. Different fitting approaches for the same model and data can give materially different answers, especially in the circumstances where there is a finite amount of data.

4.23. This issue was not explored in the previous calibrations and could have resulted in a misinterpretation of the certainty of the resultant calibration.

4.24. The fitting algorithm used was the least squares approach which is most usually regarded as appropriate when the underlying distribution is a Normal distribution – when the least squares estimator is the same as the maximum likelihood estimator. The distributional assumptions in the standard formula are LogNormal, as would be considered more appropriate for the right skewed nature of claims development.

Model Assumptions

4.25. The approach used a single set of model assumptions. Different, but similar, model assumptions fitted to the same data can give materially different answers.

4.26. This issue was not explored in the previous calibrations and could have resulted in a misinterpretation of the certainty of the resultant calibration.

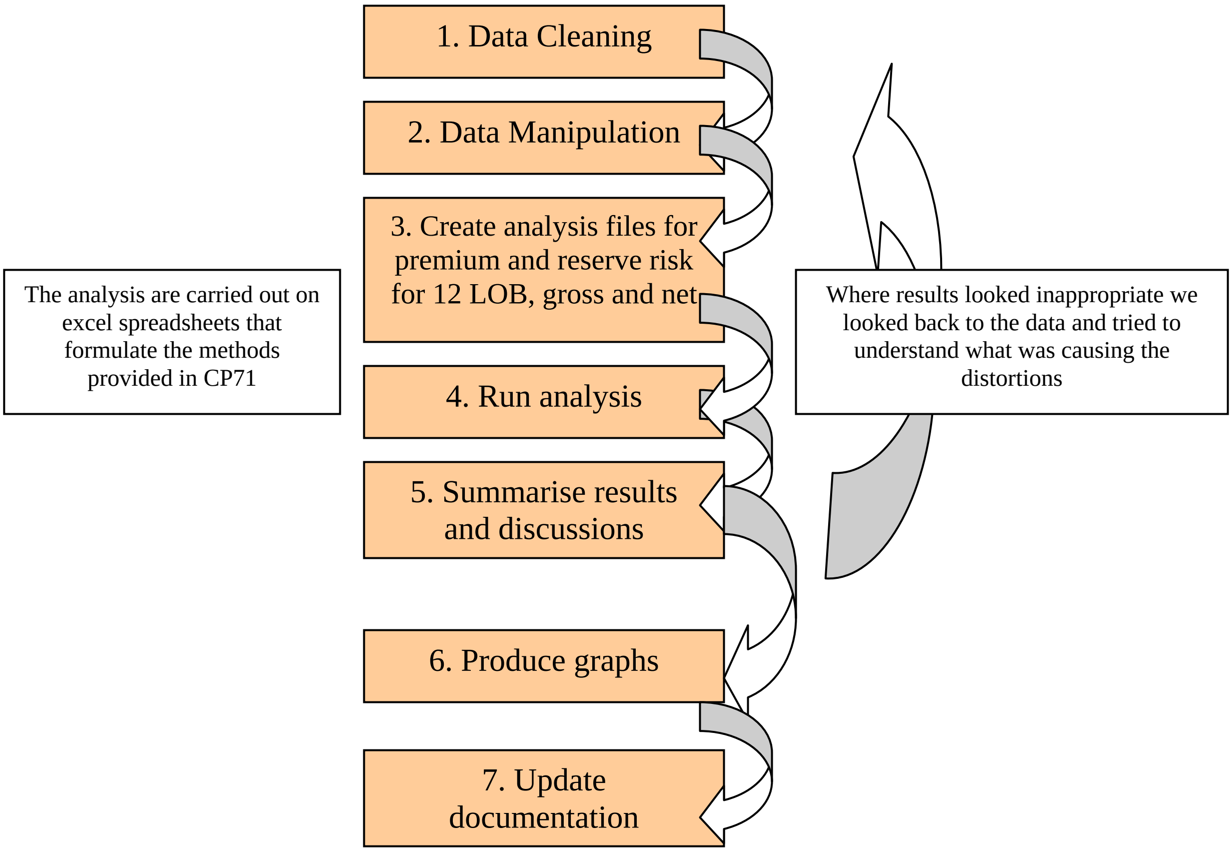
Over-fitting

4.27. The previous calibrations estimated standard deviations by undertaking. With regards to premium risk this also involved an estimation of the mean loss ration by company.

4.28. This involves estimating a wide variety of parameters in order to derive, in the end, the single parameter. The effect of this is to over-fit the model and understate the resultant market volatility.

Process followed for non life calibration

4.29. This section provides some general information regarding the process followed:



- There are four examples in Annex 7.2 which provide in depth detail regarding the process followed for all LOBs.
- Data:
    - o The data used for the analysis relates to the period from 1999 to 2008.
    - o Only a limited amount of data was available net of reinsurance. As a result CEIOPS based the analysis on gross of reinsurance data, and this is also consistent with the industry feedback. If CEIOPS had done the analysis based on the net data, the results would have only been representative of 5 member states. Section 7.2 of the Annex provides a list of the countries that provided data by LoB gross and net of reinsurance compared to the first version on CP71..

- There were issues around confidentiality which required standardisation of the data, as explained in the Data Request in the section 7.1 of the Annex. In order to use the standardised data CEIOPS had to unstandardise it making some broad assumptions regarding the size of the firms. In general this should have had little impact upon the calibration. However, there were some occurrences where companies were growing very quickly where the resultant gearing of the broad assumptions led to infeasible data and such companies had to be excluded from the analysis to avoid any material distortions in the overall calibration.
- Diversity of data from different member states as a result of different regulatory systems or accounting regimes.
- The historic posted reserves are on an undiscounted best estimate basis rather than discounted best estimate basis.
- The level of prudence embedded in the historic posted reserves is different among different undertakings (even undertakings from the same member state).
- Catastrophe double counting. The industry was concerned about the impact of including catastrophe data within the analysis. CEIOPS has attempted to remove catastrophe claims where possible. Furthermore CEIOPS has requested from member states that data should be clean of catastrophes. CEIOPS has further carried out a filtering process to remove observations that could suggest being related to a catastrophe event.
- Historic premium provisions as defined under Solvency 2 are not necessarily readily available. Only data on an accident year basis was available. Therefore given that there is a potential for deterioration in the premium provision (although this would be much smaller than the associated earned exposure) over the one year time horizon, but premium provision is not included in the volume measure, the premium risk calibration will be slightly understated.
- There are no risk margins in the data. The calibration should cover the change in risk margin over the year. However for the purpose of this calibration CEIOPS has assumed the risk margin does not change. This will lead to understanding the factors.

- Adjustment to net:
  - Gross volatilities will need to be adjusted to allow for reinsurance before they can be used in the Standard Formula. For premium risk CEIOPS has proposed to use an approach based on the experience of individual undertakings, as this will allow for the particular features of their reinsurance protections. This is covered in section 4.2.5 below. For reserve risk, CEIOPS has proposed to use a more general industry wide adjustment factor, which is explained in section 4.3.5 below.

4.2 Premium risk

4.30. This section describes the premium risk calibration and results.

4.2.1. Data

4.31. By line of business, undertaking and accident year:

- Earned premium net of reinsurance costs, but gross of acquisition costs
- Posted ultimate claims after one year gross of reinsurance recoveries, comprising the claims paid over the year and the posted outstanding claims provision posted after the one year gross of expected reinsurance recoveries.
- Paid claims triangle gross of reinsurance recoveries

4.32. These data are judgementally filtered to remove problem data points:

- Distortions due to mergers and acquisitions
- Typographic mistakes
- Apparent inconsistencies between different years and between opening reserve and closing reserve for the same company
- Catastrophe losses
- As well as other features which were considered to be incorrect based on expert judgement.

4.33. See annex 7.3 for an illustration of the process followed.

4.2.2. Assumptions

4.34. For practical reasons net earned premium is used as the volume measure in the calibration (as opposed the maximum of net earned premium, net written premium, etc as in the standard formula).

4.35. The calibration is based on the assumption that the expenses (excluding allocated claims handling expenses) are a deterministic percentage of premium and hence do not affect the volatility of the result. The largest component of these expenses is likely to be the acquisition expenses and this assumption would appear to be relatively reasonable in these circumstances.

4.36. No explicit allowance was made for inflation in the calibration process. Implicitly therefore it assumed that the inflationary experience in the period 1999 to 2008 was representative of the inflation that might occur. The period analysed was a relatively benign period with low inflation in the countries supplying data and without unexpected inflation shocks which would be expected to increase the factors significantly. Thus as the data excludes significant inflationary shocks, it may underestimate the uncertainty in the provisions.

4.37. An average level of geographical diversification is implicitly allowed for in the calibration because the volatility of the undertaking's time series reflects the geographical diversification of their business.

4.38. The risk margin does not change after stressed conditions.

The SCR is the difference between the economic balance sheets over the one year time horizon in the distressed scenario. This implicitly suggests the difference between all component parts should be analysed which includes the risk margin. CEIOPS has assumed for the purpose of the standard formula that there is no change in the risk margin.

4.2.3. Analysis

4.39. The analysis is performed using the net earned premiums as the volume measure and the net posted ultimate claims after one year to derive a standard deviation.

4.40. This figure is then adjusted to allow for the effect of discounting. These adjustments are applied on a bulk basis, ie not on a company by company basis, to ensure that the resultant calculations are manageable.

4.41. The adjustment for discounting involves projecting the aggregate triangle of paid claims (summed across undertakings) to derive a payment profile for the claims. It is assumed that the claims are paid in the middle of the corresponding year and use a discount rate of 4% to derive a resultant overall discount factor that we could apply to the posted ultimate in one year's time to discount to today's money. This adjustment is applied on a bulk basis, ie not on an undertaking by undertaking basis, for reasons of practicability.

4.42. The constant discount rate is used to avoid double counting the risk of the effect of changing yield curves which is covered within market risk in the standard formula.

4.43. The level of the discount rate is chosen judgementally. The rate of 4% is not intended to reflect current risk-free rates but rather a long-time average of risk-free rates.

4.2.4. Methodology

4.44. A variety of methods was used to estimate the factors a set of pan European factor for each line of business.

4.45. CEIOPS carried out the following methods for the estimation of the premium risk standard deviations:

Method 1

4.46. This approach is intended to follow as closely as possible the approach detailed in “CEIOPS- FS-14/07 QIS3, Calibration of the underwriting risk, market risk and MCR”.

4.47. This involves the firm calculating the average net earned premium and the standard deviation of the loss ratios posted after the first development year.

4.48. The process involves two stages. The first stage fits a separate model of each undertaking’s mean and standard deviations of loss ratio and allows for more diversification credit within larger volumes of earned premium per line of business in the same way across all years within a single undertaking.

4.49. This stage uses a least squares fit of the loss ratio and an associated variance estimator. This estimator is optimal when the underlying distribution is Normal, as opposed to the assumptions within the standard formula of Log Normality.

4.50. The second stage fits the premium risk factor to these resultant undertaking specific models.

4.51. The use of a two stage process, clearly introduces a large number of parameters that need to be calibrated which translates to a significant risk of over-fitting. The effect of this would be to understate the resultant premium risk factor, but it is not entirely clear by how much.

4.52. Furthermore, the second stage puts significantly more weight to those undertakings which write larger volumes of a specific line of business, therefore any result will be biased towards factors most appropriate for larger portfolios.

4.53. Specifically if the following terms are defined:

$U_{C,Y,lob}$	=	The posted ultimate after one year by undertaking, accident year and LoB
$V_{C,Y,lob}$	=	Earned premium by undertaking, accident year and LoB
$\sigma_{C,lob}$	=	Standard deviation of loss ratio by undertaking and LoB
$N_{C,lob}$	=	The number of years of data available by undertaking and LoB
$V_{C,lob}$	=	Average earned premium by undertaking and LoB

4.54. The following relationships are obtained:

$$\sigma_{C,lob} = \sqrt{\frac{1}{V_{C,lob}}} \sqrt{\frac{1}{N_{C,lob} - 1} \left(\sum_{Y} \frac{1}{V_{C,Y,lob}} \left(U_{C,Y,lob} - V_{C,Y,lob} \sum_{Y} \frac{U_{C,Y,lob}}{V_{C,Y,lob}} \right)^2 \right)}$$ and

$$V_{C,lob} = \frac{1}{N_{C,lob}} \sum_{Y} V_{C,Y,lob}$$

4.55. The factors are then determined using least squares optimisation across the undertakings within the LoB.

4.56. If following term is defined:

$\sigma_{(prem,lob)}$	=	Standard deviation for premium risk by LoB

4.57. Then a value for $\sigma_{(prem,lob)}$ can be derived by taking a volume weighted average of the fitted undertaking specific standard deviations as below:

$$\hat{\sigma}_{(prem,lob)} = \frac{\sum_{C} V_{C,lob}\sigma_{C,lob}}{\sum_{C} V_{C,lob}}$$

Method 2

4.58. This approach is consistent with the undertaking specific estimate assumptions from the Technical Specifications for QIS4.

4.59. The assumptions are that for any undertaking, any year and any LoB:

- The expected loss is proportional to the premium
- Each undertaking has a different, but constant expected loss ratio
- The variance of the loss is proportional to the earned premium
- The distribution of the loss is lognormal and
- The maximum likelihood fitting approach is appropriate

4.60. The process involves two stages. The first stage fits a separate model of each undertaking's mean but fits a single model for the standard deviations across all undertakings simultaneously. Thus the standard deviations by undertaking take into account the experience of all the other undertakings when assessing this particular undertaking.

4.61. This stage also allows for more diversification credit within larger volumes of earned premium per line of business in the same way across all years and all undertakings.

4.62. This stage uses a maximum likelihood for a lognormal to fit the expected loss ratio and an associated variance estimator. As opposed to method 1 this fitting approach is optimal is aligned to the assumptions within the standard formula of LogNormality.

4.63. As an attempt to derive a single factor per line of business, across all firms a linearly weighted average of the standard deviations by undertaking has been taken.

4.64. Effectively this assumes that the sample of undertakings used in the fitting process is representative of all of Europe in terms of associated premium volumes as well as putting significantly more weight to those undertakings which write larger volumes of a specific line of business, therefore any result will be biased towards factors most appropriate for larger portfolios.

$U_{C,Y,lob}$	=	The posted ultimate after one year by undertaking, accident year and LoB
$\mu_{C,lob}$	=	Expected loss ratio by undertaking and by LoB
β_{lob}^2	=	Constant of proportionality for the variance of loss by LoB
$\varepsilon_{C,Y,lob}$	=	An unspecified random distribution with mean zero and unit variance
$V_{C,Y,lob}$	=	Earned premium by undertaking, accident year and LoB
$M_{C,Y,lob}$	=	The mean of the logarithm of the posted ultimate after one year by undertaking, accident year and LoB
$S_{C,Y,lob}$	=	The standard deviation of the logarithm of the posted ultimate after one year by undertaking, accident year and LoB
$V_{C,lob}$	=	Average earned premium by undertaking and LoB

Then the distribution of losses can be formulated as:

$$U_{C,Y,lob} \sim V_{C,Y,lob}\mu_{C,lob} + \sqrt{V_{C,Y,lob}}\beta_{lob}\varepsilon_{C,Y,lob}$$

4.65. This allows to formulate the parameters of the lognormal distributions as follows:

$$S_{C,Y,lob} = \sqrt{\log\left(1 + \frac{\beta_{lob}^2}{V_{C,Y,lob}\mu_{C,lob}^2}\right)}$$

$$M_{C,Y,lob} = \log\left(V_{C,Y,lob}\mu_{C,lob}\right) - \frac{1}{2}S_{C,Y,lob}^2$$

4.66. The resultant simplified log Likelihood becomes

$$\log L = \sum_{C,Y}\left(-\log\left(S_{C,Y,lob}\right) - \frac{\left(\log\left(U_{C,Y,lob}\right) - M_{C,Y,lob}\right)^2}{2S_{C,Y,lob}^2}\right)$$

4.67. The parameter values β_{lob} and $\mu_{C,lob}$ are chosen to maximise this likelihood.

4.68. The following term is defined:

$\sigma_{(C,prem,lob)}$	=	Standard deviation for premium risk by Undertaking

		by LoB

4.69. The $\sigma_{(C,prem,lob)}$ then becomes :

$$\sigma_{C,prem,lob} = \frac{\hat{\beta}_{lob}}{\sqrt{V_{C,lob}}} \quad \text{where}$$

$$V_{C,lob} = \frac{1}{N_{C,lob}} \sum_{Y} V_{C,Y,lob}$$

4.70. If the following term is defined:

$\sigma_{(prem,lob)}$	=	Standard deviation for premium risk by LoB

4.71. Then a value for $\sigma_{(prem,lob)}$ can be derived by taking a volume weighted average of the fitted undertaking specific standard deviations as below:

$$\hat{\sigma}_{(prem,lob)} = \frac{\sum_{C} V_{C,lob} \sigma_{C,prem,lob}}{\sum_{C} V_{C,lob}}$$

<u>Method 3</u>

4.72. This approach is consistent with the undertaking specific estimate assumptions from the Technical Specifications for QIS4, but assumes that the expected loss ratio is industry wide rather than undertaking specific.

4.73. The assumptions are that for any undertaking, any year and any LoB:

- The expected loss is proportional to the premium
- Each undertaking within a single LoB has the same constant expected loss ratio
- The variance of the loss is proportional to the earned premium
- The distribution of the loss is lognormal and
- The maximum likelihood fitting approach is appropriate

4.74. The process involves two stages. The first stage fits a single model for the mean and standard deviations across all undertakings simultaneously. Thus the means and standard deviations by undertaking take into account the experience of all the other undertakings when assessing this particular undertaking.

4.75. Compared to methods 1 and 2, only two parameters are fitting per line of business. The consequences of this will result in a less over-fitting and as a result is likely to lead to an overall higher volatility. However, this will also result in a worse fit to the data.

4.76. This stage also allows for more diversification credit within larger volumes of earned premium per line of business in the same way across all years and all undertakings.

4.77. This stage uses a maximum likelihood for a lognormal to fit the expected loss ratio and an associated variance estimator. As opposed to method 1 this fitting approach is optimal is aligned to the lognormal distribution within the standard formula.

4.78. As an attempt to derive a single factor per line of business, across all firms a linearly weighted average of the standard deviations by undertaking has been taken.

4.79. If the following terms are defined:

$U_{C,Y,lob}$	=	The posted ultimate after one year by undertaking, accident year and LoB
μ_{lob}	=	Expected loss ratio by LoB
β_{lob}^2	=	Constant of proportionality for the variance of loss by LoB
$\varepsilon_{C,Y,lob}$	=	An unspecified random distribution with mean zero and unit variance
$V_{C,Y,lob}$	=	Earned premium by undertaking, accident year and LoB
$M_{C,Y,lob}$	=	The mean of the logarithm of the posted ultimate after one year by undertaking, accident year and LoB
$S_{C,Y,lob}$	=	The standard deviation of the logarithm of the posted ultimate after one year by undertaking, accident year and LoB

Then distribution of losses can be formulated as follows:

$$U_{C,Y,lob} \sim V_{C,Y,lob}\mu_{lob} + \sqrt{V_{C,Y,lob}}\beta_{lob}\varepsilon_{C,Y,lob}$$

4.80. The parameters of the lognormal distributions are formulated as follows:

$$S_{C,Y,lob} = \sqrt{\log\left(1 + \frac{\beta_{lob}^2}{V_{C,Y,lob}\mu_{lob}^2}\right)}$$

$$M_{C,Y,lob} = \log\left(V_{C,Y,lob}\mu_{lob}\right) - \frac{1}{2}S_{C,Y,lob}^2$$

4.81. The resultant simplified log Likelihood becomes

$$\log L = \sum_{C,Y}\left(-\log\left(S_{C,Y,lob}\right) - \frac{\left(\log\left(U_{C,Y,lob}\right) - M_{C,Y,lob}\right)^2}{2S_{C,Y,lob}^2}\right)$$

4.82. The parameter values β_{lob} and μ_{lob} are chosen to maximise this likelihood.

4.83. If the following term is defined as:

$\sigma_{(C,prem,lob)}$	=	Standard deviation for premium risk by Undertaking by LoB

4.84. The $\sigma_{(C,prem,lob)}$ then becomes :

$$\sigma_{C,prem,lob} = \frac{\hat{\beta}_{lob}}{\sqrt{V_{C,lob}}} \quad \text{where}$$

$$V_{C,lob} = \frac{1}{N_{C,lob}} \sum_{Y} V_{C,Y,lob}$$

4.85. If the following term is defined as:

$\sigma_{(prem,lob)}$	=	Standard deviation for premium risk by LoB

4.86. Then a value for $\sigma_{(prem,lob)}$ can be derived by taking a volume weighted average of the fitted undertaking specific standard deviations as below:

$$\hat{\sigma}_{(prem,lob)} = \frac{\sum_{C} V_{C,lob} \sigma_{C,prem,lob}}{\sum_{C} V_{C,lob}}$$

<u>Method 4</u>

4.87. This approach is essentially consistent with the standard formula representation of the relationship between volatility of future losses and volume.

4.88. The assumptions are that for any undertaking, any year and any LoB:

- The expected loss is proportional to the premium
- Each undertaking has a different, but constant expected loss ratio
- The variance of the loss is proportional to the square of the earned premium
- The distribution of the loss is lognormal and
- The maximum likelihood fitting approach is appropriate

4.89. The process involves fitting a single model for the standard deviations across all undertakings simultaneously. Thus the standard deviations by undertaking take into account the experience of all the other undertakings when assessing this particular undertaking.

4.90. This method allows for no diversification credit unlike methods 1, 2 and 3.

4.91. This method uses a maximum likelihood for a lognormal to fit the expected loss ratios and an associated variance estimator. As opposed to method 1 this fitting approach is optimal is aligned to the lognormal distribution assumptions within the standard formula.

4.92. If the following terms are defined as:

$U_{C,Y,lob}$	=	The posted ultimate after one year by undertaking, accident year and LoB
$\mu_{C,lob}$	=	Expected loss ratio by undertaking and by LoB
β_{lob}^2	=	Constant of proportionality for the variance of loss by LoB
$\varepsilon_{C,Y,lob}$	=	An unspecified random distribution with mean zero and unit variance
$V_{C,Y,lob}$	=	Earned premium by undertaking, accident year and LoB
$M_{C,Y,lob}$	=	The mean of the logarithm of the posted ultimate after one year by undertaking, accident year and LoB
$S_{C,Y,lob}$	=	The standard deviation of the logarithm of the posted ultimate after one year by undertaking, accident year and LoB

Then the distribution of losses can be formulated as:

$$U_{C,Y,lob} \sim V_{C,Y,lob}\mu_{C,lob} + V_{C,Y,lob}\beta_{lob}\varepsilon_{C,Y,lob}$$

4.93. The parameters of the lognormal distributions can be formulated as follows:

$$S_{C,Y,lob} = \sqrt{\log\left(1 + \frac{\beta_{lob}^2}{\mu_{C,lob}^2}\right)}$$

$$M_{C,Y,lob} = \log\left(V_{C,Y,lob}\mu_{C,lob}\right) - \frac{1}{2}S_{C,Y,lob}^2$$

4.94. The resultant simplified log Likelihood becomes

$$\log L = \sum_{C,Y}\left(-\log\left(S_{C,Y,lob}\right) - \frac{\left(\log\left(U_{C,Y,lob}\right) - M_{C,Y,lob}\right)^2}{2S_{C,Y,lob}^2}\right)$$

4.95. The parameter values β_{lob} and $\mu_{C,lob}$ are chosen to maximise this likelihood.

4.96. If the following term is defined as:

$\sigma_{(prem,lob)}$	=	Standard deviation for premium risk by LoB

4.97. The $\sigma_{(prem,lob)}$ then becomes :

$$\sigma_{(prem,lob)} = \hat{\beta}_{lob}$$

4.2.5. Premium Risk Results

4.98. CEIOPS has presented the results of the analysis though a combination of tables and graphs.

4.99. The table presents the results of methods 1 to 4 described in section 4.2.4. :

- The analysis includes a column of fitted factors by method based on an estimated volume weighted average of the standard deviation estimates by undertaking. Effectively this assumes that the sample of undertakings used in the fitting process is representative of all of Europe in terms of associated premium volumes as well as putting significantly more weight to those undertakings which write larger volumes of a specific line of business, therefore any result will be biased towards factors most appropriate for larger portfolios.
- The table includes the percentage of undertakings which would have a gross standard deviation, as assessed under Method 1, greater than the selected technical result.

4.100. The individual estimates of the standard deviations by undertaking that result from the application of Method 1 are plotted against the prediction model for comparison. The individual estimates can be used as evidence of the existence of diversification credit for volume. Where such an effect does exist the graph would be expected in general to be decreasing.

4.101. Where there are signs of diversification, this implies that capital requirements are significantly higher for smaller than larger portfolios. This arises for two reasons:

- Larger accounts are usually less volatile than smaller accounts. Thus expressed as a percentage of premiums a larger account often has smaller theoretical capital requirements than a smaller account.
- Larger insurers often have a greater degree of diversification of risks than smaller insurers.

4.102. For methods 2 and 3, where diversification credit is assumed to exist, an illustration of what the factor could be for 3 sizes is presented: small, which equates to a 25th percentile of the sample observations, medium a 50th percentile, large 75th percentile.

4.103. The appropriateness of methods 2, 3 and 4 are tested and presented by showing the results of a goodness of fit test through a PP plot.

4.104. Results varied across methods because each method uses different underlying assumptions. For example:

- Some methods will place more weight on volatilities estimated for larger companies which tend to have lower standard deviations thus producing a lower overall result.
- Other methods will give an equal weight to each undertaking and as a result will tend to produce a higher overall result.
- Others will test different fitting techniques (least squares vs maximum likelihood).

4.105. The selection of the final fitted factors was based on the following:

- The evidence of diversification by size has not been given full allowance. i.e. no consideration has been given to the fact that volatilities by size of portfolio may be significantly different. Therefore more focus has been placed on the fitted factors.
- Factors have been selected as the average of those methods which were considered to produce an acceptable fit according to the goodness of fit plots shown

4.106. CEIOPS would like to highlight that the selection was not conservatively selected, but rather based on the goodness of fit results. Furthermore by taking an average across methods, CEIOPS is ensuring that the factors are not biased towards factors most appropriate for larger portfolios (and hence lower).

4.107. A more thorough illustration of the process followed is outlined in the Annex 7.3.

Motor, vehicle liability

4.108. CEIOPS recommendation is that for the motor vehicle liability lob the gross factor for premium risk should be 11.5%.

4.109. The data sample included data from 209 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, PT, SI, SK, IS, IT, LT, FI, DK, SE and HU.

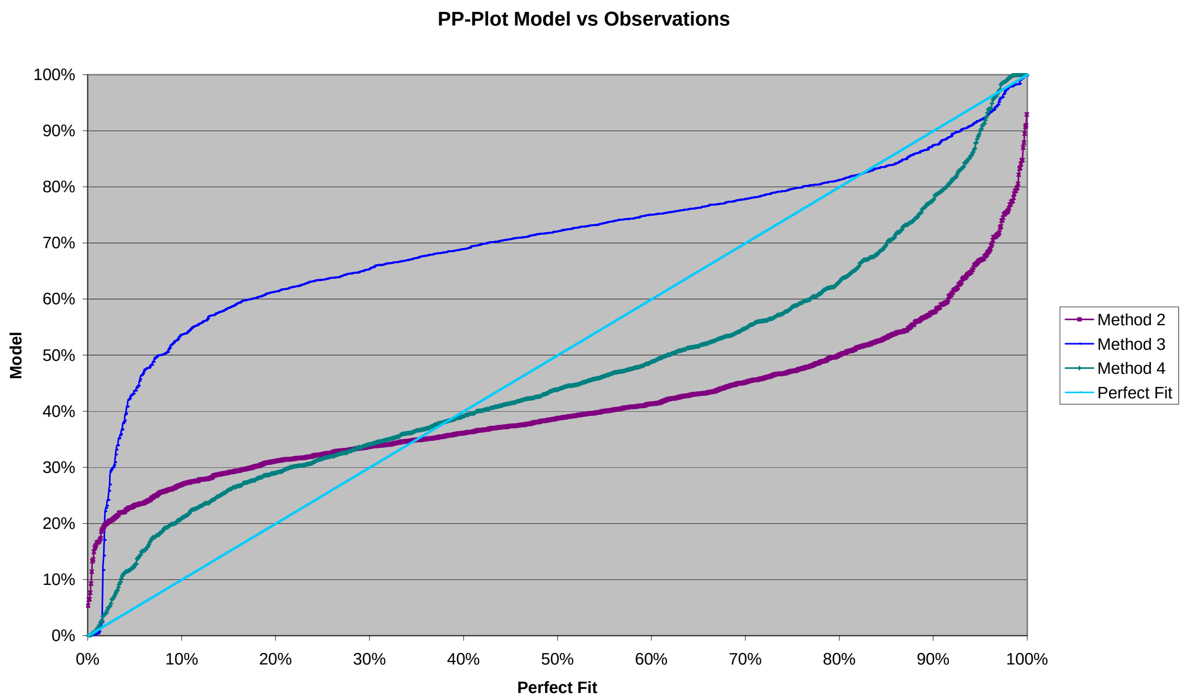
Reference co	Small	Medium	Large
Motor, third-party liability	12,500	48,879	134,604

GROSS Standard Deviations

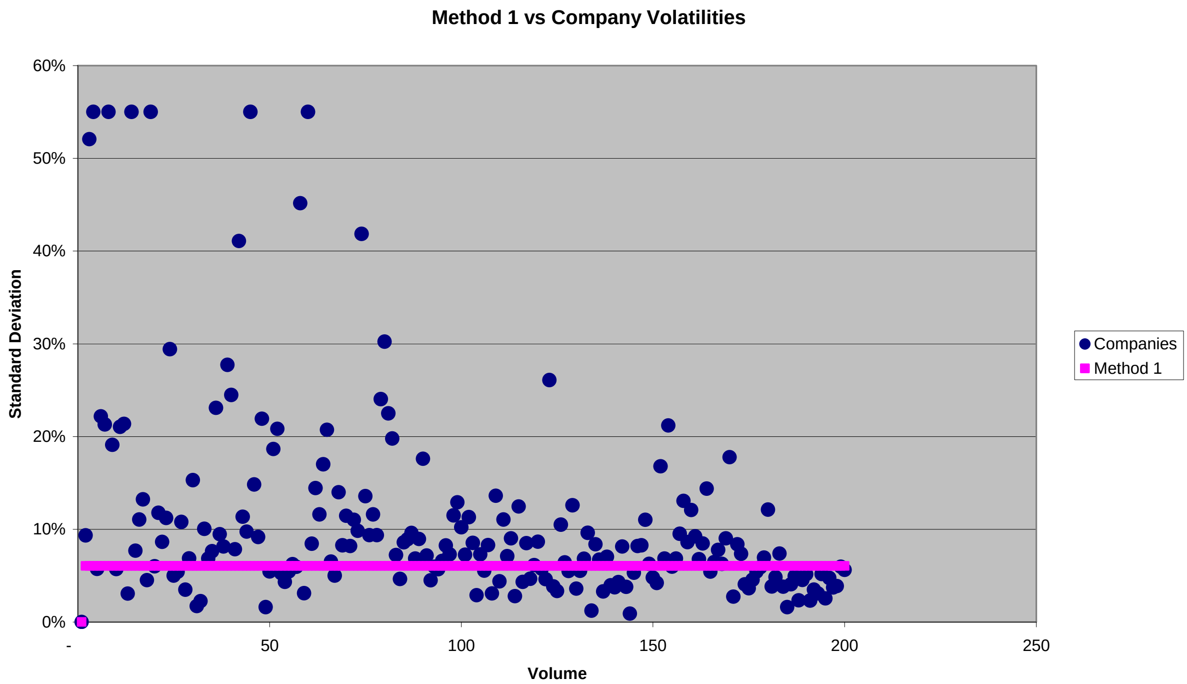
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				6%	**11.3%**	**26.1%**
Method 2	129%	65%	39%	25%		
Method 3	96%	49%	29%	18%		
Method 4				17%		

4.110.The graph below shows a pp plot of the fit of the models. None of the methods fit particularly well, but method 4 is probably the best.



4.111.The result on the graph below shows signs of diversification credit. It also shows the volatility of the individual observation compared to the fitted selection for method 1.



Overall conclusions:

4.112. The selected technical factor was chosen as the average of the results from methods 1 and 4 – result 11.3%

Motor, other classes

4.113.CEIOPS recommendation is that for the motor other lob the gross factor for premium risk should be 8.5%.

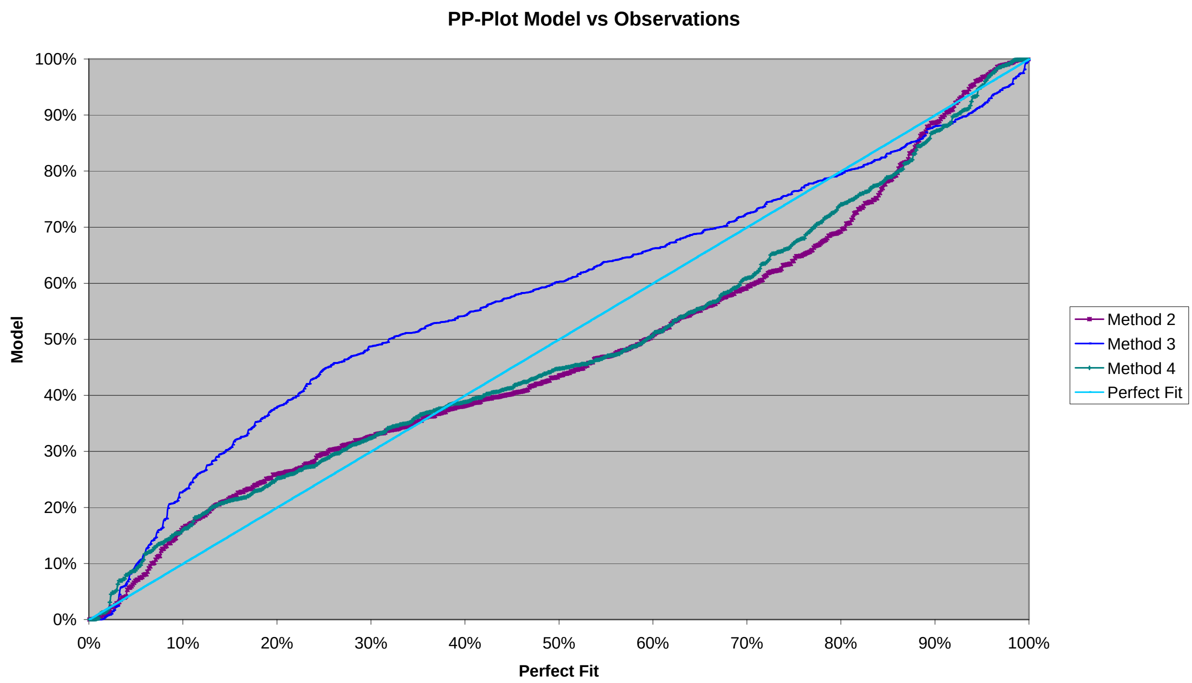
4.114.The data sample included data from 107 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, SI, PT, SK, IS, LT, FI, DK and SE.

Reference co	Small	Medium	Large
Motor, other classes	9,112	16,225	49,698

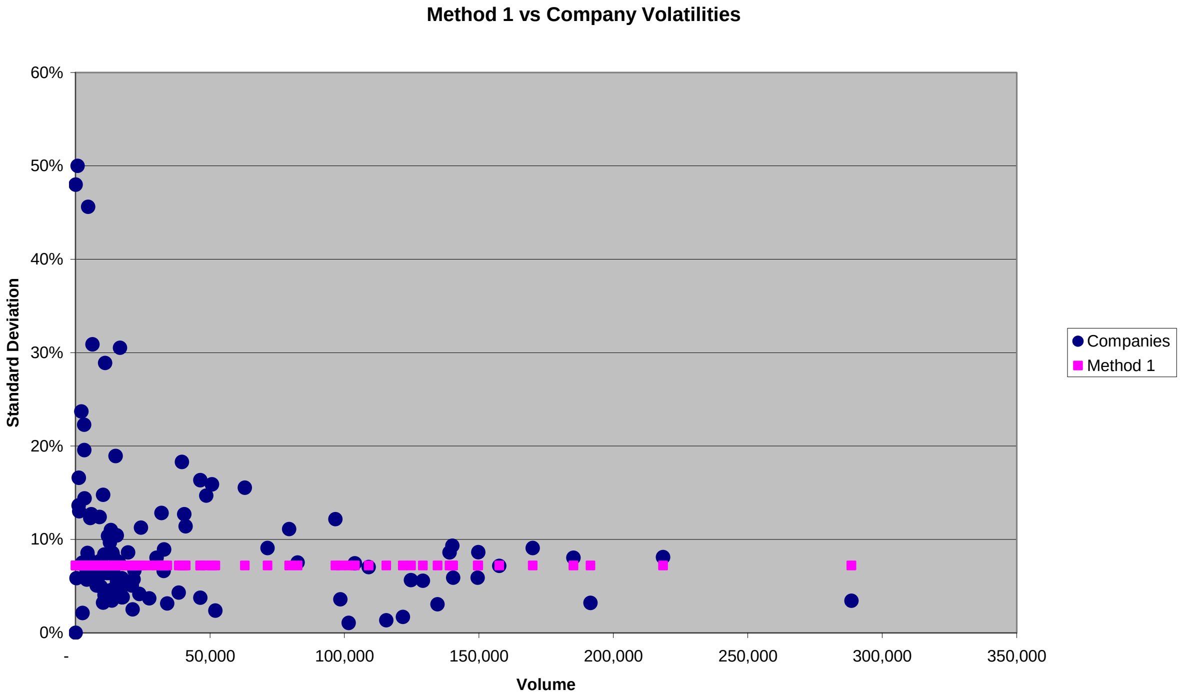
GROSS Standard Deviations
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				7%	8.3%	43.0%
Method 2	18%	14%	8%	7%		
Method 3	51%	38%	22%	19%		
Method 4				11%		

4.115.The result on the graph below shows that method 2 and 4 provide the best fits to the model, although neither is that good.



4.116.The result on the graph below shows signs of diversification credit. The graph also shows for method 1, the observations that lie above and below the fitted factor.



Overall conclusions:

4.117.The selected technical factor was chosen as the average of the results from methods 1, 2 and 4 – result 8.3%

Marine, aviation, transport (MAT)

4.118.CEIOPS recommendation is that for the MAT lob the gross factor for premium risk should be 23%.

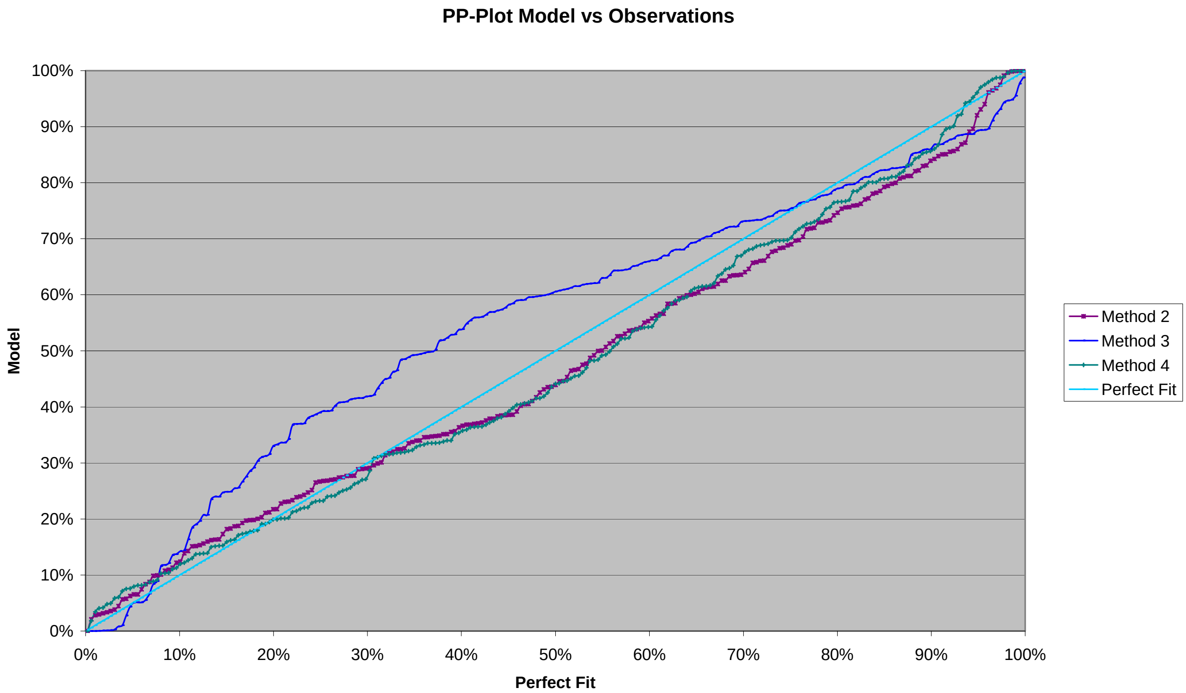
4.119.The data sample included data from 37 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, SK, IS, DK and SE.

Reference co	Small	Medium	Large
Marine, aviation, transport (MAT)	414	3,343	6,077

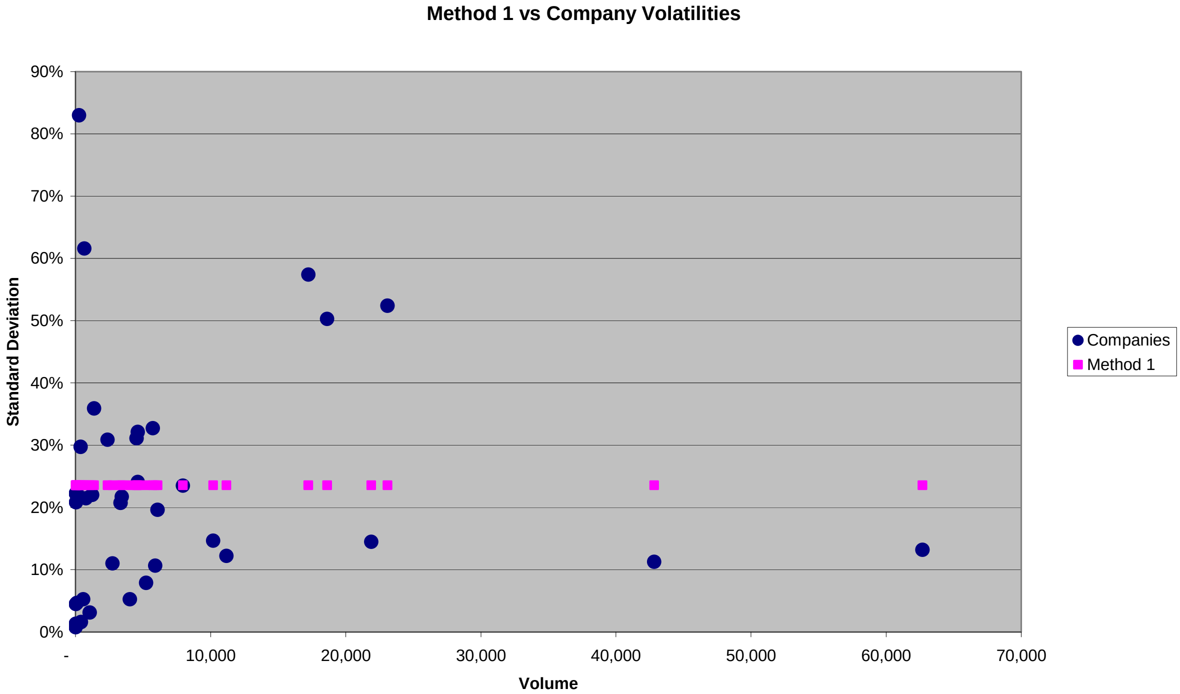
GROSS Standard Deviations
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				22%	**22.8%**	**35.1%**
Method 2	109%	38%	28%	19%		
Method 3	334%	117%	87%	59%		
Method 4				27%		

4.120.The graph below shows a pp plot of the fit of the models. It is clear that methods 2 and 4 are the best fits to the models, with little to choose between them.



4.121.The result on the graph below does show some signs of diversification credit, but it is not clear from the presence of some outliers.



Overall conclusions:

4.122.The selected technical factor was chosen as the average of the results from methods 1, 2 and 4 – result 22.8%

Fire and other property damage

4.123.CEIOPS recommendation is that for the fire and other property damage lob the gross factor for premium risk should be 15%.

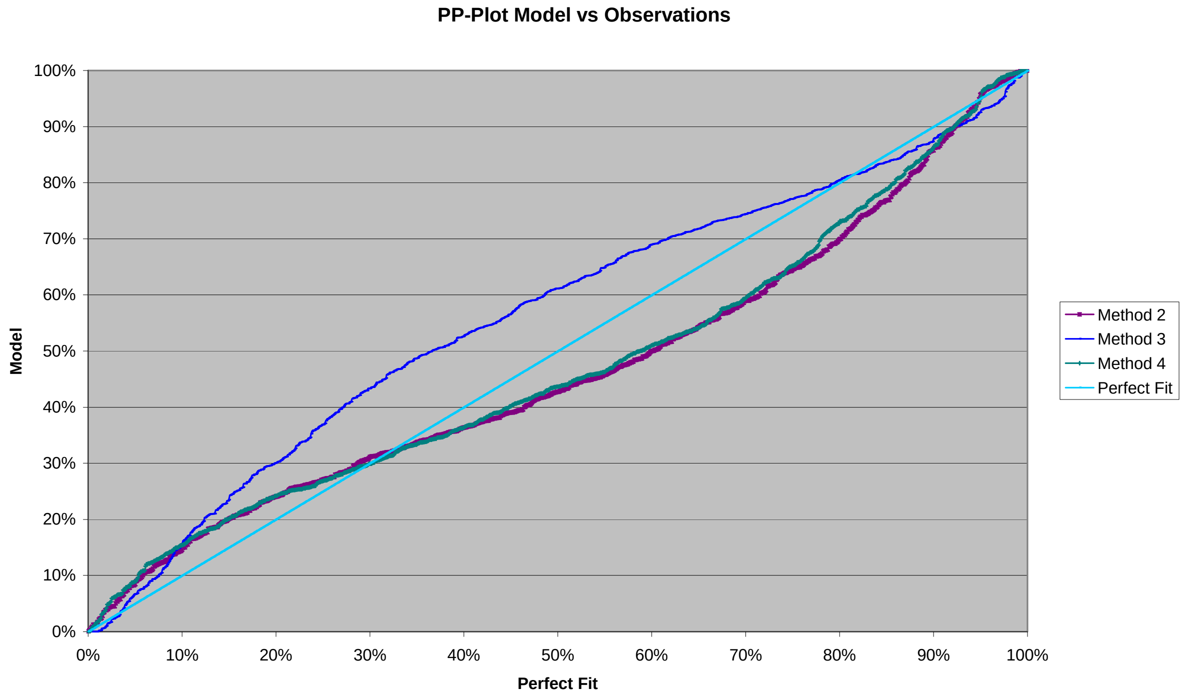
4.124.The data sample included data from 138 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, UK, SK, IS, FI, DK and SI.

Reference co	Small	Medium	Large
Fire and other property damage	6289	33,919	95,277

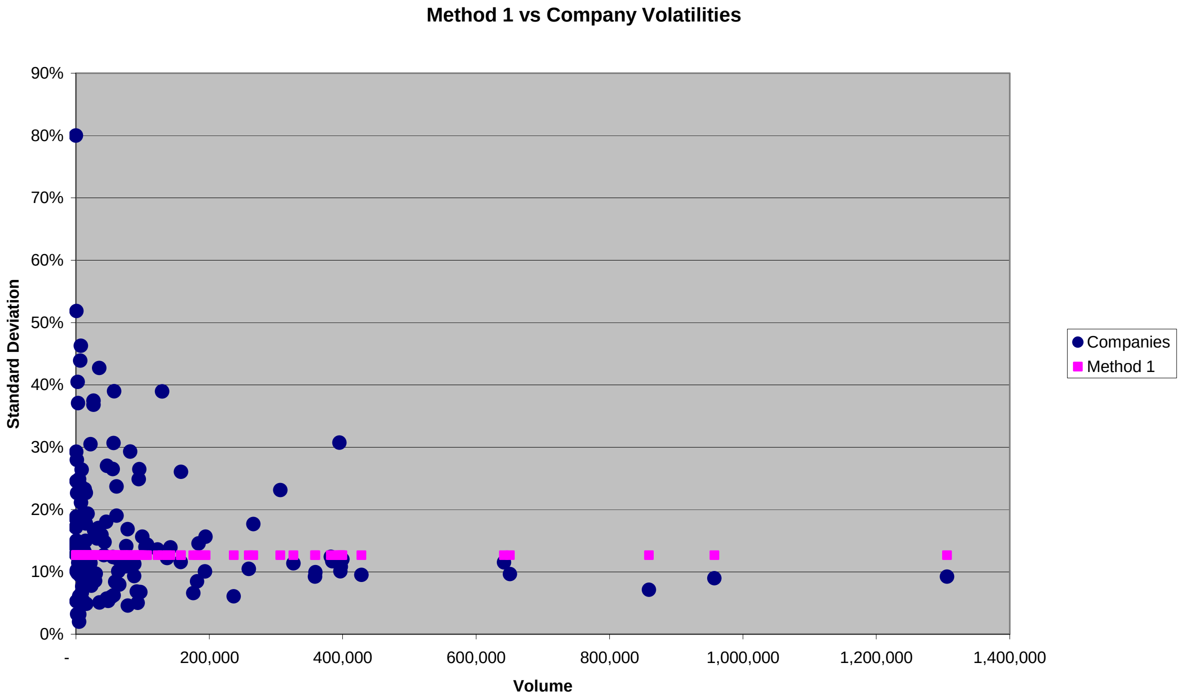
GROSS Standard Deviations
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				12%	15.2%	38.7%
Method 2	61%	26%	16%	11%		
Method 3	96%	41%	25%	18%		
Method 4				20%		

4.125.The graph below shows a pp plot of the fit of the methods. None of the methods fits particularly well, and there is little to choose between them.



4.126.The result on the graph below shows significant evidence of diversification credit.



Overall conclusions:

4.127.The selected technical factor was chosen as the average of the results from all four of the methods – result 15.2%.

Third-party liability

4.128.CEIOPS recommendation is that for the third party liability lob the gross factor for premium risk should be 17.5%.

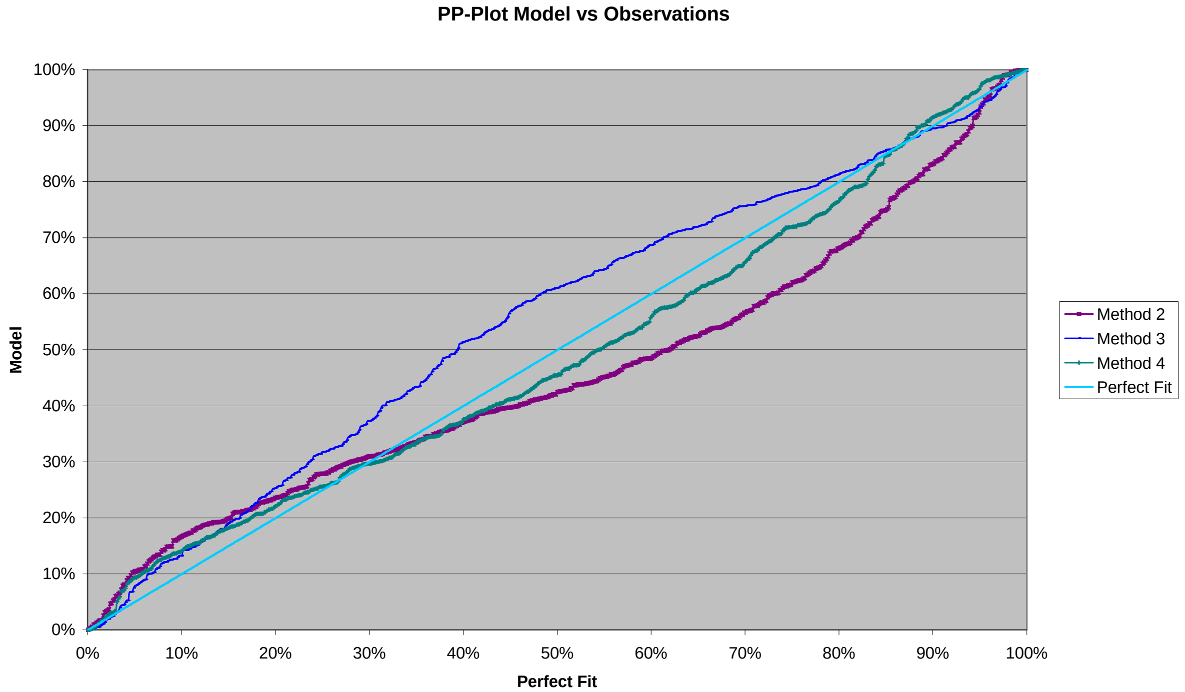
4.129.The data sample included data from 101 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, UK, SK, IS, DK and SI.

Reference co	Small	Medium	Large
Third-party liability	1466	8,850	21,276

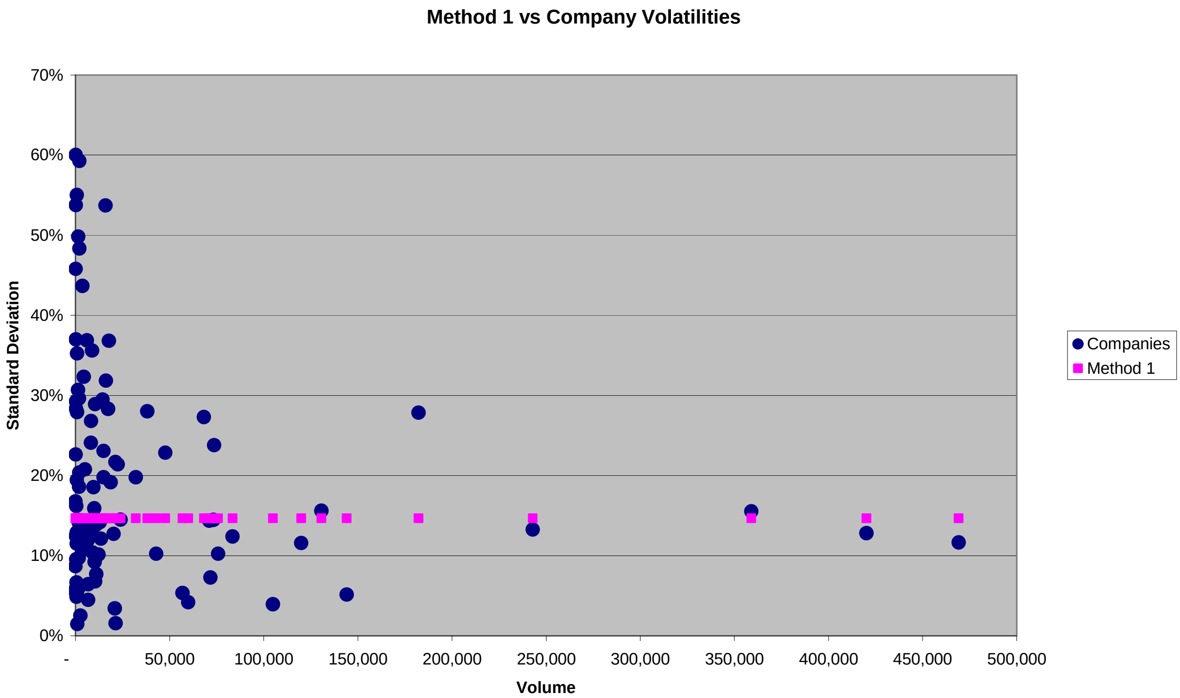
GROSS Standard Deviations
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				13%	**17.2%**	**42.6%**
Method 2	86%	35%	23%	12%		
Method 3	140%	57%	37%	20%		
Method 4				21%		

4.130.The graph below shows a pp plot of the fit of the methods. Method 4 is probably the best fit.



4.131.The result on the graph below shows evidence of diversification credit.



Overall conclusions:

4.132.The selected technical factor has been taken as the average of methods 1 and 4 – result 17.2%.

Credit and suretyship

4.133.CEIOPS recommendation is that for the credit and suretyship lob the gross factor for premium risk should be 28%.

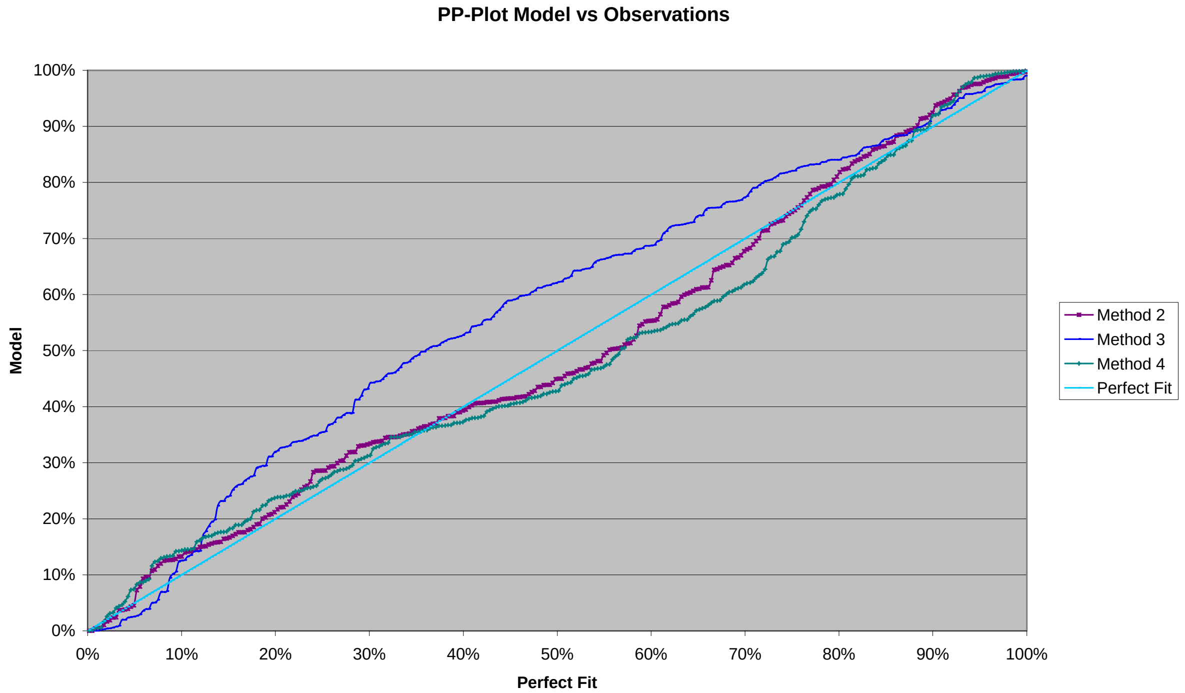
4.134.The data sample included data from 58 undertakings, was gross of reinsurance and included data from the following member states: PO, UK, SK, DK, SE and SI.

Reference co	Small	Medium	Large
Credit and suretyship	861	4,069	8,297

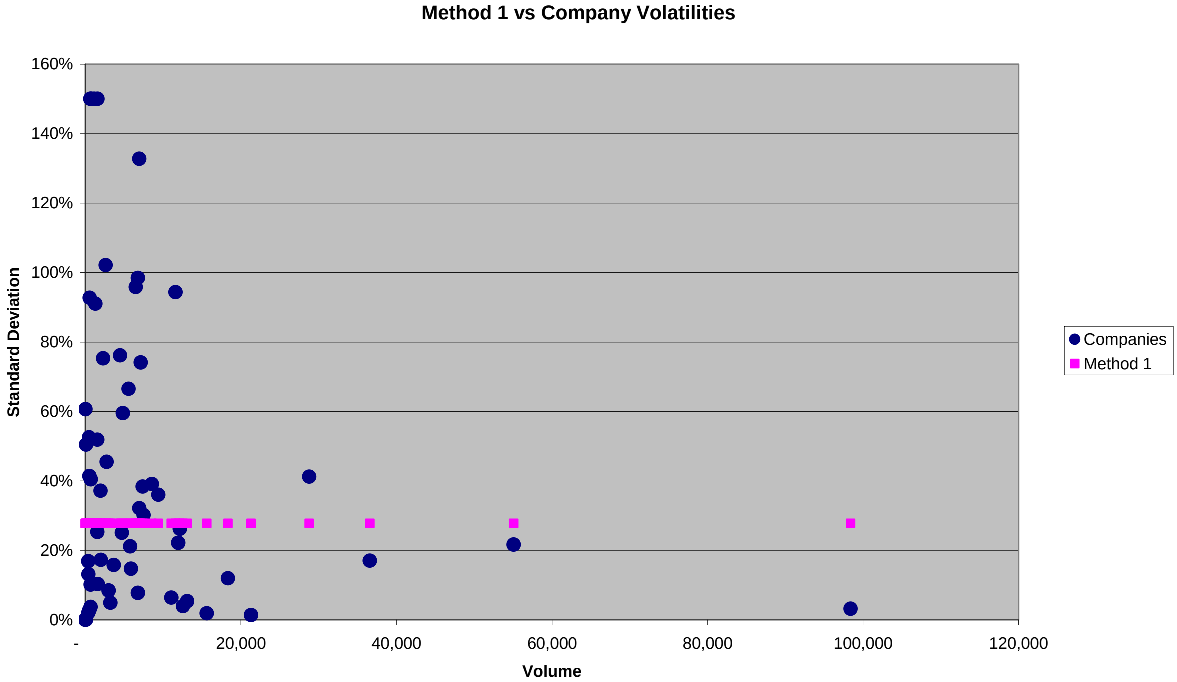
GROSS Standard Deviations
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				25%	**28.1%**	**51.7%**
Method 2	124%	57%	40%	31%		
Method 3	313%	144%	101%	79%		
Method 4				66%		

4.135.The graph below shows a pp plot of the fit of the methods. Methods 2 and 4 are the best fits, but method 2 appears to be better than method 4.



4.136.The graph below shows evidence of diversification credit.



Overall conclusions:

4.137.The selected technical factor has been taken as the average of methods 1 and 2 – result 28.1%

Legal expenses premium risk

4.138.CEIOPS recommendation is that for the legal expenses lob the gross factor for premium risk should be 8%.

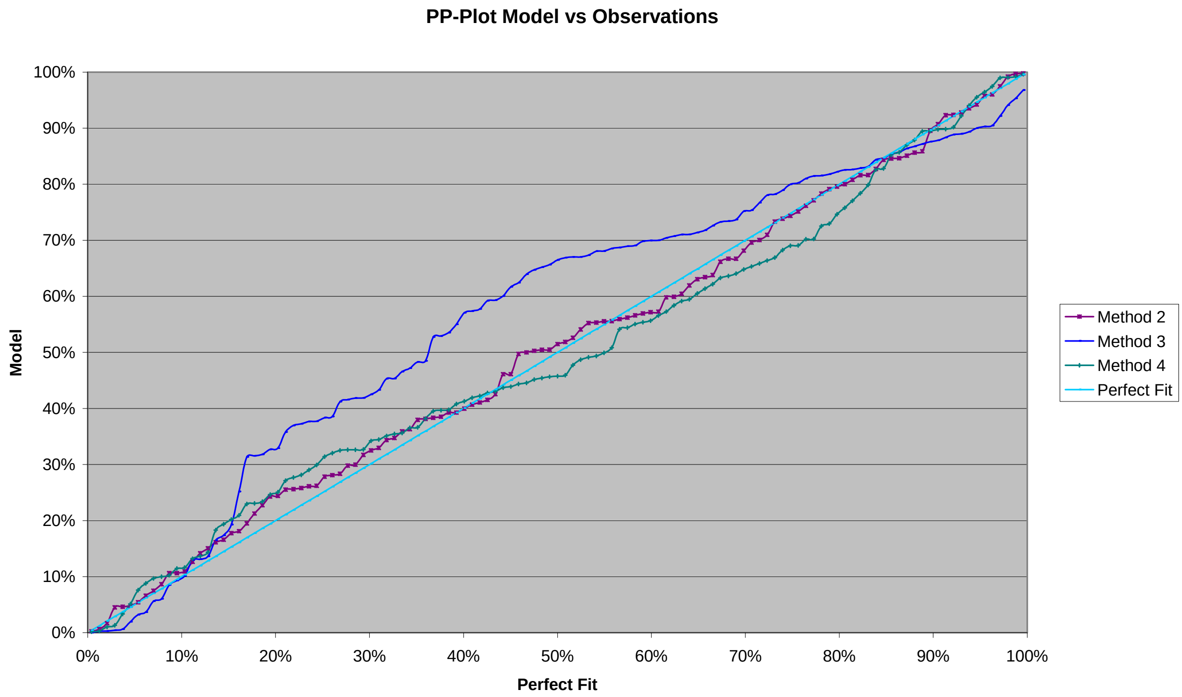
4.139.The data sample included data from 18 undertakings, was gross of reinsurance and included data from the following member states: PO, SK, FI and UK.

Reference co	Small	Medium	Large
Legal expenses	4,099	14,873	26,990

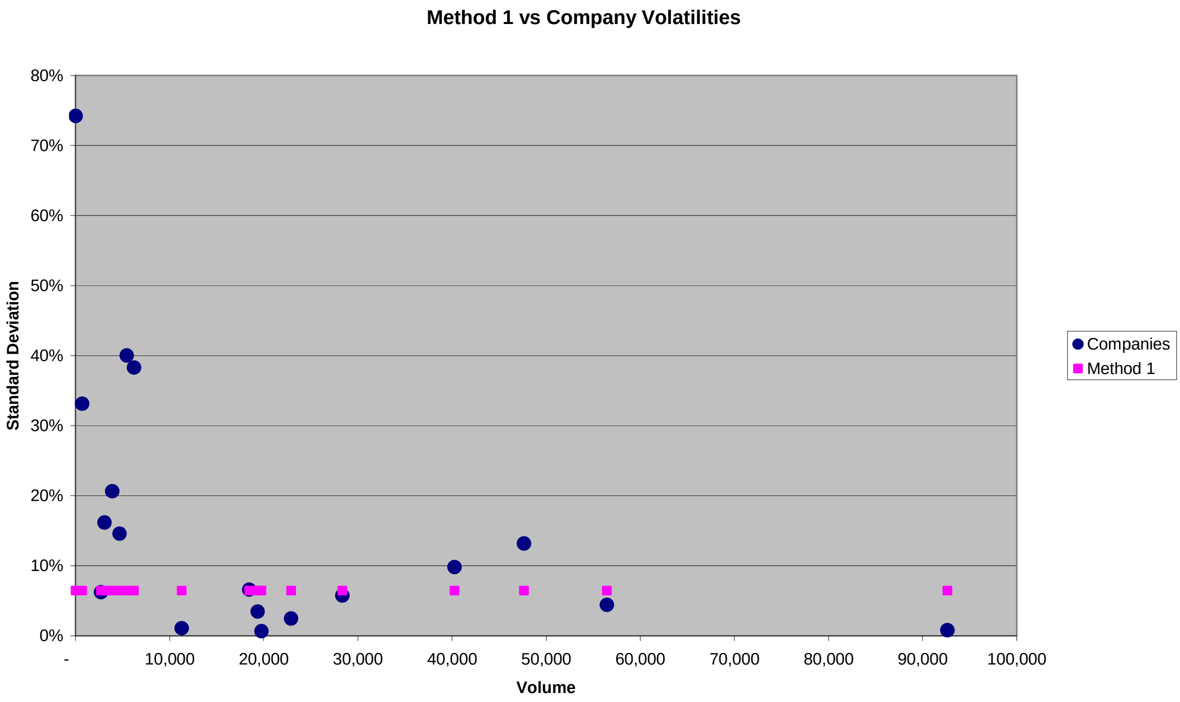
GROSS Standard Deviations
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				6%	**8.0%**	**50.0%**
Method 2	27%	14%	11%	10%		
Method 3	280%	147%	109%	104%		
Method 4				27%		

4.140.The graph below shows a pp plot of the fit of the methods. Methods 2 and 4 are the best fits, but method 2 appears to be better than method 4.



4.141.The graph below shows evidence of diversification credit.



Overall conclusions:

4.142. The selected technical factor has been taken as the average of methods 1 and 2 – result 8.0%

Assistance

4.143. CEIOPS recommendation is that for the assistance lob the gross factor for premium risk should be 5%.

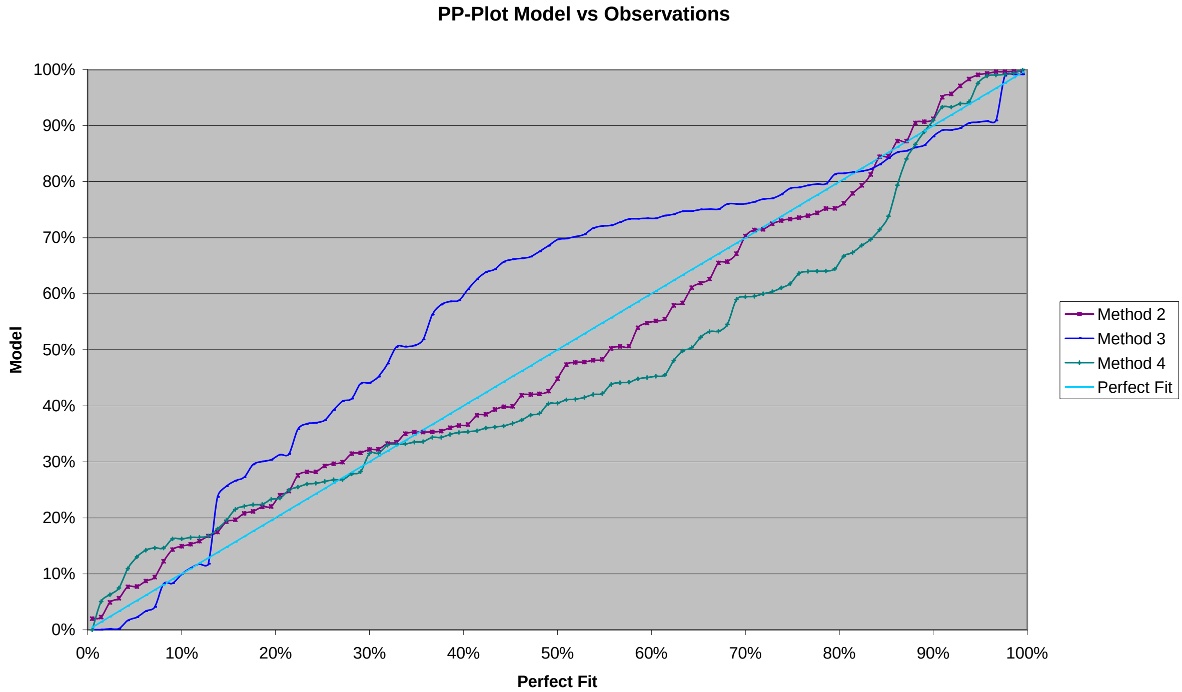
4.144. The data sample included data from 20 undertakings, was gross of reinsurance and included data from the following member states: PO, SK, DK and UK.

Reference co	Small	Medium	Large
Assistance	4,245	7,018	23,823

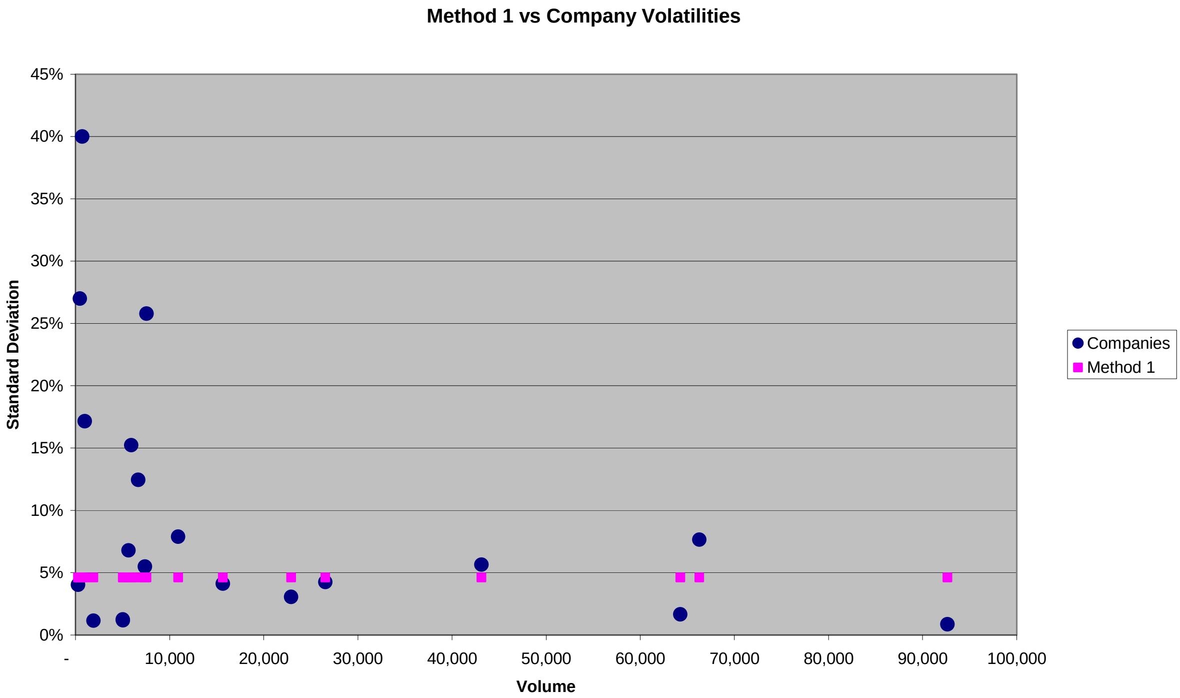
GROSS Standard Deviations
Discounted

Method	**Small (75th perc)**	**Medium (50th perc)**	**Large (25th perc)**	**VWA**	**Technical result based on VWA**	**% firms with higher sd**
Method 1				4%	**4.9%**	**55.0%**
Method 2	14%	11%	6%	5%		
Method 3	59%	46%	25%	22%		
Method 4				14%		

4.145. The graph below shows a pp plot of the fit of the methods. Method 2 is the best fit.



4.146.The graph below shows evidence of diversification credit.



Overall conclusions:

4.147.The selected technical factor has been taken as the average of methods 1 and 2 – result 4.9%

Miscellaneous

4.148.CEIOPS recommendation is that for the miscellaneous lob the gross factor for premium risk should be 15%.

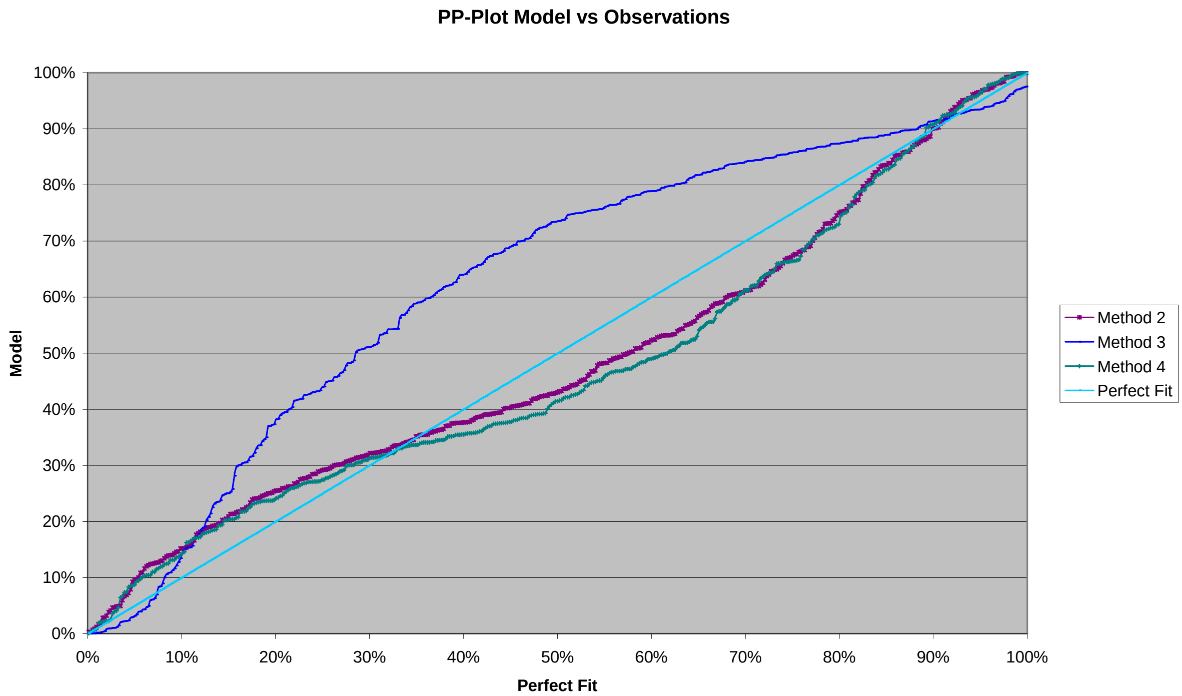
4.149.The data sample included data from 75 undertakings, was net of reinsurance and included data from the following member states: PO, DK and UK.

Reference co	Small	Medium	Large
Miscellaneous	1,486	10,603	37,819

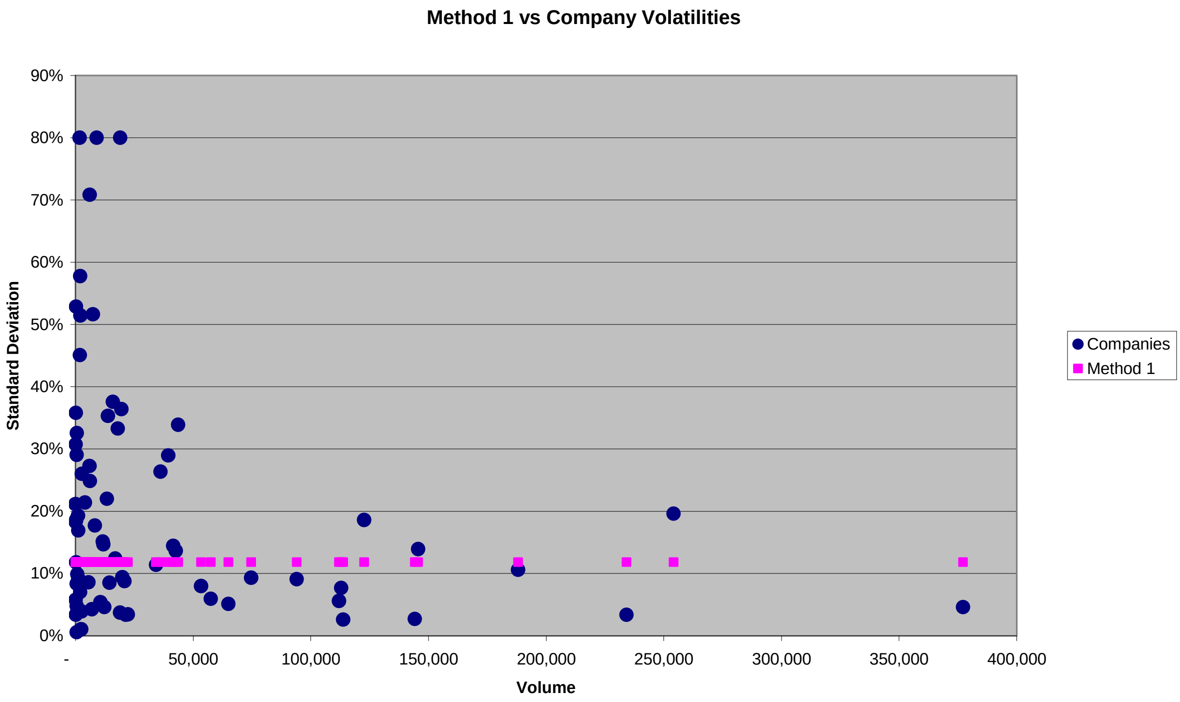
GROSS Standard Deviations
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				11%	**15.4%**	**44.0%**
Method 2	77%	29%	15%	11%		
Method 3	313%	117%	62%	45%		
Method 4				24%		

4.150.The graph below shows a pp plot of the fit of the methods. Methods 2 and 4 are the best fits, with little to choose between them.



4.151.From the graph below, we can see signs of diversification credit.



Overall conclusions:

4.152.The selected technical factor has been taken as the average of methods 1, 2 and 4 – result 15.4%.

Non-proportional reinsurance – property

4.153.CEIOPS recommendation is that for the non-proportional reinsurance - property lob the gross factor for premium risk should be 37.5%.

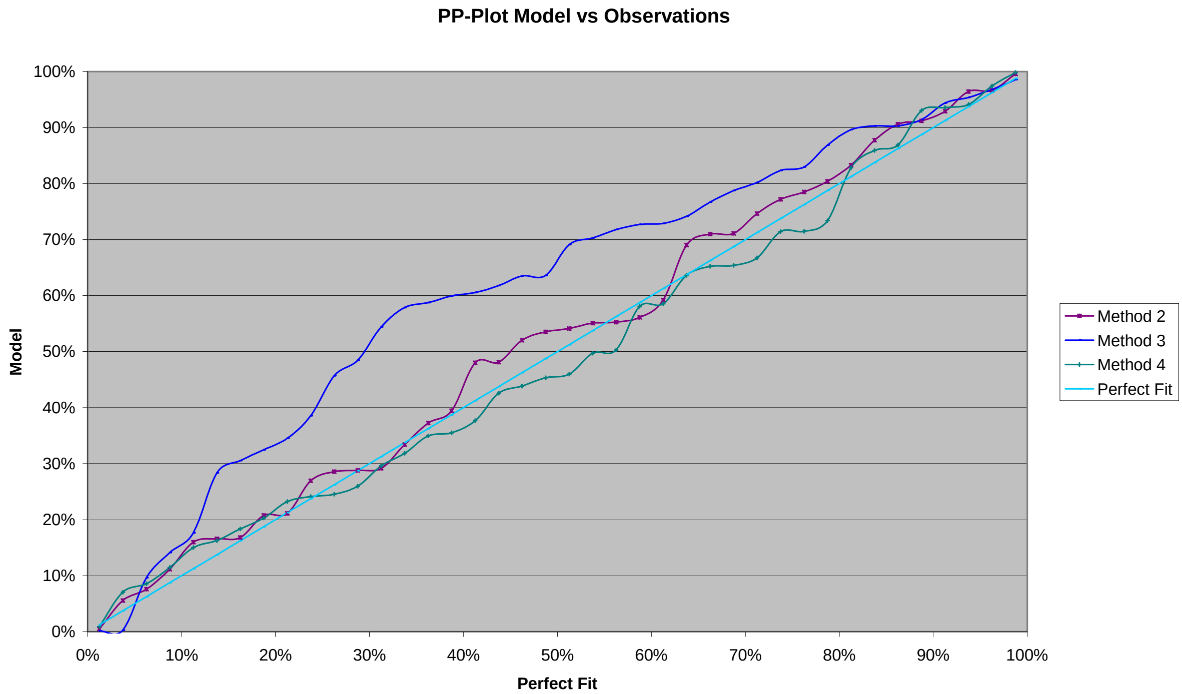
4.154.The data sample included data from 9 undertakings, was gross of reinsurance and included data from the following member states: UK.

Reference co	Small	Medium	Large
NPL Property	3,724	6,339	16,497

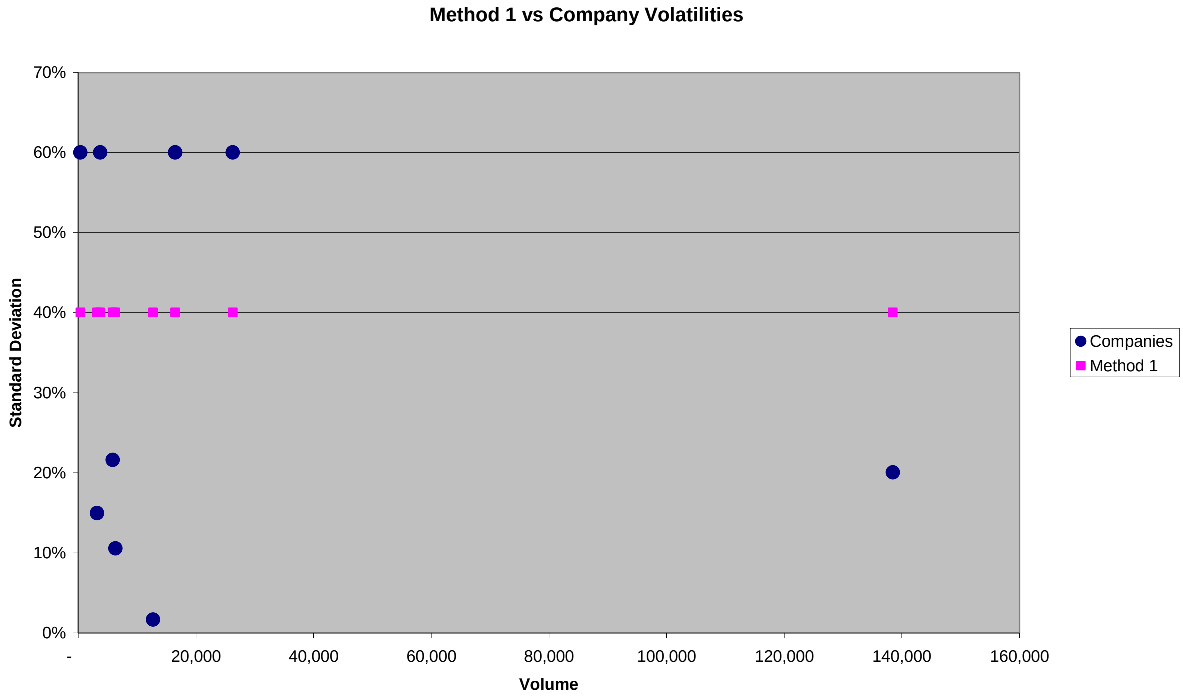
GROSS Standard Deviations
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				37%	**37.0%**	**44.4%**
Method 2	161%	124%	77%	49%		
Method 3	304%	233%	145%	93%		
Method 4				79%		

4.155.The graph below shows a pp plot of the fit of the methods. Methods 2 and 4 show the best fit with little to choose between them, although the fit is not great.



4.156.Because there not many observations the graph below does not show clear evidence of diversification credit. It also shows that 4 undertakings out of 9 are above the fitted factor under method 1.



Overall conclusions:

4.157.It is difficult to draw conclusions based on the lack of data and the volatility of the results provided by the analysis. The selected technical factor has been taken just from Method 1 – result 37.0%

Non-proportional reinsurance – casualty

4.158.CEIOPS recommendation is that for the non-proportional reinsurance - casualty lob the gross factor for premium risk should be 18%.

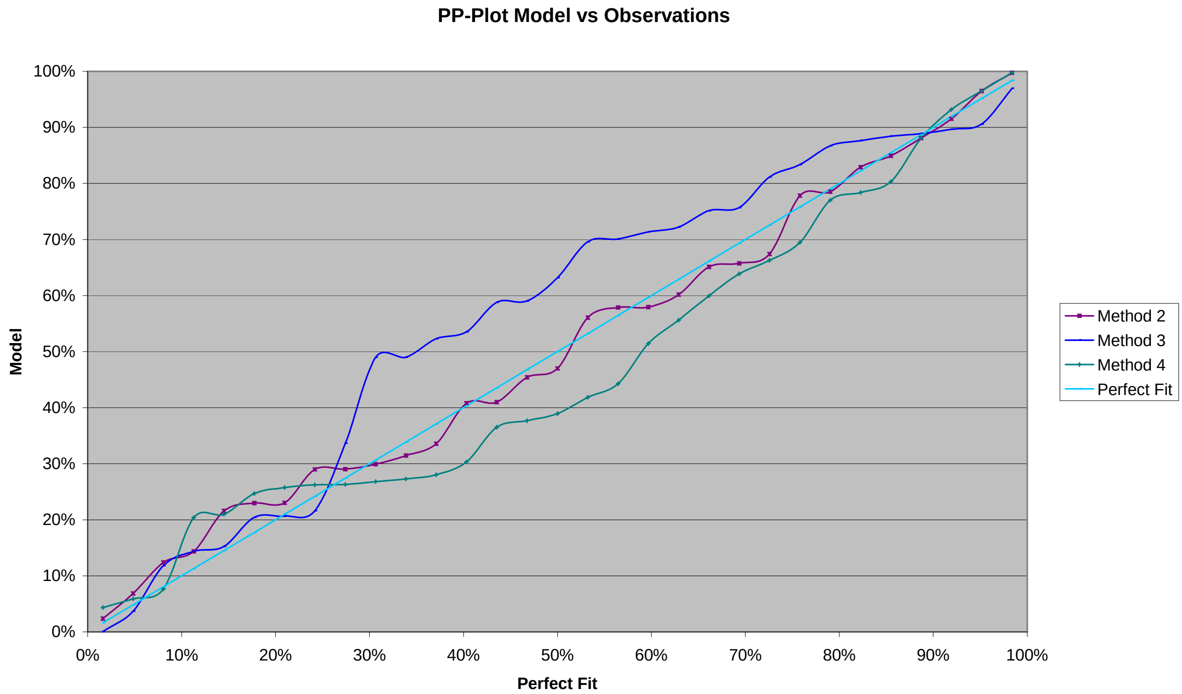
4.159.The data sample included data from 6 undertakings, was gross of reinsurance and included data from the following member states: UK.

Reference co	Small	Medium	Large
NPL Casualty	5,500	13,939	18,919

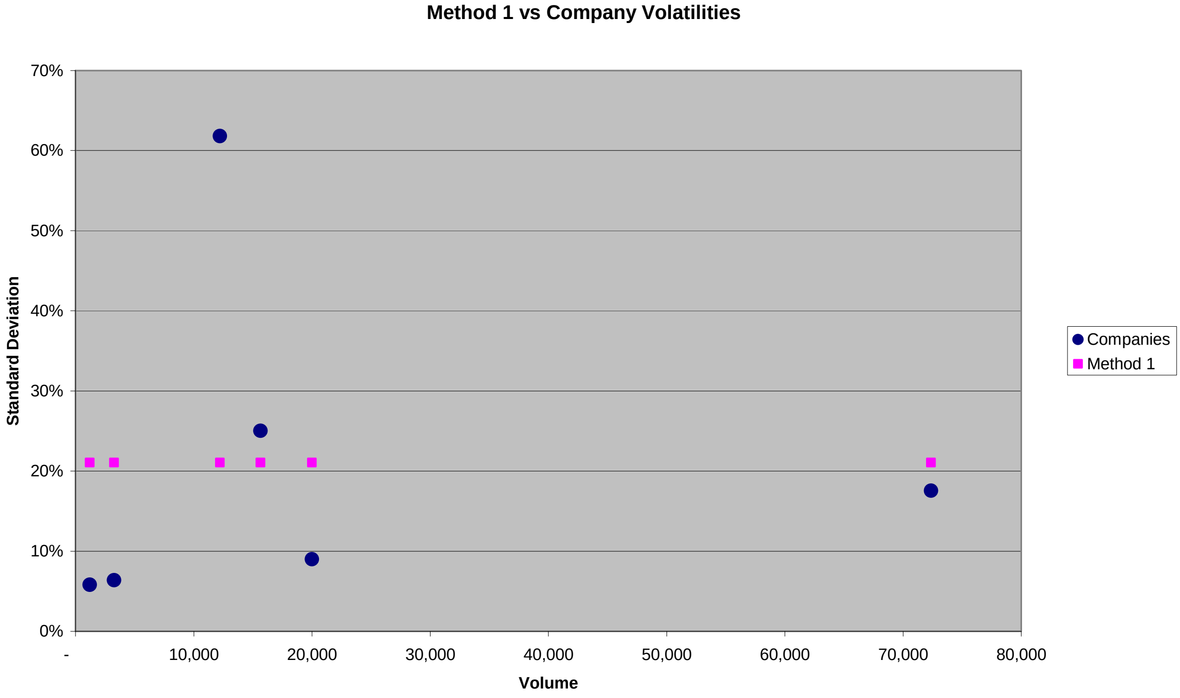
GROSS Standard Deviations
Discounted

Method	**Small (75th perc)**	**Medium (50th perc)**	**Large (25th perc)**	**VWA**	**Technical result based on VWA**	**% firms with higher sd**
Method 1				18%	**18.4%**	**33.3%**
Method 2	42%	27%	23%	19%		
Method 3	77%	48%	41%	34%		
Method 4				23%		

4.160.The graph below shows a pp plot of the fit of the methods. Method 2 shows the best fit.



4.161.Because there not many observations the graph below does not show clear evidence of diversification credit. It also shows that 2 undertakings out of 6 are above the fitted factor under method 1.



Overall conclusions:

4.162.It is difficult to draw conclusions based on the lack of data and the volatility of the results provided by the analysis. The selected technical factor has been taken as the average of methods 1 and 2 – result 18.4%.

Non-proportional reinsurance – MAT

4.163.CEIOPS recommendation is that for the non-proportional reinsurance - MAT lob the gross factor for premium risk should be 16%.

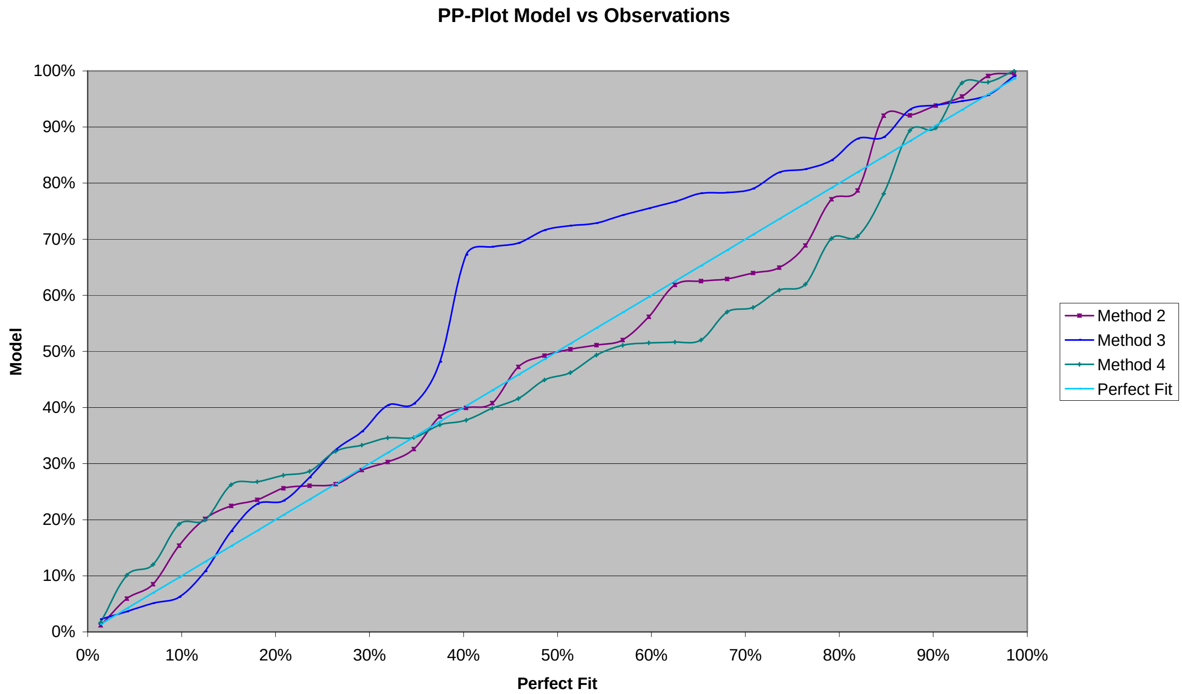
4.164.The data sample included data from 10 undertakings, was gross of reinsurance and included data from the following member states: UK.

Reference co	Small	Medium	Large
NPL MAT	1,046	2,780	8,259

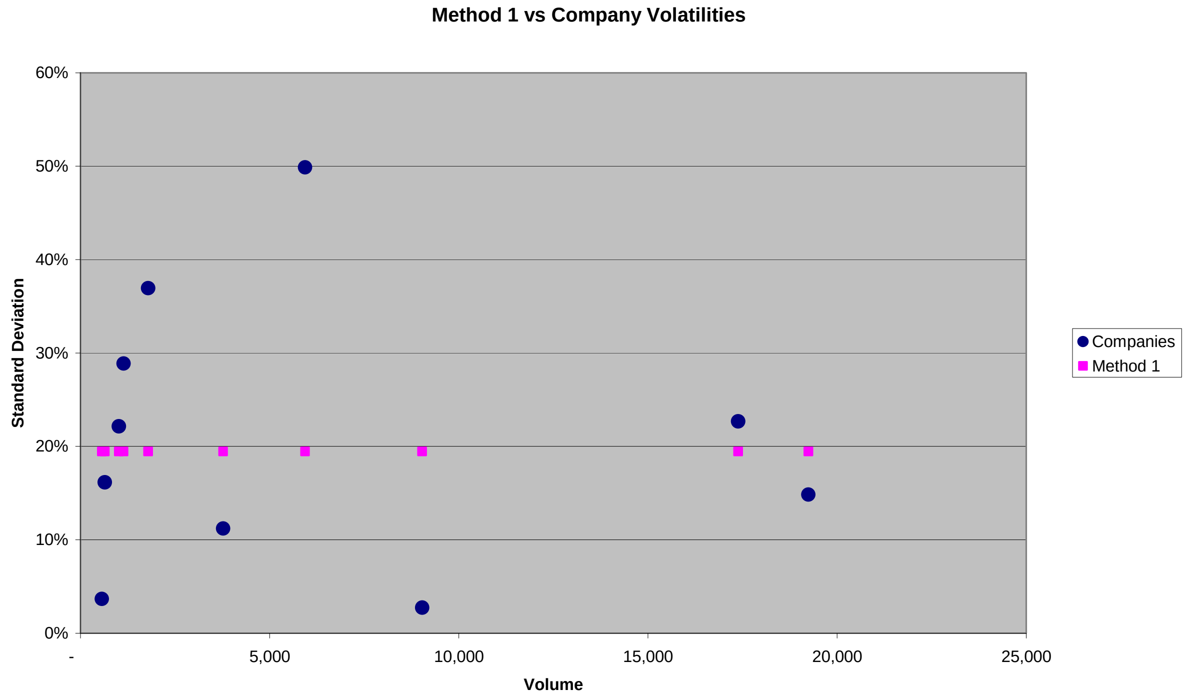
GROSS Standard Deviations
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result based on VWA	% firms with higher sd
Method 1				18%	**16.5%**	**50.0%**
Method 2	41%	25%	15%	15%		
Method 3	51%	31%	18%	18%		
Method 4				26%		

4.165.The graph below shows a pp plot of the fit of the methods. Methods 2 and 4 are the best fits, with method 2 being the best fit.



4.166.The graph below shows little evidence of diversification, but again there are very few observations. Furthermore 5 undertakings lie above the method 1 fitted factor.



Overall conclusions:

4.167.It is difficult to draw conclusions based on the lack of data and the volatility of the results provided by the analysis. The selected technical factor has been taken as the average of methods 1 and 2 – result 16.5%.

4.2.6. Adjusting gross to net for premium risk

4.168. CEIOPS considers that it is important that the standard capital charge for premium and reserve risk adequately takes into account the risk mitigation effect of reinsurance covers. To improve the risk-sensitivity of the standard formula in this respect, CEIOPS suggests to introduce a company-specific adjustment factor which translates the gross standard deviation observed in a line of business into a net standard deviation which is aligned to the risk profile of the insurer's portfolio. CEIOPS notes that in the context of the standard formula this is a technically challenging task, considering on the one hand the diversity and complexity of reinsurance covers (especially in the case of non-proportional reinsurance) and on the other hand the necessity to provide a standardised calculation which is technically feasible for all undertakings.

4.169. CEIOPS has discussed with the industry the design of such a gross-to-net adjustment factor, and has welcomed and fully considered the industry proposal for a gross-to-net adjustment[5], which focuses on a specific type of non-proportional reinsurance cover. CEIOPS has developed an approach which aims to provide a more simple and generally applicable solution to this issue. However, CEIOPS is aware of the limitations of the proposals that are on the table today, and further work may be needed to achieve a design and calibration of a gross-to-net factor which is both sufficiently risk-sensitive and also appropriate for the purposes of a standard formula calculation.

4.170. The calibration (gross) has been performed using data gross of reinsurance. However, the standard formula uses premiums net of reinsurance as a volume measure. The volatility of net claims will be lower than the volatility of gross claims, however, the net premiums will also be lower than the gross premiums.

4.171. Our provisional analysis has shown that the reduction in claims volatility due to the presence of reinsurance may be less than the reduction in premium for many undertakings due to the cost of the reinsurance, ie the appropriate net factor may often be larger than the gross factor.

4.172. Initially this may appear counter-intuitive, since it is common understanding that there are capital benefits through the purchase of reinsurance. However, we need to consider the following:

- An increase in factor (net vs gross) is not inconsistent with a lower capital requirement, since this is being driven by a lower volume measure (net premium vs gross premium). Indeed, we would clearly expect a lower net capital requirement than the comparable gross capital requirement.
- The reinsurance protection is on a “to ultimate” basis, whilst the calibration is performed on a “1 year” basis. As a result, over the one

[5] See annex 7.6

year, not all the benefit of the reinsurance is realised. However, the reinsurance cost is all charged up front (other than reinstatements). As a result there is a mis-match between the benefit of the reinsurance that emerges over the one year and the change in the premium.

- The difference between the gross and net premiums is not purely due to the claims benefits of the protection, but also used to fund the reinsurance expenses such as broker commissions, underwriting costs, etc and also to give the reinsurer an appropriate level of recompense for the level of risk they are accepting, ie risk loading, profit loading, etc.

4.173.Undertakings will be required to adjust the gross volatilities for reinsurance as follows:

- The ratio of the net combined ratio at financial year end and the gross combined ratio at financial year end can be viewed as a transformation factor for performing gross-net transitions by accident year.
- This ratio is exact in the case of quota-share reinsurance and should be viewed as a convenient approximation for surplus and non-proportional reinsurance.
- Basing the ratio on the most recent 3 financial years, will create some stability of the ratio.
- At the same time the ratio will be responsive to changes in reinsurance programs in a 3-year moving average way.
- The inputs for determining the net-gross ratio should be purified of any catastrophe effect on premiums, losses and costs. ie both gross and net claims should exclude any catastrophe claims, and catastrophe reinsurance premiums should not be deducted from gross premiums when determining net premiums. When deciding which claims should be considered as catastrophe claims, undertakings should refer to the report of the Catastrophe Task Force.

4.174.The net-gross ratio, by line of business, is determined in three steps:

$$\text{gross combined ratio} = \frac{\text{gross loss}}{\text{gross earned premium}} + \frac{\text{gross costs}}{\text{gross written premium}}$$

$$\text{net combined ratio} = \frac{\text{net loss}}{\text{net earned premium}} + \frac{\text{net costs}}{\text{net written premium}}$$

$$\text{net-gross ratio} = \frac{\text{net combined ratio}}{\text{gross combined ratio}}$$

with the following definitions of the terms:

gross losses	total best estimate ultimate claims for the last three accident years gross of reinsurance, net of salvage and subrogation, but gross of ALAE. The ultimate claims

amounts are as booked as at the end of each accident year, without allowing for any subsequent development. These figures should not include any catastrophe claims.

gross earned premium	total ultimate premium earned over the last three accident years gross of reinsurance
gross costs	total expenses (ULAE and other company expenses appropriately allocated to the LoB) excluding ALAE paid over the last three financial years.
gross written premium	total ultimate premium written over the last three financial years
net losses	total best estimate ultimate claims for the last three accident years net of reinsurance of reinsurance, net of salvage and subrogation, but gross of ALAE. The ultimate claims amounts are as booked as at the end of each accident year, without allowing for any subsequent development (to be consistent with the definition of gross losses). These figures should not include any catastrophe claims and similarly there should be no allowance for the reinsurance recoveries associated with those claims.
net earned premium	total ultimate premium earned over the last three accident years net of reinsurance. The net earned premium should include the cost of the catastrophe reinsurance protections, ie these should not be deducted from the associated gross figures.
net costs	total expenses (ULAE and other company expenses appropriately allocated to the LoB) excluding ALAE paid over the last three financial years, but including outwards reinsurance commissions. The outwards reinsurance commissions should not include any of the costs of the catastrophe protections.
net written premium	total ultimate premium written over the last three financial years net of reinsurance. The net written premium should include the cost of the catastrophe reinsurance protections, i.e. these should not be deducted from the associated gross figures.

4.175.The CEIOPS proposal has the advantages of:

- It is undertaking specific
- It is a simple and objective approach, which is produced using information that will already be supplied to the supervisor – so is less open to manipulation by undertakings.

- If a company has significant reinsurance recoveries it should produce commensurate adjustments
- The factor does not lead to over reduction in capital requirements.

4.176. Potential drawbacks are:

- Let us consider the situation where the reinsured company has just had a bad year. In this instance we would expect the effect of reinsurance to have been relatively large. As a consequence when the calculation is performed, as per the proposal from the Netherlands, the reinsurer loss ratio will be very large and thus the capital benefit the reinsured company will gain from its reinsurance will be very large. This would have the effect of reducing capital requirements after a company has a bad year, which although beneficial to companies (whose available capital is likely to have been reduced) does not appear to be sensible dynamics form a regulator's perspective. However the proposal to average experience over the last 3 years goes some way to address this issue.
- There is no evidence that this will represent the reduction equivalent to the mitigation effect over a one year time horizon.

4.3 Reserve Risk

4.177. The reserve risk calibration and results are presented below:

4.3.1. Data

4.178. The data was provided by line of business, undertaking and accident year:

- Paid claims triangle net of reinsurance recoveries
- Incurred claims triangle net of reinsurance recoveries
- Posted reserves claims triangle net of reinsurance recoveries (including case estimates, IBNR and IBNER)
- The data was judgementally filtered to remove problem data points. Examples of such adjustments include:
- Negative values in any of the data.
- Zero values for the data – since all the models used assume that this is impossible.
- Massive implied development ratios where these appear to be "errors" in the data – since these completely distort some of the methods used.
- Typographic mistakes
- Apparent inconsistencies between different years and between opening reserve and closing reserve for the same company
- Catastrophe losses
- As well as other features which were considered to be incorrect based on expert judgement.

4.179. See annex 7.3 for an illustration of the process followed.

4.180. Data available for some lines of business was still limited despite collecting further data. The analysis produced for these lines of business is thus naturally not as robust as that for lines of business with more data.

4.181. The analysis was performed directly using the data available. Thus dependent upon the data in question, implicit assumptions were made.

4.3.2. Assumptions

4.182. The expenses (excluding allocated claims handling expenses) will be a fixed proportion of the future claims reserve, i.e. these expenses will be 100% correlated to the claims reserve. Our analysis ignores the impact of expenses to derive the reserve risk standard deviation, but in the standard formula this will be applied to the reserves including these expenses. We would expect these expenses to be less volatile than the claims and for these expenses to less than 100% correlated to the claims. As a result, in theory, we would expect the estimate we derive to be conservative in this respect. CEIOPS was limited to what it could do due to lack of expense data. CEIOPS does not consider that this would be material enough to justify an adjustment to the resultant volatilities produced from the analysis.

4.183. The effect of discounting will be the same in the stressed scenario as in the best estimate. As a result, no modification to our result is necessary.

4.184. No explicit allowance was made for inflation in the calibration process. Implicitly therefore it assumed that the inflationary experience in the period 1999 to 2008 was representative of the inflation that might occur. The period analysed was a relatively benign period with low inflation in the countries supplying data and without unexpected inflation shocks which would be expected to increase the factors significantly. Thus as the data excludes significant inflationary shocks, it may underestimate the uncertainty in the provisions.

4.185. An average level of geographical diversification is implicitly allowed for in the calibration because the volatility of the undertaking's time series reflects the geographical diversification of their business.

4.186. The risk margin does not change after stressed conditions. The SCR is the difference between the economic balance sheet over the one year time horizon in the distressed scenario. This implicitly suggests that the difference between all component parts should be analysed, including the risk margin. CEIOPS has assumed that the risk margin does not change and therefore no adjustment to the factors has been made for this feature.

4.3.3. Analysis

4.187. The analysis is performed using either:

- the opening value of the gross reserves as the volume measure and the gross claims development result after one year for these exposures to derive a standard deviation.
- the gross paid and incurred triangle.

4.3.4. Methodology

4.188. CEIOPS chose the following methods for the estimation of the Non life underwriting parameters for reserve risk:

<u>Method 1</u>

4.189. This approach is intended to follow as closely as possible the approach detailed in "CEIOPS- FS-14/07 QIS3, Calibration of the underwriting risk, market risk and MCR".

4.190. This method assumes that the expected reserves in one year plus the expected incremental paid claims in one year is the current best estimate for claims outstanding.

4.191. This method involves by firm calculating the average claims reserve at each historic calendar year and the standard deviation of the following ratio: reserves in the next calendar year (excluding the new accident year) and the incremental paid claims emerging over the next calendar year (excluding the new accident year) to the reserves in this calendar year.

4.192. Essentially the standard deviation will represent the uncertainty in the expected ultimate claims over the one year time horizon for the same accident years.

4.193. The fitting process involves two stages. The first stage fits a separate model of each undertaking's standard deviation of the ratio and allows for more diversification credit within larger volumes of opening claims provision per line of business in the same way across all years within a single undertaking.

4.194. This stage uses a least squares fit of the ratio and an associated variance estimator. This estimator is optimal when the underlying distribution is Normal, as opposed to the lognormal distribution assumptions within the standard formula.

4.195. The second stage fits the reserve risk factor to these resultant undertaking specific models.

4.196. The use of a two stage process, clearly introduces a large number of parameters that need to be calibrated which translates to a significant risk of over-fitting. The effect of this would be to understate the resultant premium risk factor, but it is not entirely clear by how much.

4.197. Furthermore, the second stage puts significantly more weight to those undertakings holding larger claims provision volumes of a specific line of business, therefore any result will be biased towards factors most appropriate for larger portfolios.

4.198. Specifically if the following terms are defined as:

$PCO_{C,lob,i,j}$	=	The best estimate for claims outstanding by undertaking and LoB for accident year i and development year j
$I_{C,lob,i,j}$	=	The incremental paid claims by undertaking and LoB for accident year i and development year j
$V_{C,Y,lob}$	=	Volume measure by undertaking, calendar year and LoB
$R_{C,Y,lob}$	=	The best estimate for outstanding claims and incremental paid claims for the exposures covered by the volume measure, but in one year's time by undertaking, calendar year and LoB

$\sigma_{C,lob}$	=	Standard deviation of reserve development ratio by undertaking and LoB
$N_{C,lob}$	=	The number of calendar years of data available by undertaking and LoB where there is both a value of $V_{C,Y,lob}$ and $R_{C,Y,lob}$.
$V_{C,lob}$	=	Average volume measure by undertaking and LoB

4.199.Then the following relationships can be defined as:

$$V_{C,Y,lob} = \sum_{i+j=Y+1} PCO_{C,lob,i,j}$$

$$R_{C,Y,lob} = \sum_{\substack{i+j=Y+2 \\ i\neq Y+1}} PCO_{C,lob,i,j} + \sum_{\substack{i+j=Y+2 \\ i\neq Y+1}} I_{C,lob,i,j}$$

4.200.Then, remembering that the reserve should be the expected value of future claims development,

i.e. $E\left(\frac{R_{C,Y,lob}}{V_{C,Y,lob}}\right) = 1$

4.201.the following relationships are obtained:

$$\sigma_{C,lob} = \sqrt{\frac{1}{V_{C,lob}}}\sqrt{\frac{1}{N_{C,lob}-1}\left(\sum_{Y}\frac{1}{V_{C,Y,lob}}\left(R_{C,Y,lob} - V_{C,Y,lob}\right)^2\right)}$$ and

$$V_{C,lob} = V_{C,\max(Y),lob}$$

4.202.The factors are then determined using least squares optimisation across the undertakings within the LoB.

4.203.If the following term is defined as:

$\sigma_{(res,lob)}$	=	Standard deviation for reserve risk by LoB

4.204.Then $\sigma_{(res,lob)}$ can be derived by taking a volume weighted average of the fitted undertaking specific standard deviations as below:

$$\hat{\sigma}_{(res,lob)} = \frac{\sum_{C} V_{C,lob}\sigma_{C,lob}}{\sum_{C} V_{C,lob}}$$

<u>Method 2</u>

4.205.This approach is consistent with the undertaking specific estimate assumptions from the Technical Specifications for QIS4 for reserve risk.

4.206. The assumptions are that for any undertaking, any year and any LoB:

- The expected reserves in one year plus the expected incremental paid claims in one year is the current best estimate for claims outstanding.
- The variance of the best estimate for claims outstanding in one year plus the incremental claims paid over the one year is proportional to the current best estimate for claims outstanding and
- The maximum likelihood fitting approach is appropriate.

4.207. The process involves two stages. The first stage fits a single model for the standard deviations across all undertakings simultaneously. Thus standard deviations by undertaking takes into account the experience of all the other undertakings when assessing this particular undertaking.

4.208. Compared to method 1, only one parameter is fitted per line of business. The consequences of this will be less over-fitting and as a result is likely to lead to an overall higher volatility.

4.209. This stage also allows for more diversification credit within larger volumes of opening claims provision per line of business in the same way across all years and all undertakings.

4.210. This stage uses a maximum likelihood for a lognormal to fit the variance estimator. As opposed to method 1 this fitting approach is aligned to the lognormal distribution assumptions within the standard formula.

4.211. As an attempt to derive a single factor per line of business, across all firms we have taken a linearly weighted average of the standard deviations by undertaking.

4.212. Effectively this assumes that the sample of undertakings used in the fitting process is representative of all of Europe in terms of associated claims provision volumes as well as putting significantly more weight to those undertakings which write larger volumes of a specific line of business, therefore any result will be biased towards factors most appropriate for larger portfolios.

4.213. If the following terms are defined as:

β_{lob}^2	=	Constant of proportionality for the variance of the best estimate for claims outstanding in one year plus the incremental claims paid over the one year by LoB
$\varepsilon_{C,Y,lob}$	=	An unspecified random distribution with mean zero and unit variance
$M_{C,Y,lob}$	=	The mean of the logarithm of the best estimate for claims outstanding in one year plus the incremental claims paid over the one year by undertaking, accident year and LoB
$S_{C,Y,lob}$	=	The standard deviation of the logarithm of the best estimate for claims outstanding in one year plus the

		incremental claims paid over the one year by undertaking, accident year and LoB
$PCO_{C,lob,i,j}$	=	The best estimate for claims outstanding by undertaking and LoB for accident year i and development year j
$I_{C,lob,i,j}$	=	The incremental paid claims by undertaking and LoB for accident year i and development year j
$V_{C,Y,lob}$	=	Volume measure by undertaking, calendar year and LoB
$R_{C,Y,lob}$	=	The best estimate for outstanding claims and incremental paid claims for the exposures covered by the volume measure, but in one year's time by undertaking, calendar year and LoB
N_{lob}	=	The number of data points available by LoB where there is both a value of $V_{C,Y,lob}$ and $R_{C,Y,lob}$.
$V_{C,lob}$	=	Average volume measure by undertaking and LoB

4.214.Then the following relationships can be determined as:

$$V_{C,Y,lob} = \sum_{i+j=Y+1} PCO_{C,lob,i,j}$$

$$R_{C,Y,lob} = \sum_{\substack{i+j=Y+2 \\ i\neq Y+1}} PCO_{C,lob,i,j} + \sum_{\substack{i+j=Y+2 \\ i\neq Y+1}} I_{C,lob,i,j}$$

4.215.Then the distribution of losses can be formulated as:

$$R_{C,Y,lob} \sim V_{C,Y,lob} + \sqrt{V_{C,Y,lob}}\,\beta_{lob}\varepsilon_{C,Y,lob}$$

4.216. The parameters of the lognormal distributions can be formulated as follows:

$$S_{C,Y,lob} = \sqrt{\log\left(1 + \frac{\beta_{lob}^2}{V_{C,Y,lob}}\right)}$$

$$M_{C,Y,lob} = \log(V_{C,Y,lob}) - \frac{1}{2}S_{C,Y,lob}^2$$

4.217.The resultant simplified log Likelihood becomes

$$\log L = \sum_{C,Y}\left(-\log(S_{C,Y,lob}) - \frac{(\log(R_{C,Y,lob}) - M_{C,Y,lob})^2}{2S_{C,Y,lob}^2}\right)$$

4.218.The parameter β_{lob} is chosen to maximise this likelihood.

4.219.If the following term is defined as:

$\sigma_{(C,res,lob)}$	=	Standard deviation for reserve risk by Undertaking by LoB

4.220.The $\sigma_{(C,res,lob)}$ then becomes :

$$\sigma_{C,res,lob} = \frac{\hat{\beta}_{lob}}{\sqrt{V_{C,lob}}} \quad \text{where}$$

$$V_{C,lob} = V_{C,\max(Y),lob}$$

4.221.If the following term is defined as:

$\sigma_{(res,lob)}$	=	Standard deviation for reserve risk by LoB

4.222.Then a value for $\sigma_{(res,lob)}$ is determined by taking a volume weighted average of the fitted undertaking specific standard deviations as below:

$$\hat{\sigma}_{(res,lob)} = \frac{\sum_C V_{C,lob}\sigma_{C,res,lob}}{\sum_C V_{C,lob}}$$

<u>Method 3</u>

4.223.This approach is essentially consistent with the standard formula representation of the relationship between volatility of future reserve deterioration and volume.

4.224. The assumptions are that for any undertaking, any year and any LoB:

- The expected reserves in one year plus the expected incremental paid claims in one year is the current best estimate for claims outstanding.
- The variance of the best estimate for claims outstanding in one year plus the incremental claims paid over the one year is proportional to the square of the current best estimate for claims outstanding and
- The maximum likelihood fitting approach is appropriate.

4.225. If the following terms are defined:

β^2_{lob}	=	Constant of proportionality for the variance of the best estimate for claims outstanding in one year plus the incremental claims paid over the one year by LoB
$\varepsilon_{C,Y,lob}$	=	An unspecified random distribution with mean zero and unit variance
$M_{C,Y,lob}$	=	The mean of the logarithm of the best estimate for claims outstanding in one year plus the incremental claims paid over the one year by undertaking,

		accident year and LoB
$S_{C,Y,lob}$	=	The standard deviation of the logarithm of the best estimate for claims outstanding in one year plus the incremental claims paid over the one year by undertaking, accident year and LoB
$PCO_{C,lob,i,j}$	=	The best estimate for claims outstanding by undertaking and LoB for accident year i and development year j
$I_{C,lob,i,j}$	=	The incremental paid claims by undertaking and LoB for accident year i and development year j
$V_{C,Y,lob}$	=	Volume measure by undertaking, calendar year and LoB
$R_{C,Y,lob}$	=	The best estimate for outstanding claims and incremental paid claims for the exposures covered by the volume measure, but in one year's time by undertaking, calendar year and LoB
N_{lob}	=	The number of data points available by LoB where there is both a value of $V_{C,Y,lob}$ and $R_{C,Y,lob}$.

4.226.Then the following relationships are defined:

$$V_{C,Y,lob} = \sum_{i+j=Y+1} PCO_{C,lob,i,j}$$

$$R_{C,Y,lob} = \sum_{\substack{i+j=Y+2 \\ i\neq Y+1}} PCO_{C,lob,i,j} + \sum_{\substack{i+j=Y+2 \\ i\neq Y+1}} I_{C,lob,i,j}$$

4.227.Then the distribution of losses can be formulated as:

$$R_{C,Y,lob} \sim V_{C,Y,lob} + V_{C,Y,lob}\beta_{lob}\varepsilon_{C,Y,lob}$$

4.228.This allows the parameters of the lognormal distributions to be formulated as follows:

$$S_{C,Y,lob} = \sqrt{\log\left(1+\beta_{lob}^2\right)}$$

$$M_{C,Y,lob} = \log\left(V_{C,Y,lob}\right) - \frac{1}{2}S_{C,Y,lob}^2$$

4.229. The resultant simplified log Likelihood becomes

$$\log L = \sum_{C,Y}\left(-\log\left(S_{C,Y,lob}\right) - \frac{\left(\log\left(R_{C,Y,lob}\right) - M_{C,Y,lob}\right)^2}{2S_{C,Y,lob}^2}\right)$$

4.230. The parameter β_{lob} is chosen to maximise this likelihood.

$\sigma_{(res,lob)}$	=	Standard deviation for reserve risk by LoB

4.231.Then we can derive a value for $\sigma_{(res,lob)}$ as below:

$$\hat{\sigma}_{(res,lob)} = \hat{\beta}_{lob}$$

Method 4

4.232.This approach is consistent with the undertaking specific estimate assumptions from the Technical Specifications for QIS4 for reserve risk.

4.233.This method involves a three stage process:

a. **Involves by undertaking calculating the mean squared error of prediction of the claims development result over the one year.**

- o The mean squared errors are calculated using the approach detailed in "Modelling The Claims Development Result For Solvency Purposes" by Michael Merz and Mario V Wuthrich, Casualty Actuarial Society E-Forum, Fall 2008.
- o Furthermore, in the claims triangles:
- o cumulative payments $C_{i,j}$ in different accident years i are independent
- o for each accident year, the cumulative payments $(C_{i,j})_j$ are a Markov process and there are constants f_j and s_j such that $E(C_{i,j}|C_{i,j-1})=f_jC_{i,j-1}$ and $Var(C_{i,j}|C_{i,j-1})=s_j^2C_{i,j-1}$.

b. **Involves fitting a model by undertaking to the results of the Merz method:**

- o The assumptions are that for any LoB:
- o The appropriate volume measure is the best estimate for claims outstanding as derived by the chain ladder for the undertaking.
- o The variance of the claims development result is proportional to the current best estimate for claims outstanding and
- o The least squares fitting approach, of the undertaking specific standard deviations, is appropriate.

4.234.Specifically if the following terms are defined:

$PCO_{C,lob}$	=	The current best estimate for claims outstanding as derived by the chain ladder by undertaking and LoB
$V_{C,lob}$	=	Volume measure by undertaking and LoB
$MSEP_{C,lob}$	=	The mean squared error of prediction of the claims development result in one year's time, as prescribed by the paper referenced above, by undertaking and LoB

4.235.Then the following relationship can be defined:

$$V_{C,lob} = PCO_{C,lob}$$

4.236.If the following term is defined:

β_{lob}^2	=	Constant of proportionality for the variance of the claims development result by LoB

Then the least squares estimator of the coefficients of variation is the value of β_{lob} which minimises the following function:

$$\sum_C \left(\frac{\beta_{lob}}{\sqrt{V_{C,lob}}} - \frac{\sqrt{MSEP_{C,lob}}}{V_{C,lob}} \right)^2$$

4.237.By differentiating this function with respect to β_{lob} and setting this to zero the following least squares estimator is obtained:

$$\hat{\beta}_{lob} = \frac{\sum_C \frac{\sqrt{MSEP_{C,lob}}}{V_{C,lob}^{3/2}}}{\sum_C \frac{1}{V_{C,lob}}}$$

And

$$\sigma_{C,res,lob} = \frac{\hat{\beta}_{lob}}{\sqrt{V_{C,lob}}} \quad \text{where}$$

c. **Estimating the volume weighted average across all undertakings**

4.238.If the following terms are defined:

$V_{C,lob}^{'}$		The best estimate for claims outstanding by undertaking and LoB
$\sigma_{(res,lob)}$	=	Standard deviation for reserve risk by LoB

4.239.Then a value for $\sigma_{(res,lob)}$ can be determined by taking a volume weighted average of the fitted undertaking specific standard deviations as below:

$$\hat{\sigma}_{(res,lob)} = \frac{\sum_C V_{C,lob}^{'} \sigma_{C,res,lob}}{\sum_C V_{C,lob}^{'}}$$

<u>Method 5</u>

4.240. This approach is consistent with the undertaking specific estimate assumptions from the Technical Specifications for QIS4 for premium risk.

4.241. This method involves a two stage process:

a. Involves by undertaking calculating the mean squared error of prediction of the claims development result over the one year.

- The mean squared errors are calculated using the approach detailed in "Modelling The Claims Development Result For Solvency Purposes" by Michael Merz and Mario V Wuthrich, Casualty Actuarial Society E-Forum, Fall 2008.
- Furthermore, in the claims triangles:
- cumulative payments $C_{i,j}$ in different accident years i are independent
- for each accident year, the cumulative payments $(C_{i,j})_j$ are a Markov process and there are constants f_j and s_j such that $E(C_{i,j}|C_{i,j-1})=f_jC_{i,j-1}$ and $Var(C_{i,j}|C_{i,j-1})=s_j^2C_{i,j-1}$.

b. Involves fitting a model by undertaking to the results of the Merz method:

- The appropriate volume measure is the best estimate for claims outstanding as derived by the chain ladder for the undertaking.
- The variance of the claims development result is proportional to the square of the current best estimate for claims outstanding and
- The least squares fitting approach, of the undertaking specific standard deviations, is appropriate.

4.242. Specifically if the following terms are defined:

$PCO_{C,lob}$	=	The current best estimate for claims outstanding as derived by the chain ladder by undertaking and LoB
$V_{C,lob}$	=	Volume measure by undertaking and LoB
$MSEP_{C,lob}$	=	The mean squared error of prediction of the claims development result in one year's time, as prescribed by the paper referenced above, by undertaking and LoB

4.243. Then the following relationship can be defined:

$$V_{C,lob} = PCO_{C,lob}$$

4.244. If the following term is defined:

$\sigma_{(res,lob)}$	=	Standard deviation for reserve risk by LoB

Then the least squares estimator of standard deviation is the value of $\sigma_{(res,lob)}$ which minimises the following function:

$$\sum_C \left(V_{C,lob}\sigma_{(res,lob)} - \sqrt{MSEP_{C,lob}} \right)^2$$

4.245.By differentiating this function with respect to $\sigma_{(res,lob)}$ and setting this to zero the following least squares estimator is obtained by :

$$\hat{\sigma}_{(res,lob)} = \frac{\sum_C V_{C,lob}\sqrt{MSEP_{C,lob}}}{\sum_C V_{C,lob}^2}$$

Method 6

4.246.This method involves a two stage process:

a. Involves by undertaking calculating the mean squared error of prediction of the claims development result over the one year.

- o The mean squared errors are calculated using the approach detailed in "Modelling The Claims Development Result For Solvency Purposes" by Michael Merz and Mario V Wuthrich, Casualty Actuarial Society E-Forum, Fall 2008.
- o Furthermore, in the claims triangles:
- o cumulative payments $C_{i,j}$ in different accident years i are independent
- o for each accident year, the cumulative payments $(C_{i,j})_j$ are a Markov process and there are constants f_j and s_j such that $E(C_{i,j}|C_{i,j-1})=f_jC_{i,j-1}$ and $Var(C_{i,j}|C_{i,j-1})=s_j^2C_{i,j-1}$.

b. Involves fitting a model by undertaking to the results of the Merz method:

- The appropriate volume measure is the best estimate for claims outstanding as derived by the chain ladder for the undertaking.
- The variance of the claims development result is proportional to the square of the current best estimate for claims outstanding and
- The least squares fitting approach, of the undertaking specific coefficients of variation, is appropriate.

4.247.Specifically the following terms are defined:

$PCO_{C,lob}$	=	The current best estimate for claims outstanding as derived by the chain ladder by undertaking and LoB
$V_{C,lob}$	=	Volume measure by undertaking and LoB

$MSEP_{C,lob}$	=	The mean squared error of prediction of the claims development result in one year's time, as prescribed by the paper referenced above, by undertaking and LoB
N_{lob}	=	The number of undertakings by LoB where there is both a value of $PCO_{C,lob}$ and $MSEP_{C,lob}$.

4.248. Then we can define the following relationship:

$$V_{C,lob} = PCO_{C,lob}$$

4.249. The following term is defined as follows:

$\sigma_{(res,lob)}$	=	Standard deviation for reserve risk by LoB

Then the least squares estimator of the coefficients of variation is the value of $\sigma_{(res,lob)}$ which minimises the following function:

$$\sum_C \left(\sigma_{(res,lob)} - \frac{\sqrt{MSEP_{C,lob}}}{V_{C,lob}} \right)^2$$

4.250. By differentiating this function with respect to $\sigma_{(res,lob)}$ and setting this to zero we obtain the following least squares estimator:

$$\hat{\sigma}_{(res,lob)} = \frac{\sum_C \frac{\sqrt{MSEP_{C,lob}}}{V_{C,lob}}}{N_{lob}}$$

4.3.5. Reserve Risk Results

4.251. CEIOPS has presented the results of the gross analysis through a combination of tables and graphs.

4.252. The tables present the results for all 6 methods described in section 4.3.4:

- The analysis includes a column of fitted factors by method based on an estimated volume weighted average of the standard deviation estimates by undertaking. Effectively this assumes that the sample of undertakings used in the fitting process is representative of all of Europe in terms of associated premium volumes as well as putting significantly more weight to those undertakings which write larger volumes of a specific line of business, therefore any result will be biased towards factors most appropriate for larger portfolios.

- The table includes the percentage of undertakings which would have a gross standard deviation, as assessed under Method 1, greater than the selected technical result.

4.253. Results vary across methods because each method uses different underlying assumptions. For example:

- The individual estimates of the standard deviations by undertaking that result from the application of Method 1 are plotted against the prediction model for comparison. The individual estimates can be used as evidence of the existence of diversification credit for volume. Where such an effect does exist the graph would be expected in general to be decreasing.
- This also implies that capital requirements are significantly higher for smaller than larger portfolios. This arises for two reasons:
- Larger accounts are usually less volatile than smaller accounts. Thus expressed as a percentage of premiums a larger account often has smaller theoretical capital requirements than a smaller account.
- Larger insurers often have a greater degree of diversification of risks than smaller insurers.

4.254. For those methods where diversification credit is assumed to exist, an illustration of what the factor could be for 3 sizes is presented: small, which equates to a 25th percentile of the sample observations, medium a 50th percentile, large 90th percentile.

4.255. The appropriateness of each method and the underlying assumptions are tested and presented by showing the results of a goodness of test fit through a PP plot.

4.256. The Merz methods (4, 5 and 6) are plotted in a third graph. Here we are able to observe whether there is diversification credit as well as a comparison of the individual observations versus the fitted models. Observations used for methods 1 to 3 are not necessarily included in methods 4 to 6.

4.257. The selection of the final fitted factors was based on the following:

- The evidence of diversification by size has not been given full allowance. i.e. no consideration has been given to the fact that volatilities by size of portfolio may be significantly different. Therefore more focus has been placed on the fitted factors.
- Factors have been selected as the average of those methods which were considered to produce an acceptable fit according to the goodness of fit plots shown

4.258. CEIOPS would like to highlight that the selection was not conservatively selected, but rather based on the goodness of fit results and the adequacy of the method. Furthermore by taking an average across methods, CEIOPS is ensuring that the factors are not biased towards factors most appropriate for larger portfolios (and hence lower).

4.259.A more thorough illustration of the process followed is outlined in the Annex 7.3.

Motor, vehicle liability

4.260.CEIOPS recommendation is that for the Motor TPL lob the gross factor for reserve risk should be 11%.

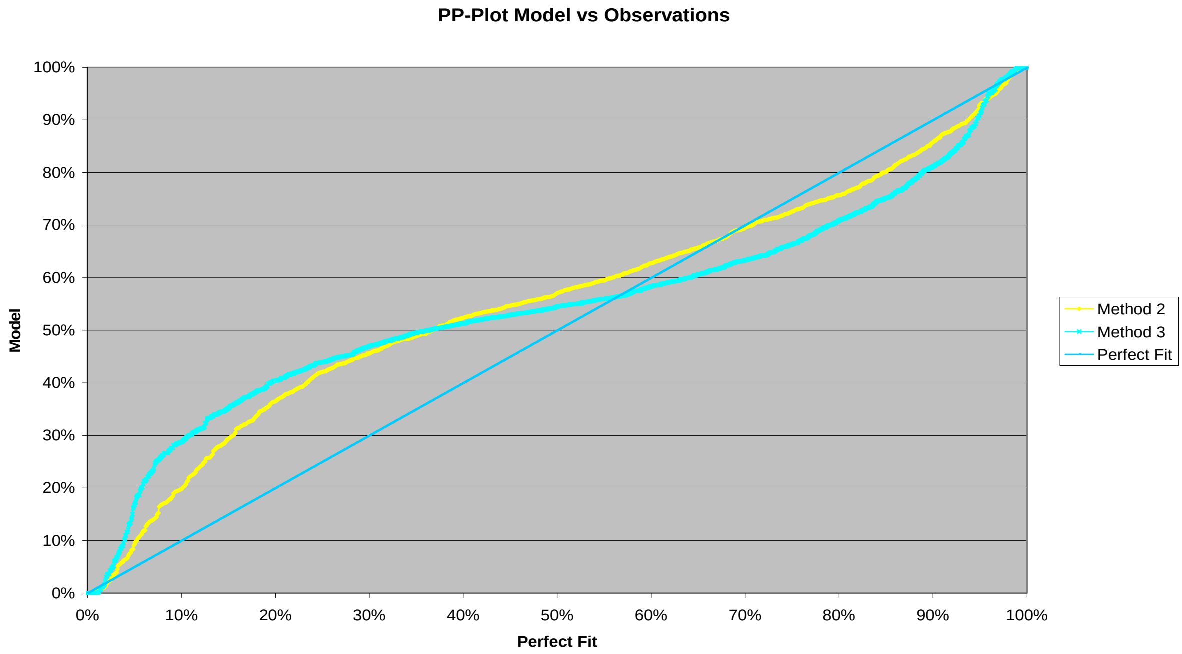
4.261.The data sample included data from 327 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, SI, PT, SK, IS, IT, LT, DK, SE, HU, FI and DE.

Reference co	Small	Medium	Large
Motor, third-party liability	15,308	68,037	219,317

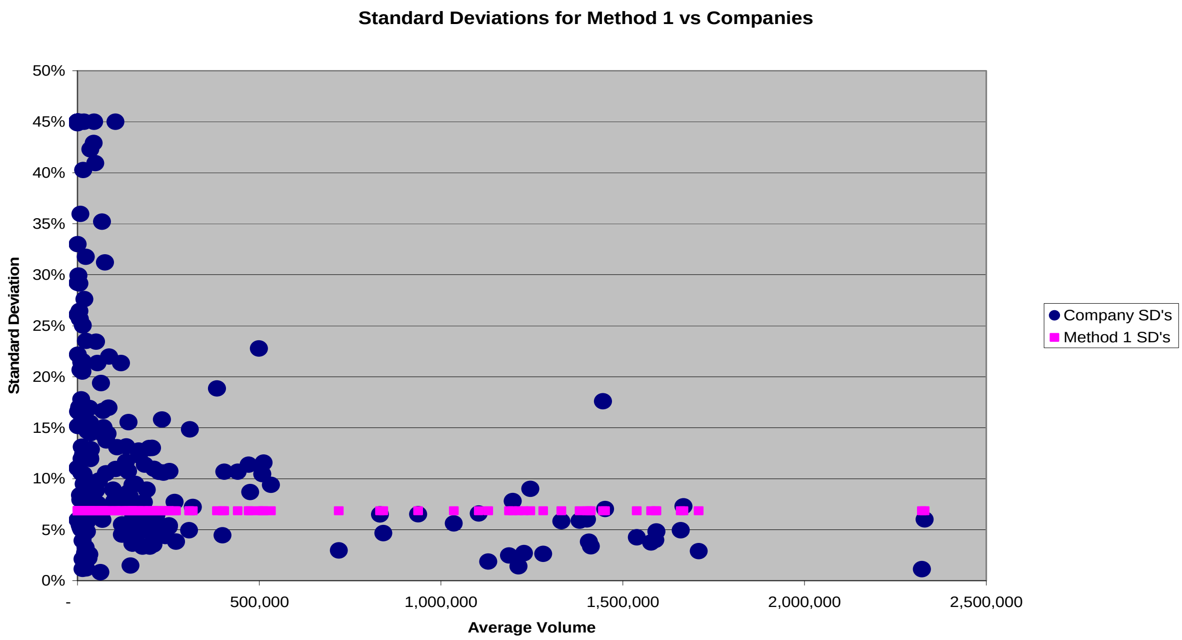
GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	17%	10%	6%	6%	**10.8%**	**44.9%**
Method 2	40%	19%	10%	7%		
Method 3				25%		
Method 4	9%	4%	2%	2%		
Method 5				6%		
Method 6				11%		

4.262.The graph below shows a pp plot of the fit of the models. Both methods provide a relatively poor fit, although there is some credibility in the tail.

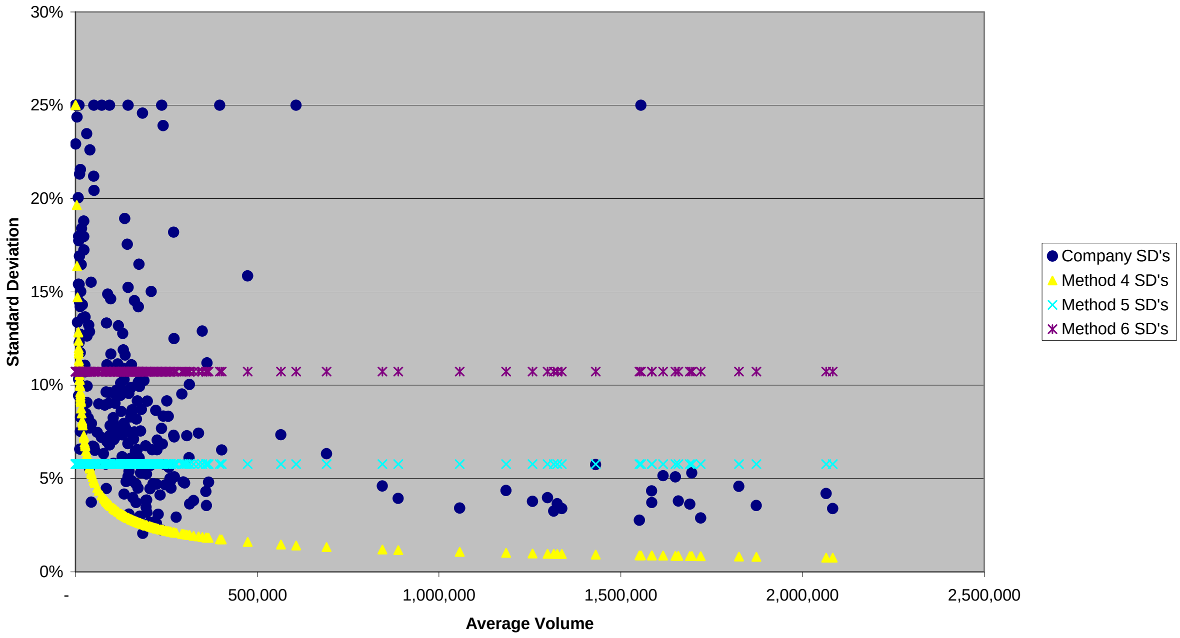


4.263. The result of the graph below shows some evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1.



4.264. The graph below shows the results for the Merz methods.

Standard Deviations for Methods 4, 5 and 6 vs Companies



Overall conclusions:

4.265. The selected technical factor was chosen as the average of methods 1, 2, 3 5 and 6 – result 10.8%.

Motor, other classes

4.266.CEIOPS recommendation is that for the Motor other the gross factor for reserve risk should be 20%.

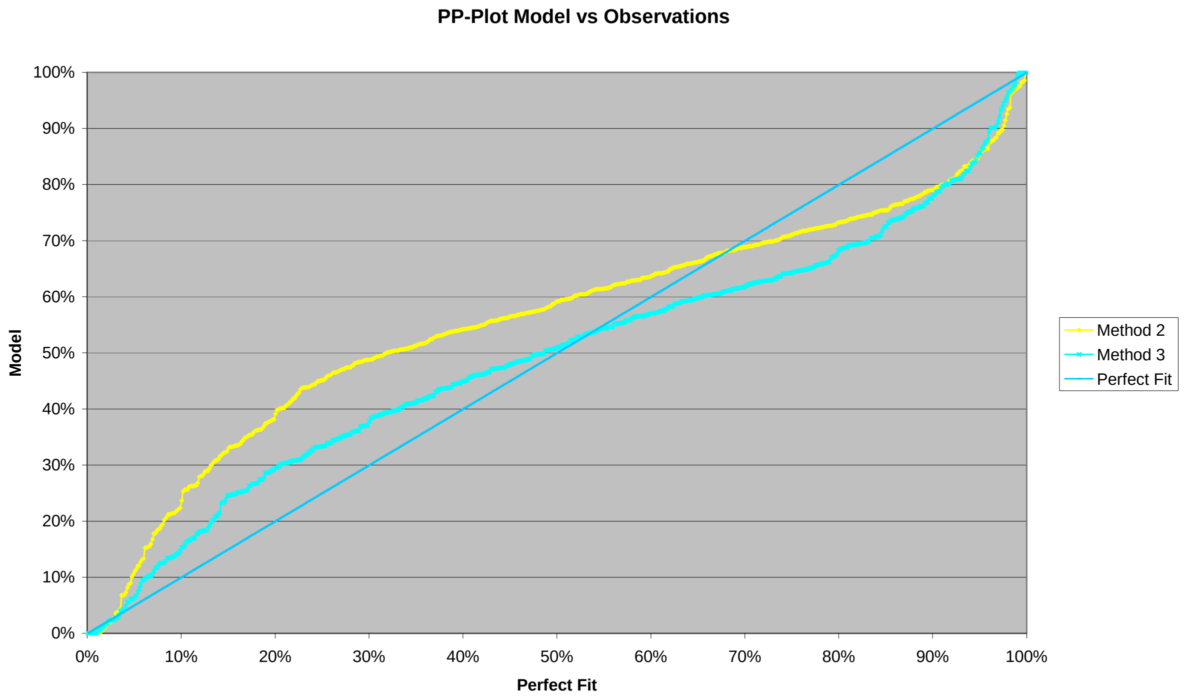
4.267.The data sample included data from 106 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, SI, PT, SK, IS, LT, FI, DK and SE.

Reference co	Small	Medium	Large
Motor, other classes	1,460	4,054	16,170

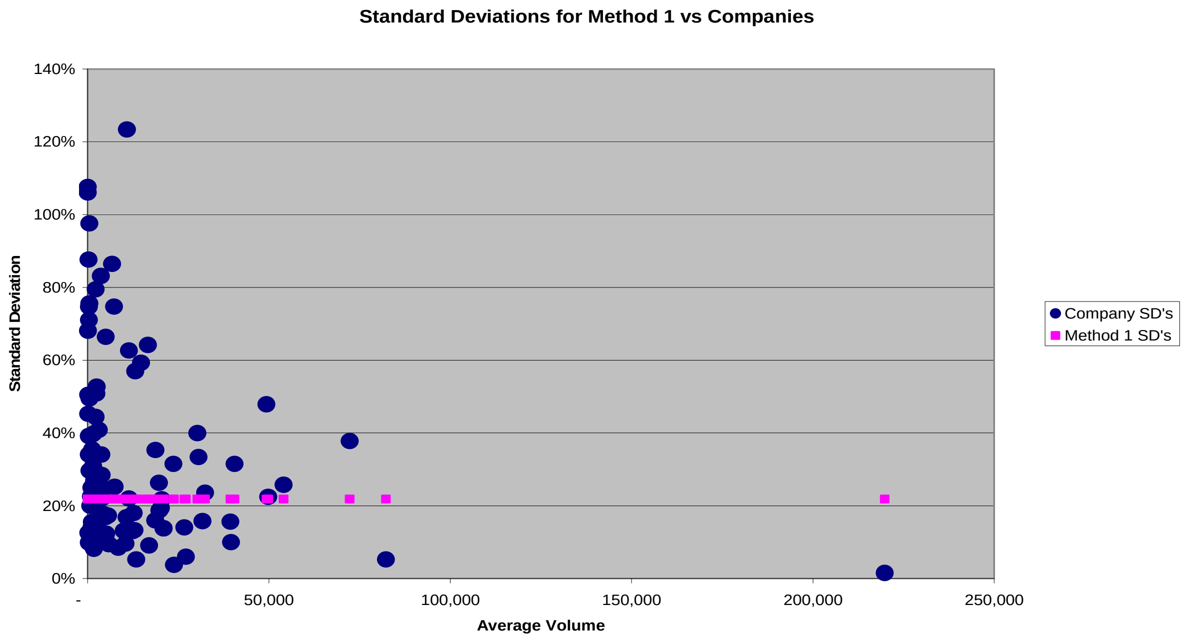
GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	40%	23%	14%	22%	**19.9%**	**59.4%**
Method 2	112%	67%	34%	29%		
Method 3				42%		
Method 4	23%	14%	7%	6%		
Method 5				12%		
Method 6				26%		

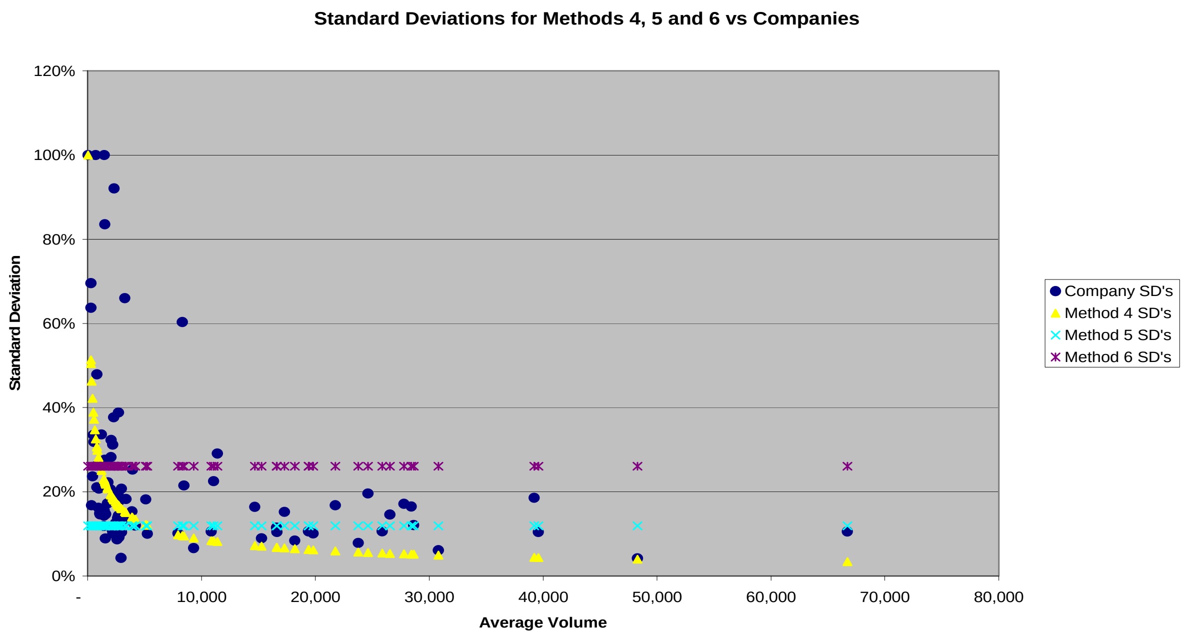
4.268.The graph below shows a pp plot of the fit of the models. Both methods provide a relatively poor fit, although method 3 appears to be a bit better than method 2.



4.269.The result of the graph below shows some evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1.



4.270.The graph below shows the results for the Merz methods.



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Overall conclusions:

4.271.The selected technical factor was chosen considering the average of methods 1, 5 and 6 – result 19.9%.

Marine, aviation, transport (MAT)

4.272.CEIOPS recommendation is that for the MAT lob the gross factor for reserve risk is 40%.

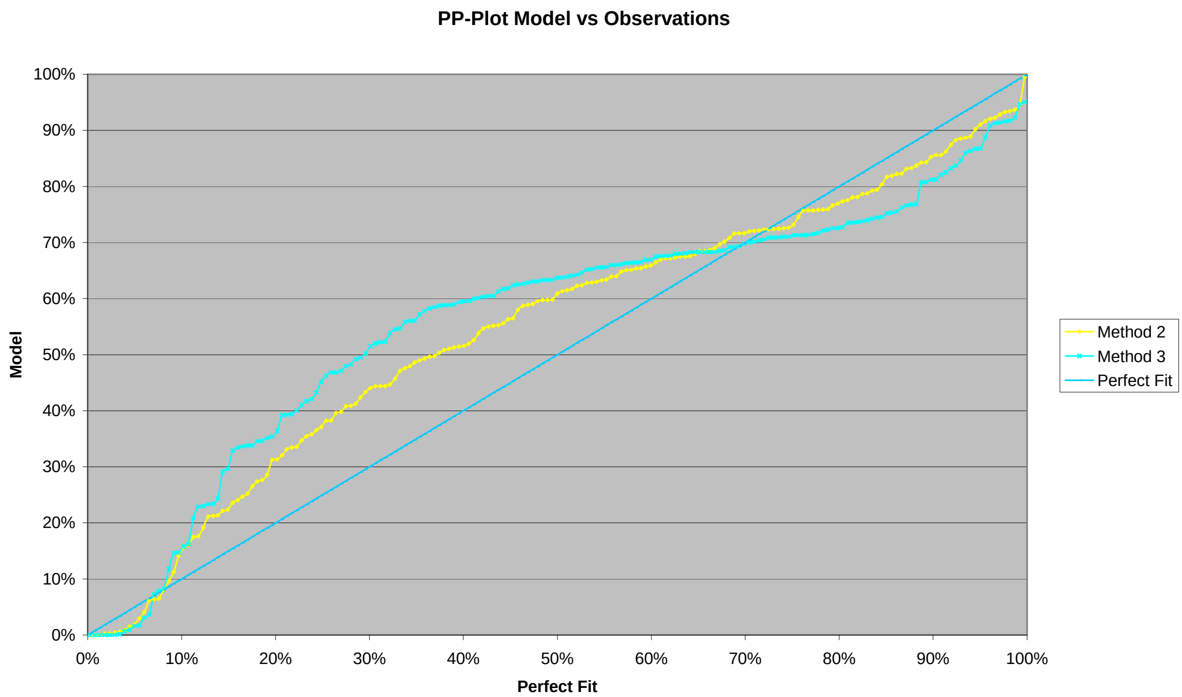
4.273.The data sample included data from 36 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, SI, IS, DK and SE.

Reference co	Small	Medium	Large
Marine, aviation, transport (MAT)	158	1,311	11,289

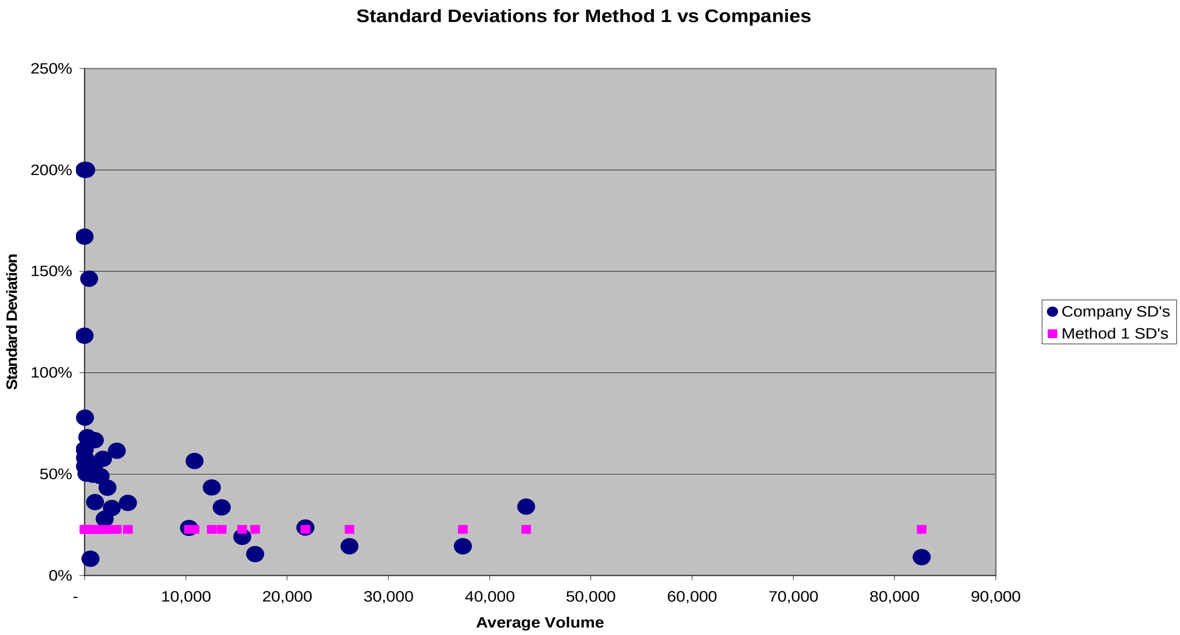
GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	63%	50%	32%	23%	**38.7%**	**61.1%**
Method 2	365%	127%	43%	33%		
Method 3				121%		
Method 4	192%	67%	23%	18%		
Method 5				31%		
Method 6				68%		

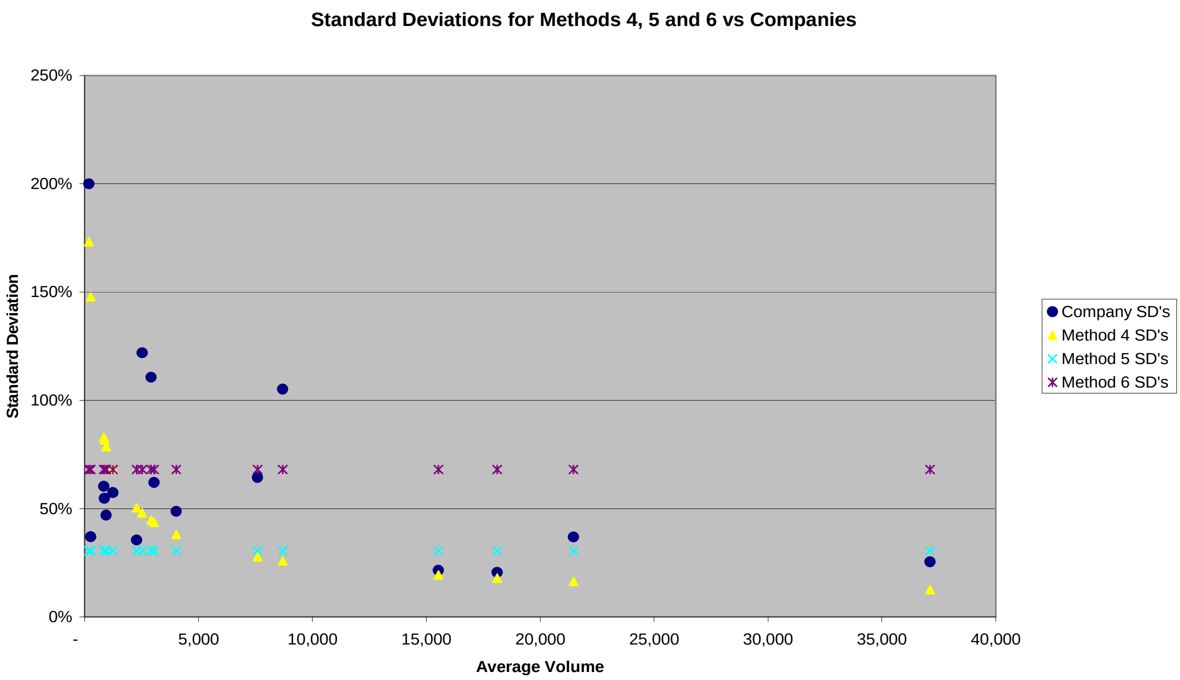
4.274.The graph below shows a pp plot of the fit of the models. Both methods provide a relatively poor fit, although method 2 appears to be just about acceptable.



4.275.The result of the graph below shows some evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1.



4.276.The graph below shows the results for the Merz methods.



Overall conclusions:

4.277.The selected technical factor was chosen considering the average of methods 1, 2, 5 and 6 – result 38.7%.

Fire and other property damage

4.278.CEIOPS recommendation is that for the Fire and other property damage lob the gross factor for reserve risk is 25%.

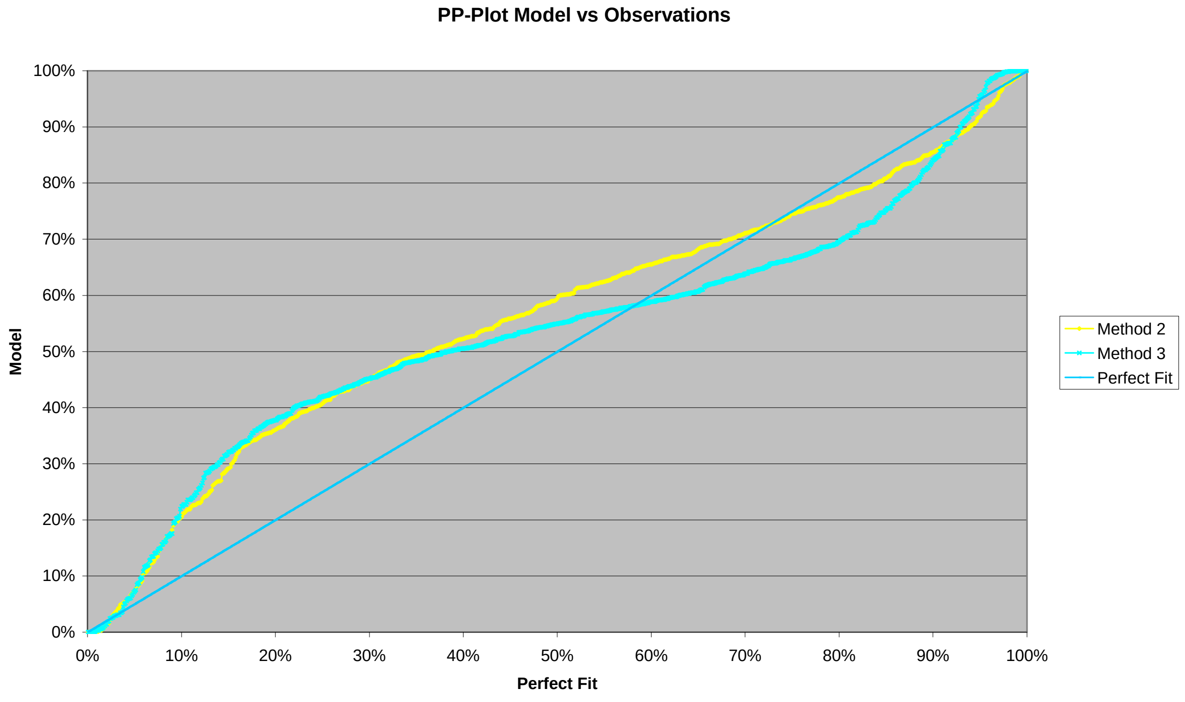
4.279.The data sample included data from 86 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, UK, SK, IS, FI, DK and SL.

Reference co	Small	Medium	Large
Fire and other property damage	7,893	35,211	89,540

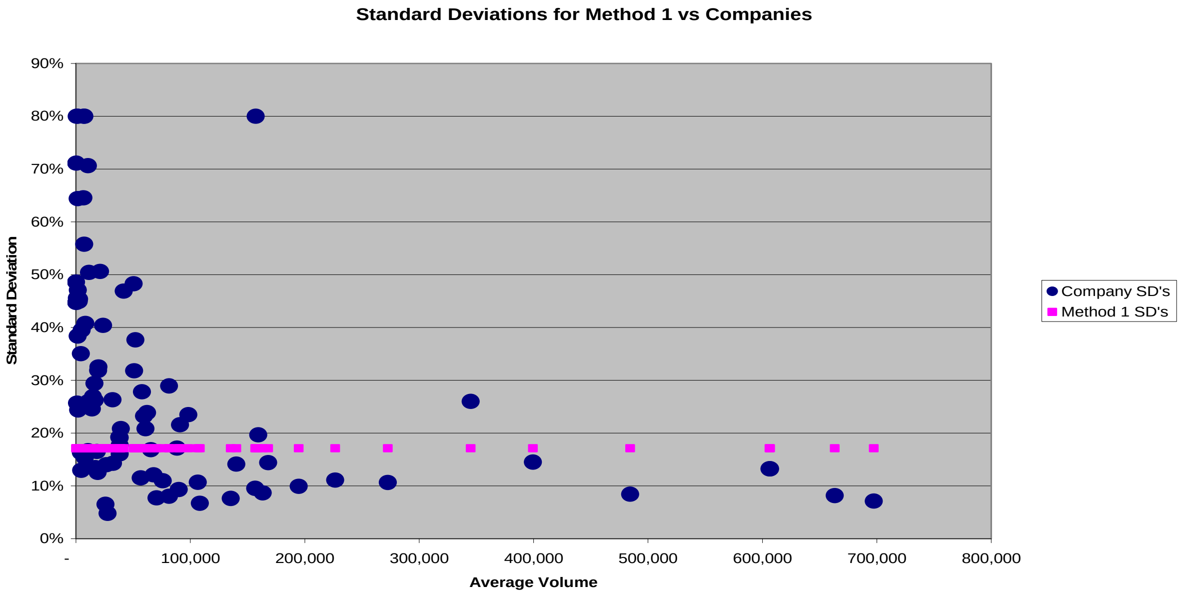
GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	40%	22%	13%	17%	**25.1%**	**45.3%**
Method 2	81%	38%	24%	18%		
Method 3				55%		
Method 4	24%	11%	7%	5%		
Method 5				21%		
Method 6				44%		

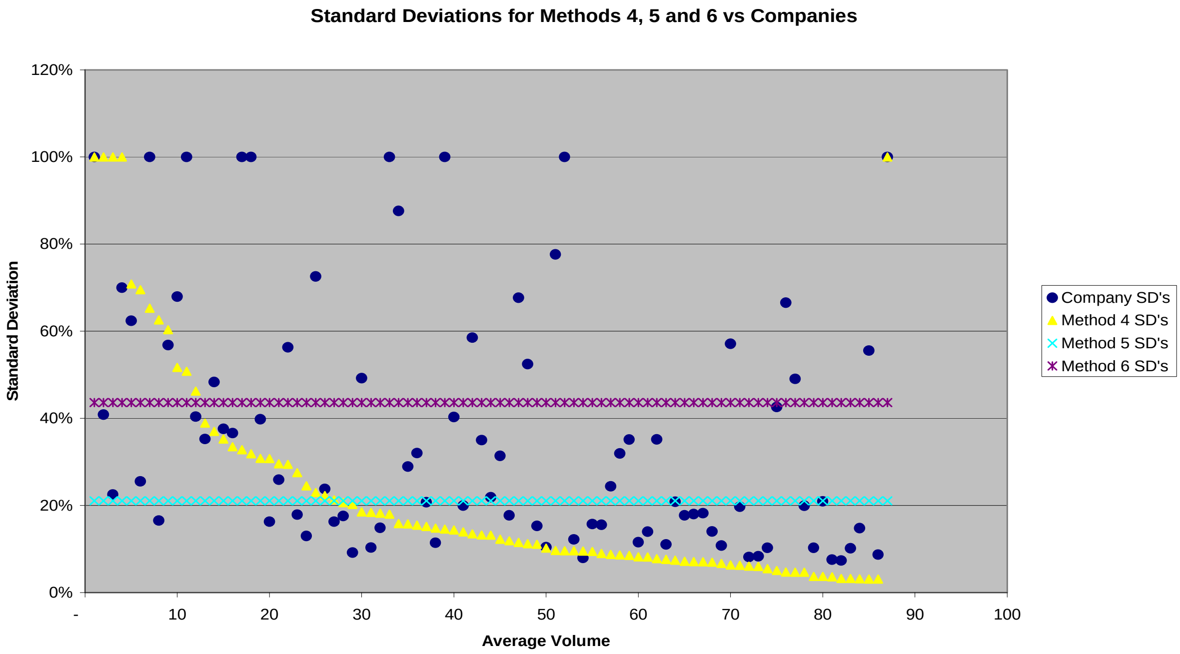
4.280.The graph below shows a pp plot of the fit of the models. Both methods provide a relatively poor fit, although method 2 appears to be just about acceptable.



4.281. The result of the graph below shows significant evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1.



4.282.The graph below shows the results for the Merz methods



Overall conclusions:

4.283.The selected technical factor was chosen considering the average of methods 1, 2, 5 and 6 – result 25.1%.

Third-party liability

4.284.CEIOPS recommendation is that for the third party liability lob the gross factor for reserve risk is 23%.

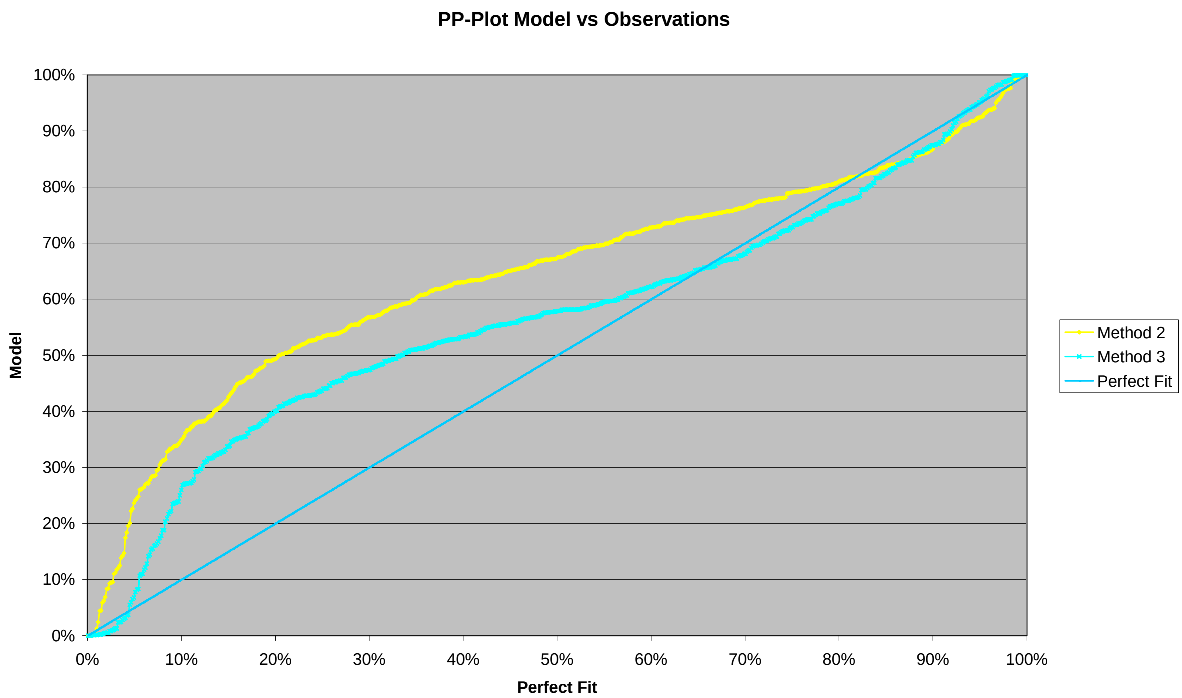
4.285.The data sample included data from 219 undertakings, was gross of reinsurance and included data from the following member states: PO, LU, DE, UK, SK, IS, DK and SI.

Reference co	Small	Medium	Large
Third-party liability	1,467	13,129	48,521

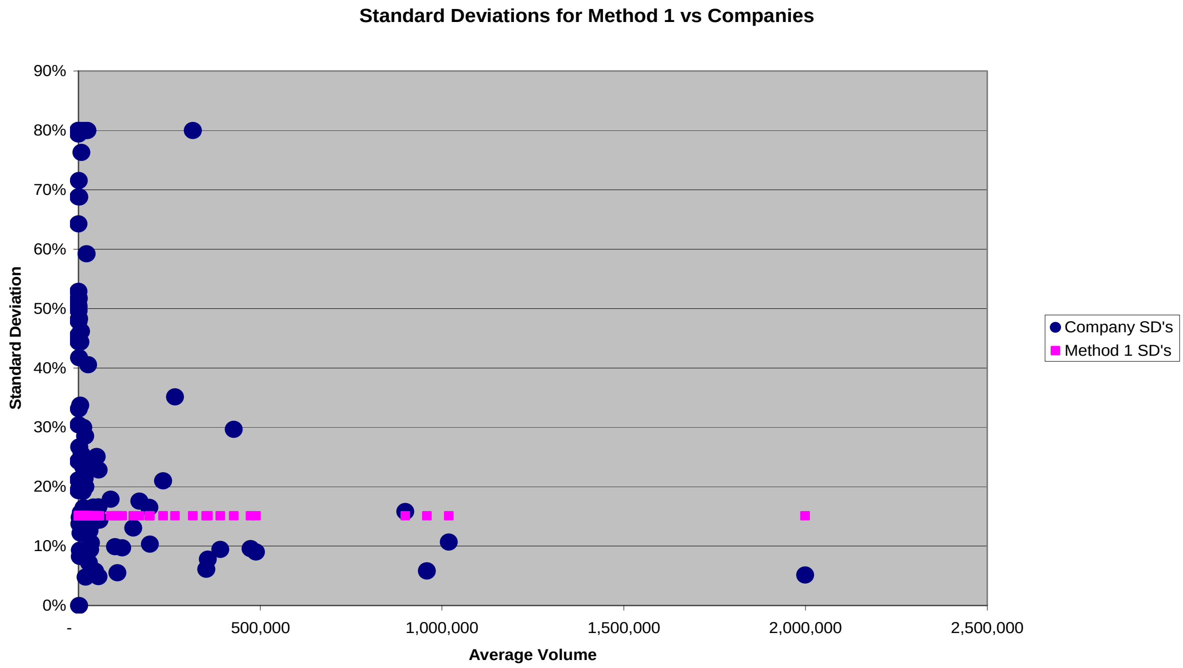
GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	50%	21%	13%	15%	**23.3%**	**47.6%**
Method 2	221%	74%	38%	17%		
Method 3				43%		
Method 4	25%	8%	4%	2%		
Method 5				14%		
Method 6				22%		

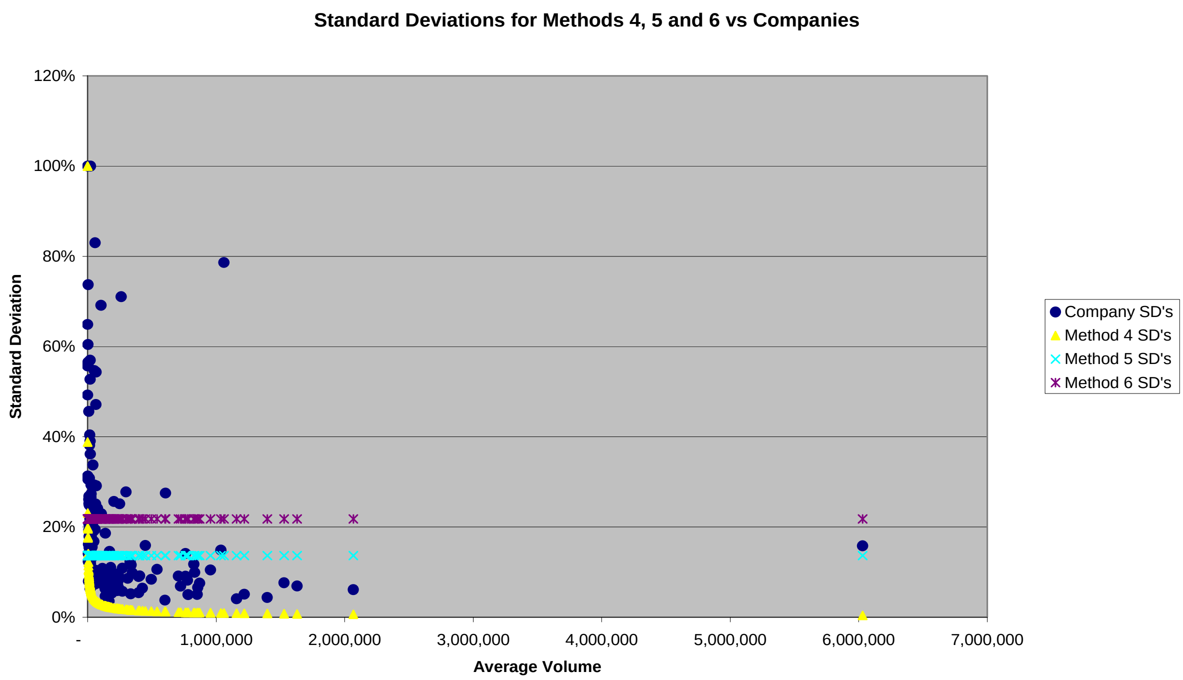
4.286.The graph below shows a pp plot of the fit of the models. Both methods provide a relatively poor fit, although method 3 appears to be just about acceptable.



4.287. The result of the graph below shows evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1



4.288. The graph below shows the results for the Merz methods



Overall conclusions:

4.289. The selected technical factor was chosen considering the average of methods 1, 3, 5 and 6 – result 23.3%.

Credit and suretyship

4.290.CEIOPS recommendation is that for the credit and suretyship lob the gross factor for reserve risk is 50%.

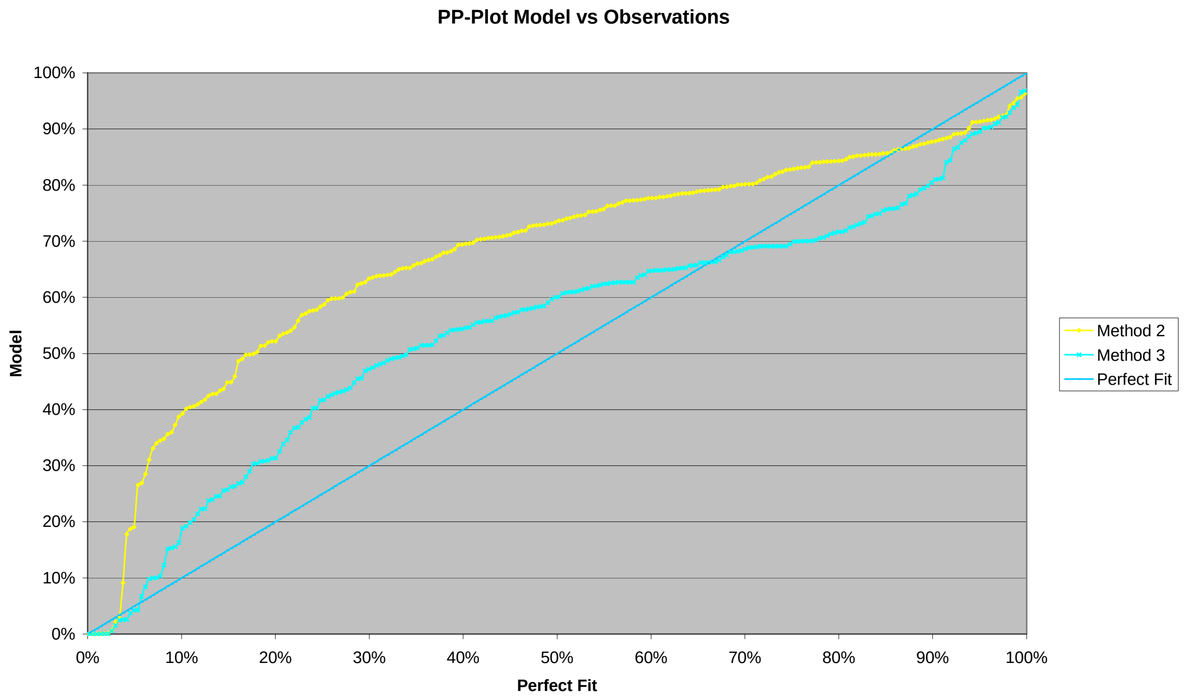
4.291.The data sample included data from 53 undertakings, was gross of reinsurance and included data from the following member states: PO, UK, SK, DK, SE, LU and SI.

Reference co	Small	Medium	Large
Credit and suretyship	560	2,695	8,626

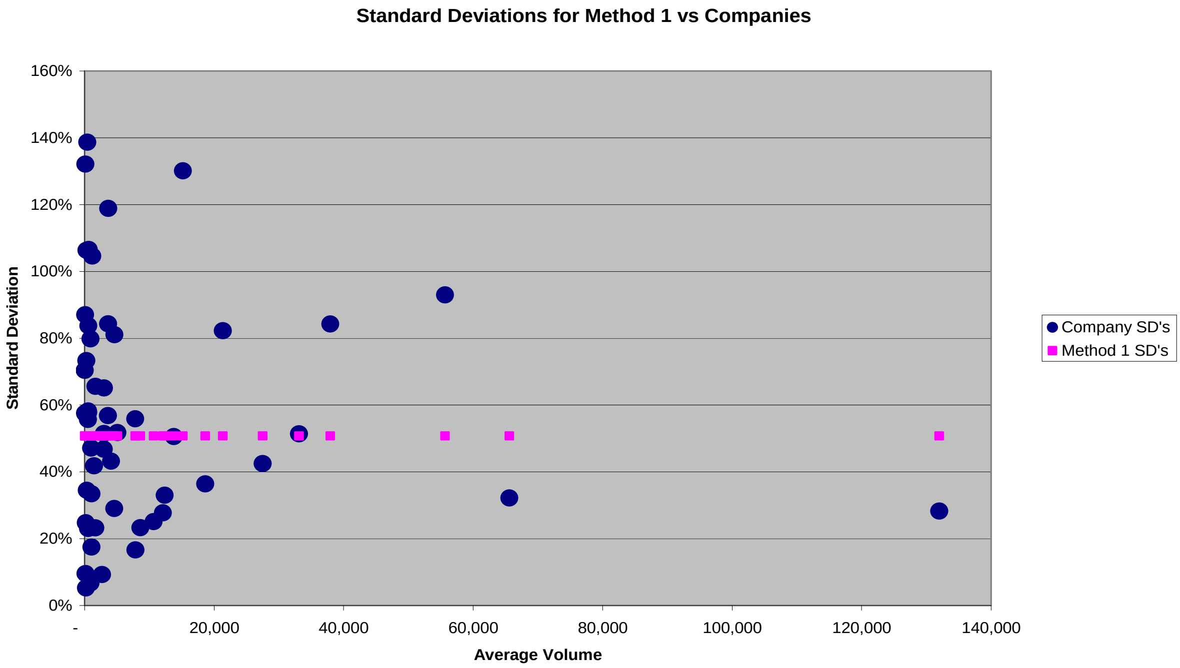
GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	81%	51%	29%	51%	**50.7%**	**52.8%**
Method 2	672%	306%	171%	112%		
Method 3				131%		
Method 4	32%	15%	8%	5%		
Method 5				49%		
Method 6				298%		

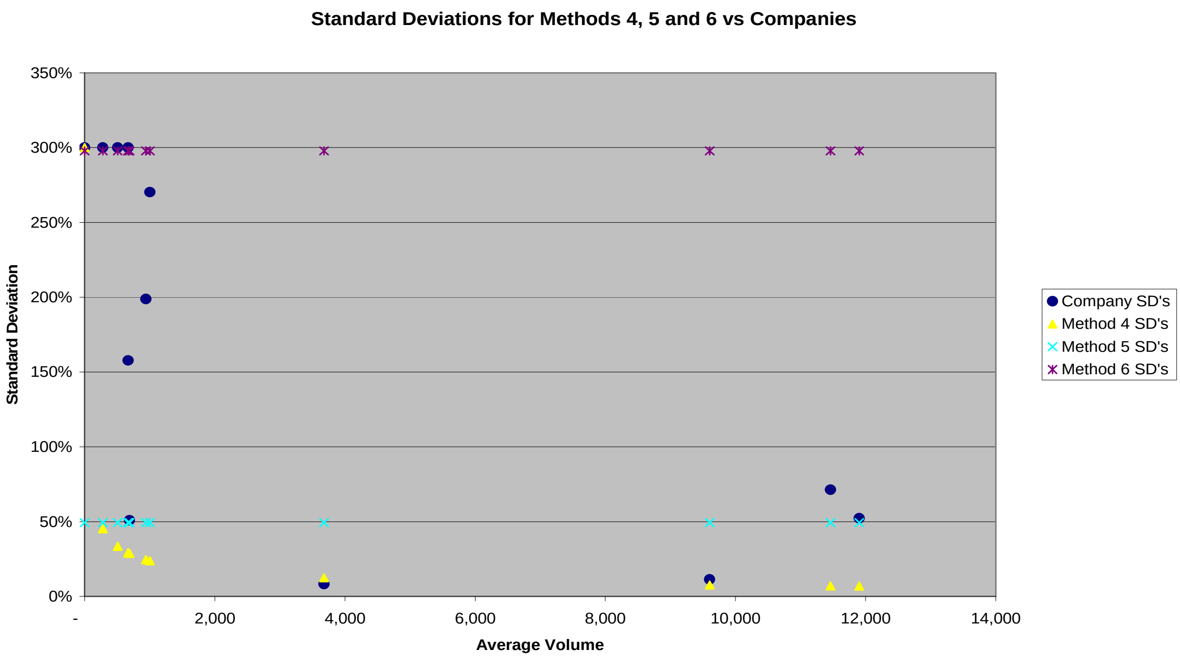
4.292.The graph below shows a pp plot of the fit of the models. Both methods provide a poor fit.



4.293. The result of the graph below shows some evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1



4.294. The graph below shows the results for the Merz methods



Overall conclusions:

4.295. The selected technical factor was chosen considering method 1 alone – result 50.7%.

Legal expenses

4.296.CEIOPS recommendation is that for the legal expenses lob the gross factor for reserve risk is 9%.

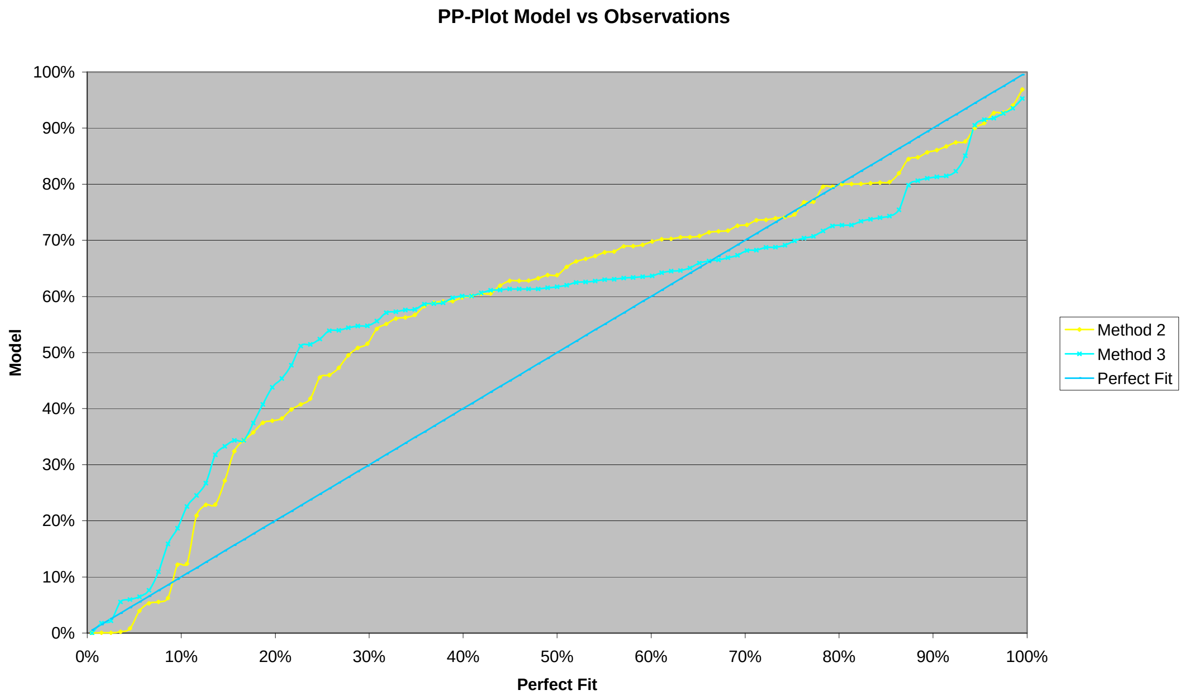
4.297.The data sample included data from 68 undertakings, was gross of reinsurance and included data from the following member states: PO, DE, SK, FI and UK.

Reference co	Small	Medium	Large
Legal expenses	556	2,892	11,541

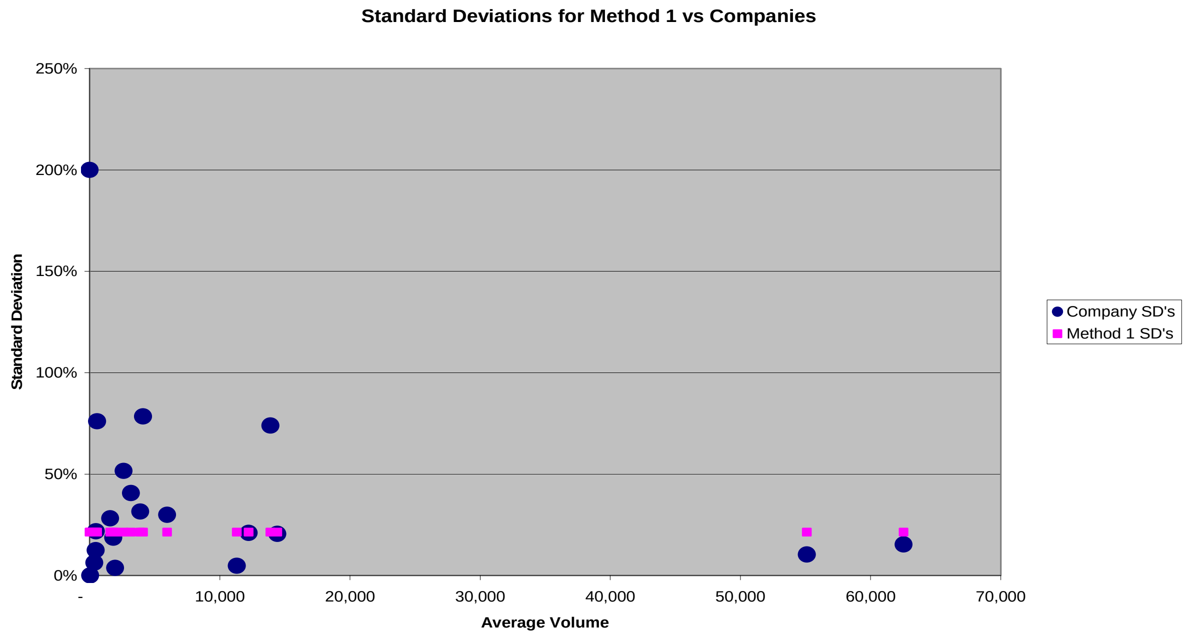
GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	43%	21%	12%	21%	**9.1%**	**80.0%**
Method 2	115%	51%	25%	20%		
Method 3				63%		
Method 4	63%	27%	14%	11%		
Method 5				4%		
Method 6				14%		

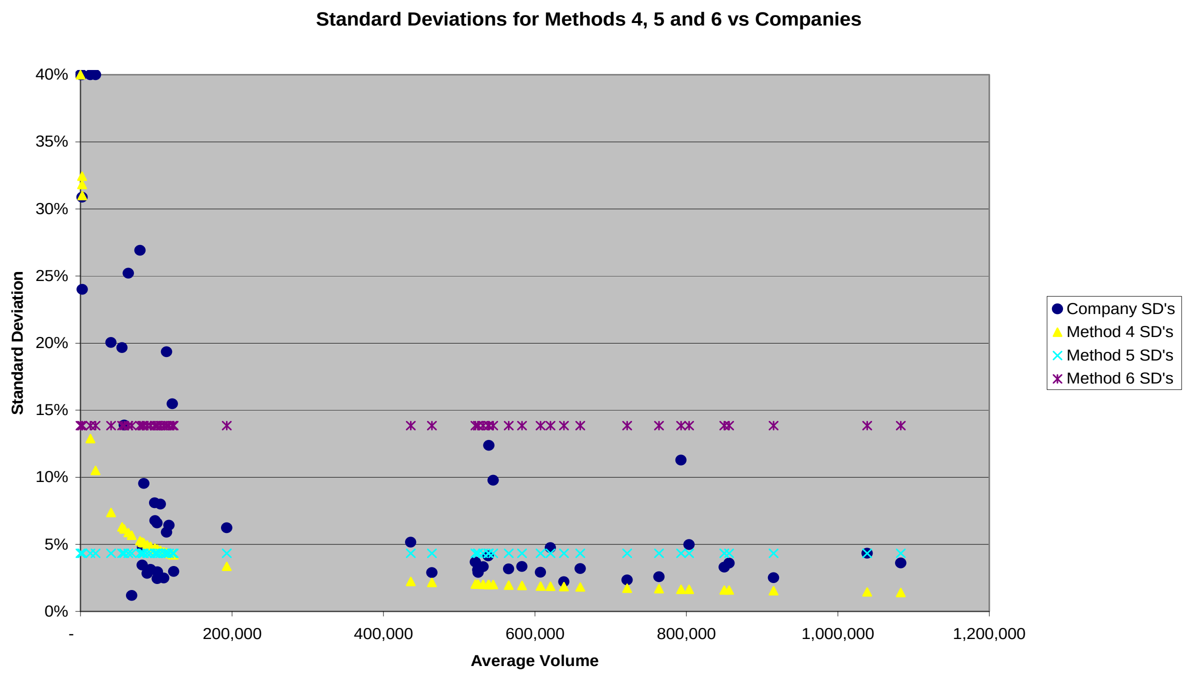
4.298.The graph below shows a pp plot of the fit of the models. Both methods provide a poor fit.



4.299.The result of the graph below shows evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1



4.300.The graph below shows the results for the Merz methods .



Overall conclusions:

4.301.The selected technical factor was chosen considering the average of methods 5 and 6 – result 9.1%.

Assistance

4.302.CEIOPS recommendation is that for the Assistance lob the gross factor for reserve risk is 45%.

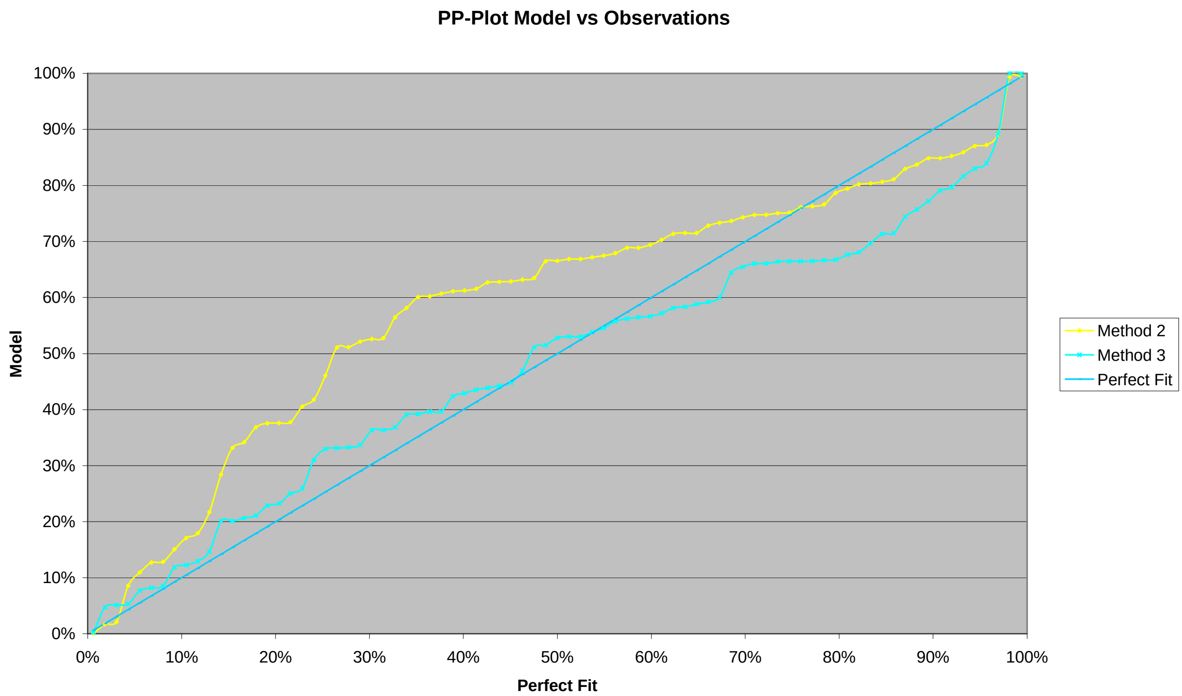
4.303.The data sample included data from 20 undertakings, was gross of reinsurance and included data from the following member states: PO and UK.

Reference co	Small	Medium	Large
Assistance	560	1,287	4,305

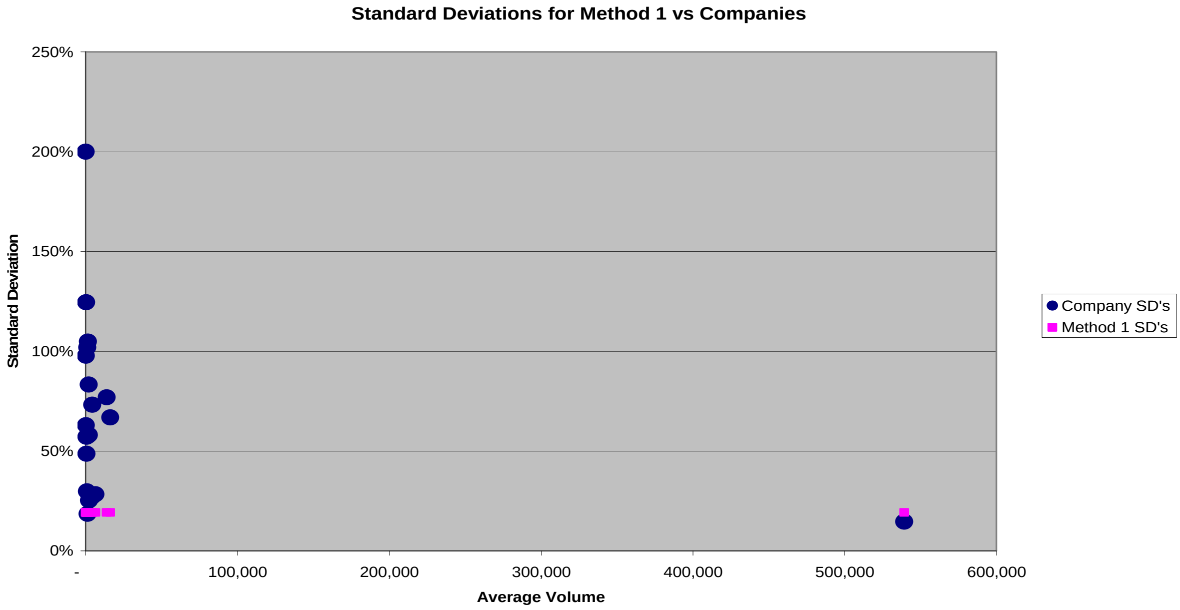
GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	87%	60%	29%	19%	**44.7%**	**70.0%**
Method 2	327%	215%	118%	20%		
Method 3	103%	103%	103%	103%		
Method 4	57%	38%	21%	4%		
Method 5	41%	41%	41%	41%		
Method 6	74%	74%	74%	74%		

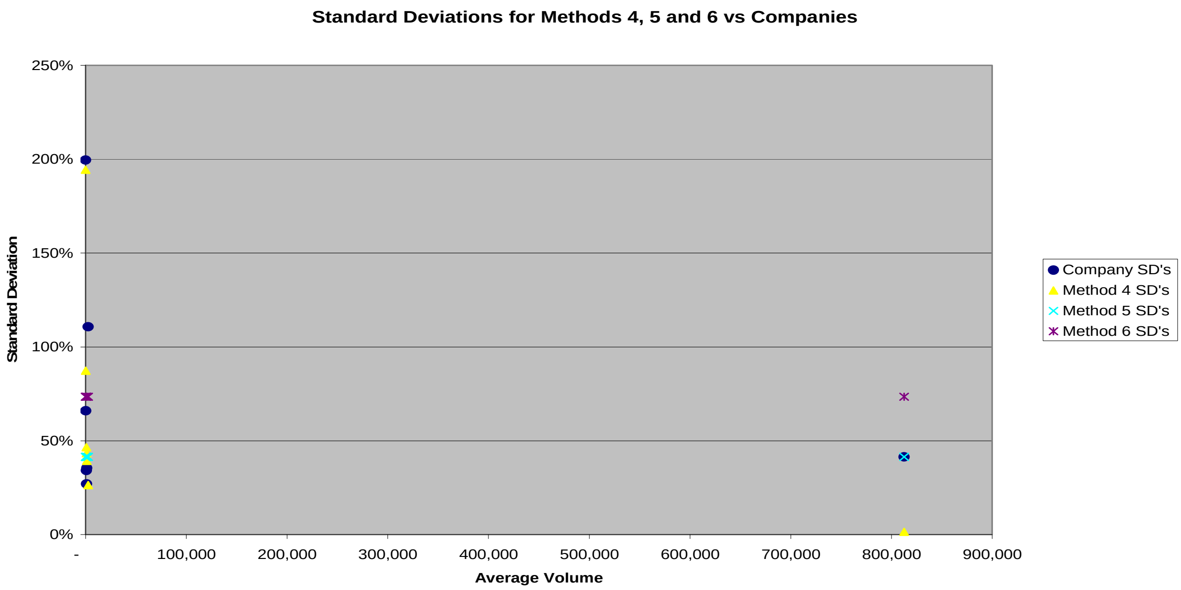
4.304.The graph below shows a pp plot of the fit of the models. Both methods provide a poor fit.



4.305. The result of the graph below shows some evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1.



4.306. The graph below shows the results for the Merz methods.



Overall conclusions:

4.307. The selected technical factor was chosen considering the average of methods 1, 5 and 6 – result 44.7%.

Miscellaneous

4.308.CEIOPS recommendation is that for the Miscellaneous lob the gross factor for reserve risk is 40%.

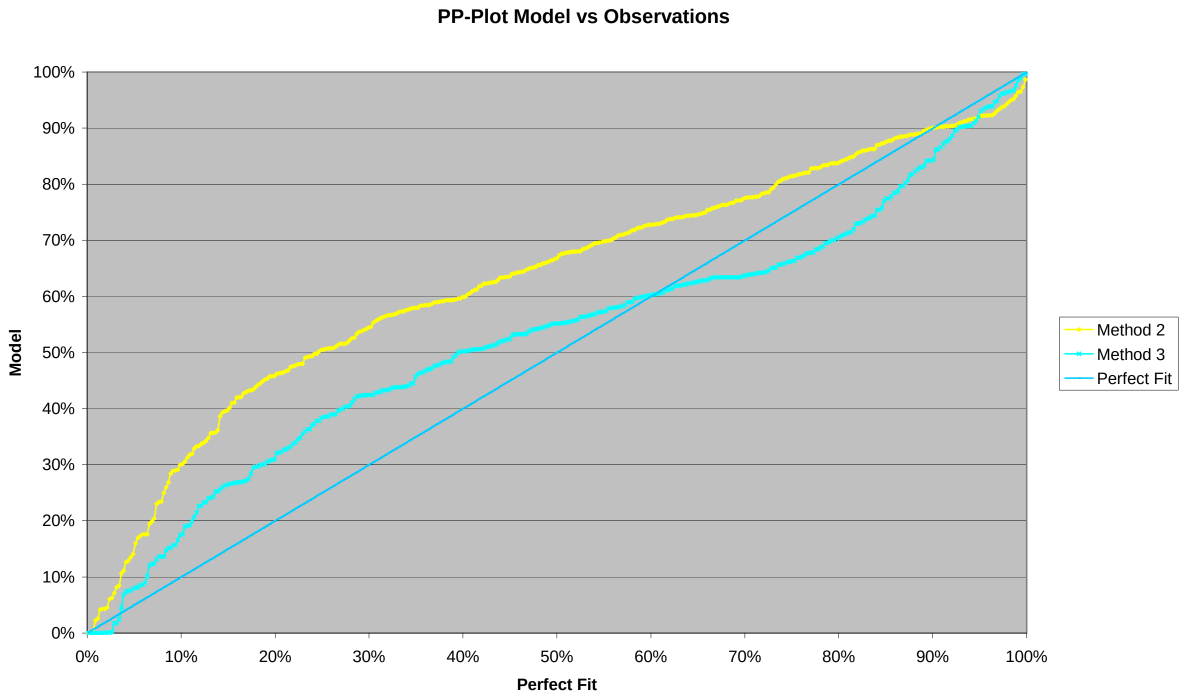
4.309.The data sample included data from 71 undertakings, was gross of reinsurance and included data from the following member states: PO, UK and DK.

Reference co	Small	Medium	Large
Miscellaneous	561	4,445	16,603

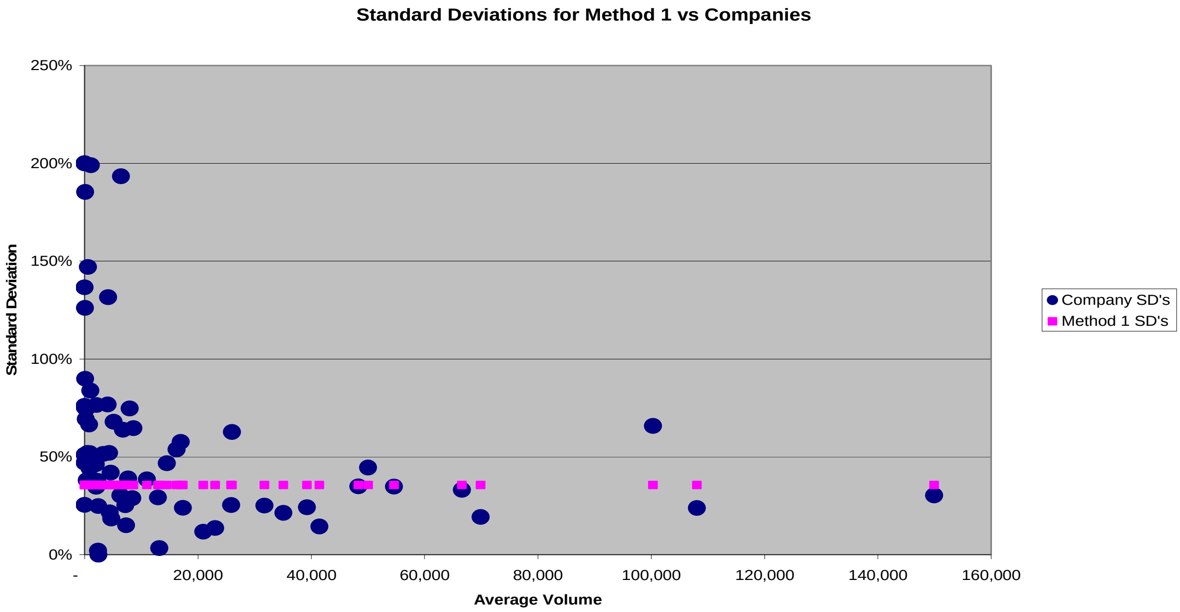
GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	72%	47%	25%	36%	**41.5%**	**56.3%**
Method 2	435%	154%	80%	59%		
Method 3				78%		
Method 4	26%	9%	5%	4%		
Method 5				36%		
Method 6				53%		

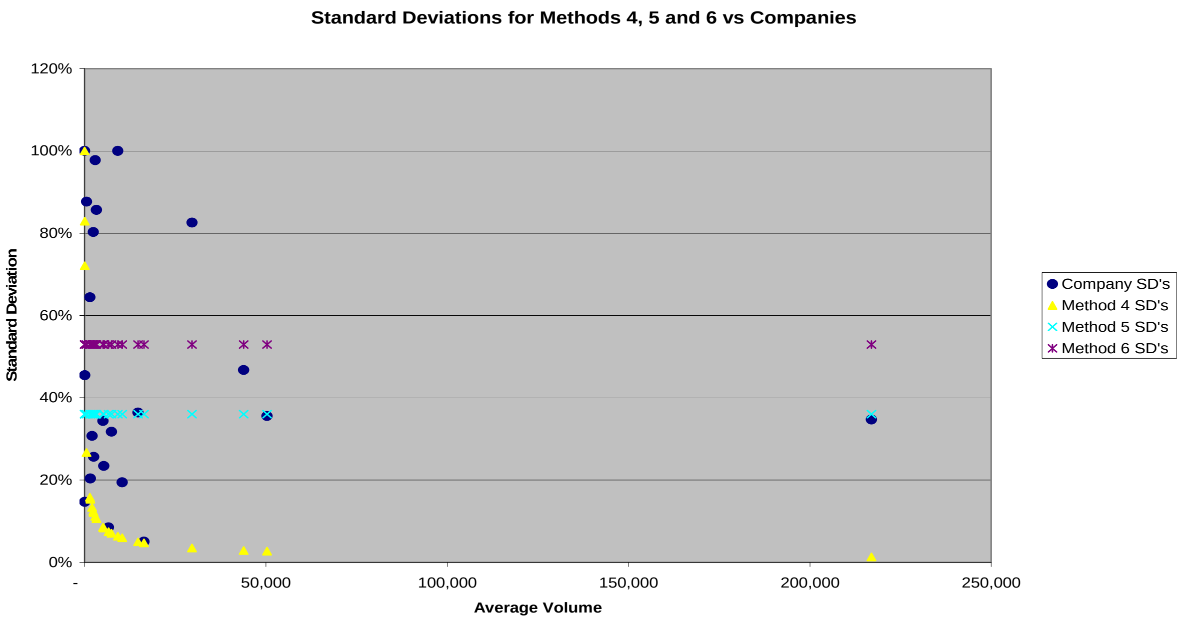
4.310.The graph below shows a pp plot of the fit of the models. Both methods provide a poor fit, but Method 3 is slightly better than Method 2.



4.311.The result of the graph below shows some evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1..



4.312.The graph below shows the results for the Merz methods.



Overall conclusions:

4.313.The selected technical factor was chosen considering the average of methods 1, 5 and 6 – result 41.5%.

Non-proportional reinsurance - property

4.314.CEIOPS recommendation is that for the non-proportional reinsurance property lob the gross factor for reserve risk is 45%.

4.315.The data sample included data from 8 undertakings, was gross of reinsurance and included data from the following member states: UK. Lack of data in respect of provisions, did not allow application of methods 4, 5 and 6.

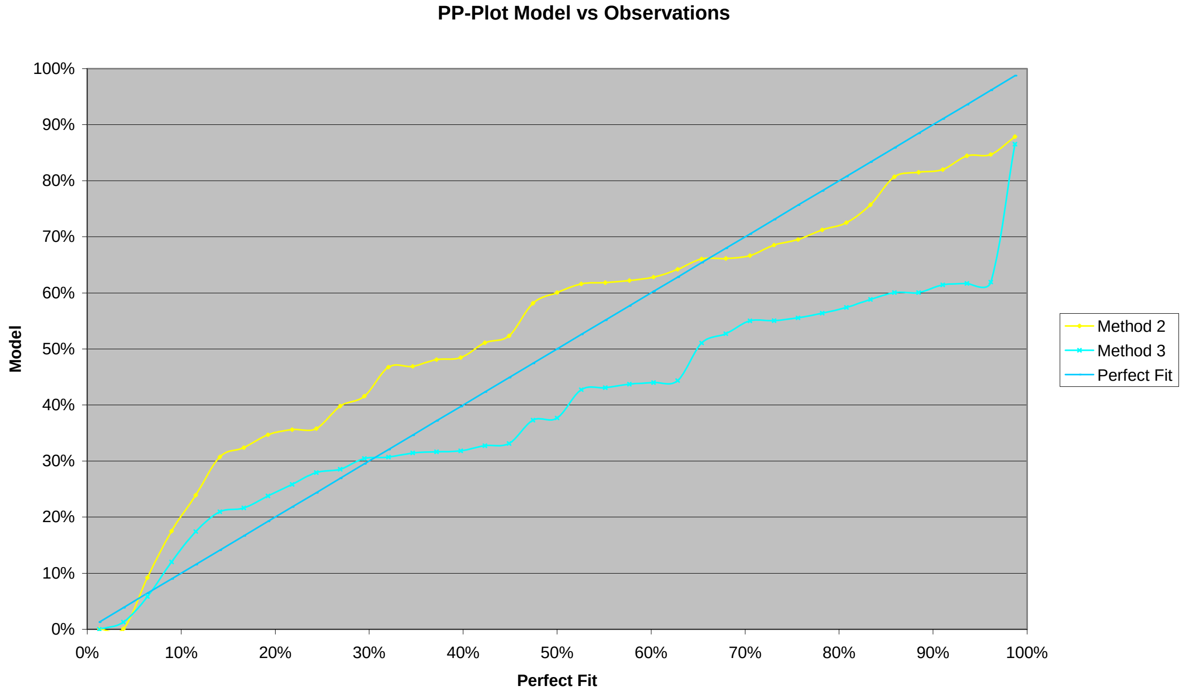
Reference co	Small	Medium	Large
Non-proportional reinsurance – property	2,631	12,516	31,254

GROSS SD

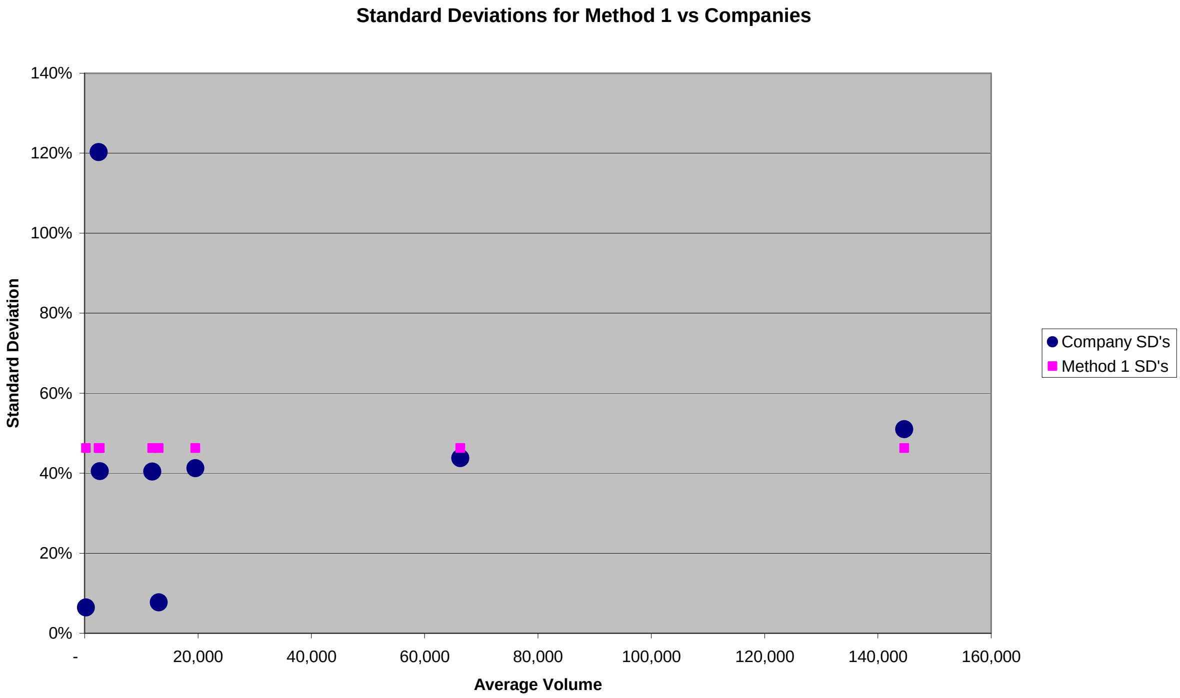
Discounted

Method	**Small (75th perc)**	**Medium (50th perc)**	**Large (25th perc)**	**VWA**	**Technical result**	**% firms with higher sd**
Method 1	46%	41%	32%	46%	**46.3%**	**25.0%**
Method 2	289%	132%	84%	64%		
Method 3				54%		
Method 4						
Method 5						
Method 6						

4.316.The graph below shows a pp plot of the fit of the models. Both methods provide a poor fit.



4.317. The result of the graph below shows no evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1.



Overall conclusions:

4.318. The selected technical factor was chosen considering method 1 – result 46.3%.

Non-proportional reinsurance - casualty

4.319.CEIOPS recommendation is that for the non-proportional reinsurance casualty lob the gross factor for reserve risk is 40%.

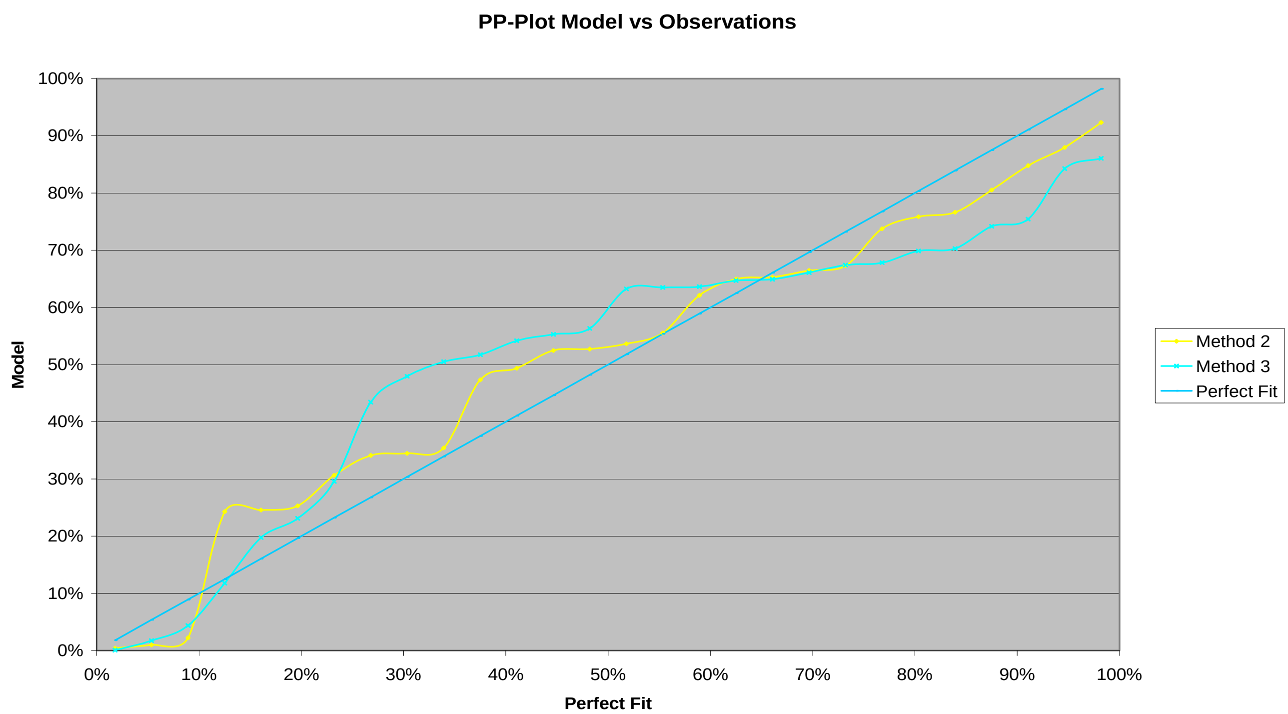
4.320.The data sample included data from 5 undertakings, was gross of reinsurance and included data from the following member states: UK. Lack of data in respect of provisions, did not allow application of methods 4, 5 and 6.

Reference co	Small	Medium	Large
Non-proportional reinsurance – casualty	32,328	34,099	92,418

GROSS SD **5 undertakings**
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	65%	40%	38%	41%	**41.1%**	**40.0%**
Method 2	93%	91%	55%	59%		
Method 3				112%		
Method 4						
Method 5						
Method 6						

4.321.The graph below shows a pp plot of the fit of the models. Both methods provide a poor fit.



4.322.The result of the graph below shows no evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1.

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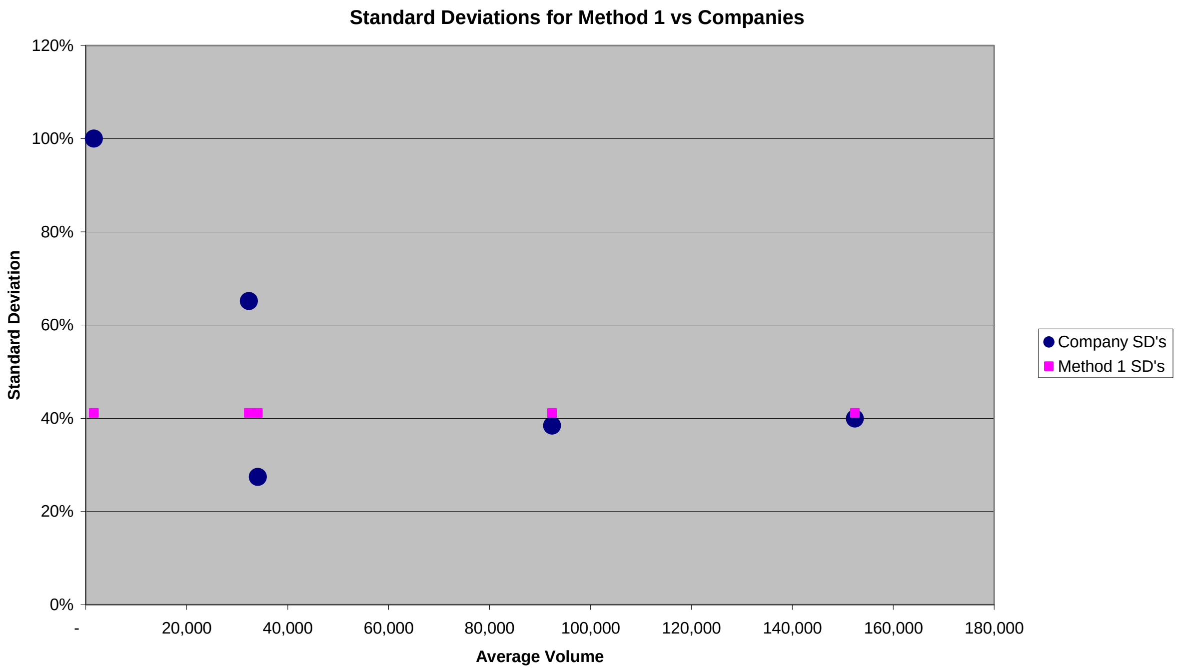

Overall conclusions:

4.323.The selected technical factor was chosen considering method 1 – result 41.1%.

Non-proportional reinsurance - MAT

4.324.CEIOPS recommendation is that for the non-proportional reinsurance MAT lob the gross factor for reserve risk is 70%.

4.325.The data sample included data from 8 undertakings, was gross of reinsurance and included data from the following member states: UK. The lack of data did not allow the application of methods 4, 5 and 6.

Reference co	Small	Medium	Large
Non-proportional reinsurance – MAT	1,659	3,718	4,947

GROSS SD
Discounted

Method	Small (75th perc)	Medium (50th perc)	Large (25th perc)	VWA	Technical result	% firms with higher sd
Method 1	71%	50%	27%	70%	**70.1%**	**25.0%**
Method 2	257%	172%	149%	119%		
Method 3				105%		
Method 4						
Method 5						
Method 6						

4.326.The graph below shows a pp plot of the fit of the models. Both methods provide a poor fit.

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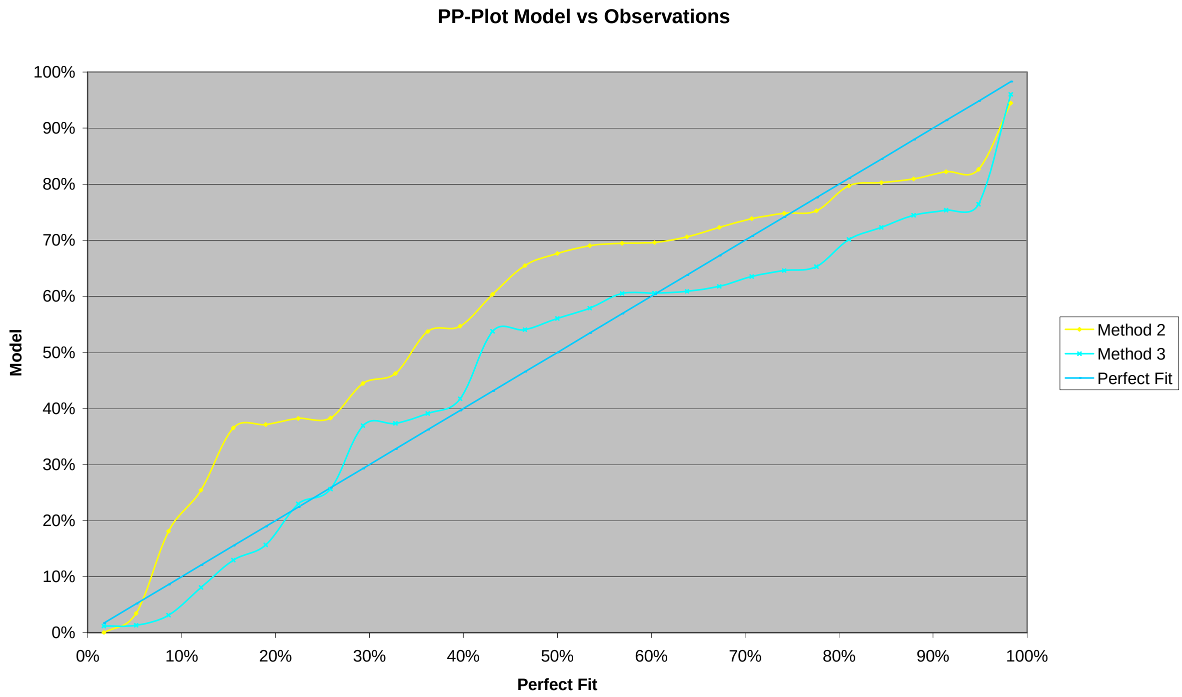
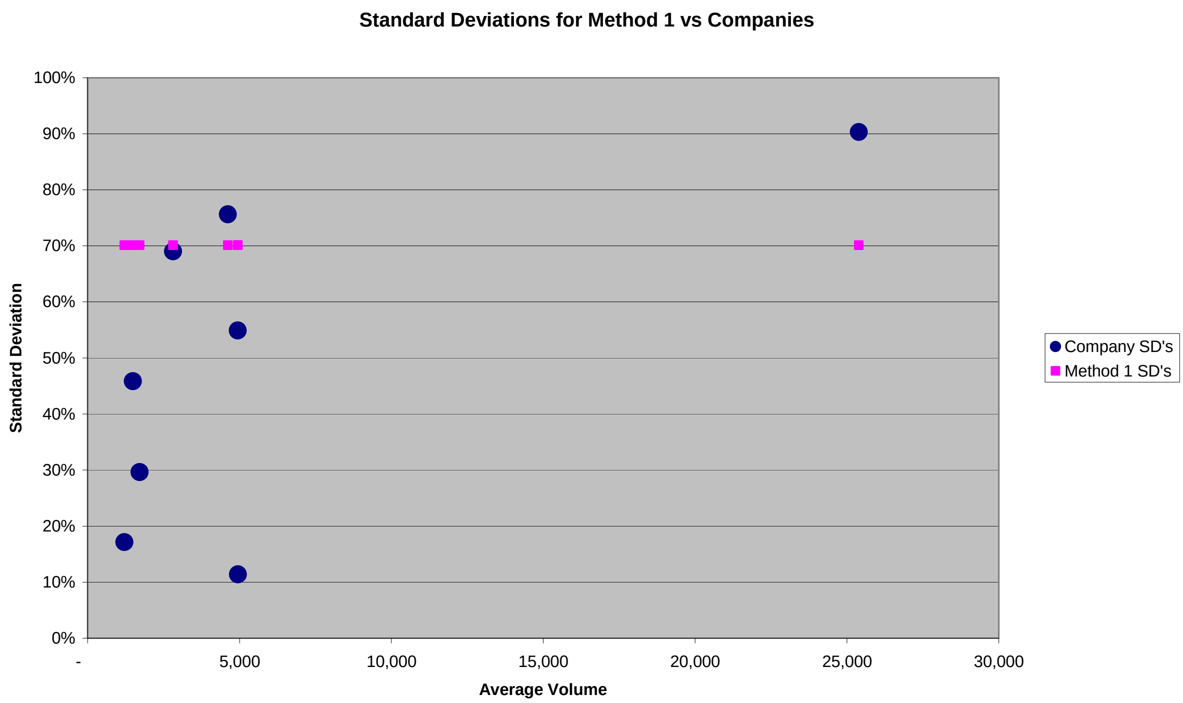


4.327.The result of the graph below shows no evidence for diversification credit. It also shows the volatility of the individual observations compared to the fitted selection for method 1.



Overall conclusions:

4.328.The selected technical factor was chosen considering method 1 – result 70.1%.

4.3.6. Adjusting gross to net for reserve risk

4.329. CEIOPS initially considered whether it was possible to derive an approach similar to the method being used in the premium risk to convert the gross reserving risk factors to an appropriate net reserving risk factor.

4.330. However, an initial impact study made it immediately clear that this resulted in a relatively small reduction in the factors for individual undertakings. This was due to undertakings having an insufficient number of years of observations of the benefit of reinsurance over one year to realistically derive a reduction that was appropriate for the 1 in 200 year scenario implicit within the gross calibration.

4.331. As a result CEIOPS felt obliged to help undertakings by using data across multiple companies and subsequently many more one year observations than available to any one undertaking to help estimate appropriate reductions in the gross calibration.

4.332. CEIOPS has selected the following net factors as the calibration for the non-life underwriting module for the purpose of the standard formula:

Line of Business	Net Factor	QIS 4	CP 71
Motor TPL	9.5%	12.0%	12.5%
Motor Other	17.5%	7.0%	12.5%
MAT	24.7%	10.0%	17.5%
Fire	12.0%	10.0%	15.0%
TPL	15.8%	15.0%	20.0%
Credit & Suretyship	25.1%	15.0%	20.0%
Assistance	25.3%	10.0%	15.0%
Legal Expenses	8.9%	10.0%	12.5%
Miscellaneous	23.0%	10.0%	20.0%
NPL - property	25.4%	15.0%	30.0%
NPL - Casualty	25.1%	15.0%	30.0%
NPL - MAT	41.2%	15.0%	30.0%

4.333. The approach used to derive the net reserving risk factor from the gross reserving risk factor involved three steps.

- The first step was to derive an uplift to the gross factor . This is needed as the original gross volatility factor was designed to be applied to gross reserves to get the gross capital amount. It is now to be applied to the net reserves, and so an uplift is needed to arrive at the same gross capital amount.

  For example: for TPL, the gross volatility factor was 23.5%. If gross reserves were 1,000, this would imply a gross capital requirement of 235. Since net reserves may only be 780, the factor needs to be uplifted to 30.1% to get the same level of capital requirement.

- The second step was to derive the benefit of the mitigating effect of the reinsurance programme on the large gross deteriorations. This was done by looking at the net to gross experience of claims development over the year, but limited to situations where claims deterioration was relatively extreme, so that the factor would reflect the experience at these levels rather than at expected levels.

  For example; for TPL, the analysis suggested that the effect of reinsurance (at the relatively more extreme levels) would be around 53% rather than 78% at the mean.

- The third step was to blend these analyses together with the results from the gross calibration. This effectively meant taking the gross volatility, applying the uplift factor obtained in step 1 and then applying the reinsurance mitigation obtained from the second step.

- The resulting net reserving factor for TPL, to be applied to net reserves, would then be 30.1% * 53%= 16%.

4.334. Essentially this approach looks at the reduction in the net to gross ratio over the one year time horizon conditioned upon the gross deterioration being relatively extreme – ie consistent with the scenario effectively identified by the gross calibration.

Data

4.335. The data used was four time series per line of business by individual companies and years.

- First time series: The opening gross reserve by company by year. (This time series was used as part of the calibration of the gross factors.)
- Second time series: The closing gross reserve after one year plus the incremental gross claims paid during the year, for the same accident years as the first time series by company by accident year. (This time series was used as part of the calibration of the gross factors.)
- Third time series: The opening net reserve by company by year.
- Fourth time series: The closing net reserve after one year plus the incremental net claims paid during the year, for the same accident years as the third time series by company by accident year.

Formulaic Filter

4.336. Due to the nature of the data collected for the calibration exercise it was necessary to apply a restrictive filter to remove apparent mismatches between the gross and net figures. This comprised the following components:

- First Filter: Only observations where a value existed for each of the time series were included in the calibration.

- Second Filter: Only observations where the net amounts were smaller than the associated gross amounts for both the opening and closing time series were included in the calibration.
- Third Filter: Only observations where the change in the net position was smaller than the associated change in the gross position were included in the calibration.

Manual Filter

4.337. Even with the formulaic filters described above there were a few observations that had to be removed from the calibration due to apparent inconsistencies between the gross and net amounts.

Calibration Step 1

4.338. The volume weighted average gross to net ratio was selected. This was the volume weighted average of the first time series divided by the third time series.

Calibration Step 2

4.339. This analysis comprised taking the observations with the largest gross deteriorations and summarising the closing net to gross ratios (ie the fourth time series divided by the second time series).

Calibration Step 3

4.340. The final step multiplied the gross calibration factor by the gross to net ratio derived in step 1 and then multiplied by the associated net to gross ratio derived in step 2.

4.4 Summary results

4.341. CEIOPS has selected the following gross factors as the calibration for the Non life underwriting module for the purpose of the standard formula:

LOB	Gross Premium factor	Gross Reserve Factor
Motor, Vehicle Liability	11.5%	11%
Motor, Other Classes	8.5%	20%
MAT	23%	38.5%
Fire and Other damage	15%	25%
Third party liability	17%	23.5%
Credit and suretyship	28%	50.5%
Legal expenses	8%	9%
Assistance	5%	44.5%
Miscellaneous	15.5%	41.5%
NPL Property	37%	46.5%
NPL Casualty	18.5%	41%
NPL MAT	16.5%	70%

4.342. After adjusting for reinsurance as recommended above, the net technical factors for the calibration for the Non life underwriting module for the purpose of the standard formula would be as follows:

LOB	Net premium factor[6]	Net reserve factor
Motor vehicle liability	$11.5\%*(NCR_i/GCR_i)$	9.5%
Motor Other	$8.5\%*(NCR_i/GCR_i)$	17.5%
MAT	$23\%*(NCR_i/GCR_i)$	25%

[6] CEIOPS has recommended an adjustment factor for Premium Risk that is undertaking specific, and so it is not possible to provide a net premium factor. NCR and GCR stand for net combined ratio and gross combined ratio respectively

Fire and Other damage	$15\%*(NCR_i/GCR_i)$	12%
Third party liability	$17\%*(NCR_i/GCR_i)$	16%
Credit and suretyship	$28\%*(NCR_i/GCR_i)$	25%
Legal expenses	$8\%*(NCR_i/GCR_i)$	9%
Assistance	$5\%*(NCR_i/GCR_i)$	25.5%
Miscellaneous	$15.5\%*(NCR_i/GCR_i)$	23%
NPL Property	$37\%*(NCR_i/GCR_i)$	25.5%
NPL Casualty	$18.5\%*(NCR_i/GCR_i)$	25%
NPL MAT	$16.5\%*(NCR_i/GCR_i)$	41%

4.343. CEIOPS members have considered the technical results produced from the analysis along with results and other evidence produced by individual CEIOPS members and other interested parties. These are discussed in section 4.5 (Other Analyses).

4.344. CEIOPS believes it is important to consider this additional evidence, along with other judgements made with the benefits of a wider understanding of the business along with the pure technical analysis described above. Particularly in cases where the volume of data is not as large as might be desired for such an analysis, it is then desirable to take this other information into account before arriving at the final recommendations.

4.345. In general, CEIOPS members have not identified any significant issues with the proposed net technical factors for premium risk apart from the non-proportional reinsurance property lob. For reserve risk, there are 5 lines of business where particular concerns were raised over the results produced by the pure technical analysis. These were mainly associated with issues around the volume of data available for analysis. However, not all lines of business with smaller volumes raised particular concerns.

4.346. In these instances, CEIOPS members have taken into account the factors used for QIS4, those proposed as part of the earlier CP71 analysis[7], the information from the other analyses (as noted in section 4.5), as well their wider knowledge of the underlying business characteristics and its performance. This assessment has taken into account the known shortcomings in those analyses so as to not put undue weight on any one source.

[7] CEIOPS-CP71-09

4.347.The results of this assessment are as follows:

LOB	Technical net premium risk factor	Recommended net premium risk factor
NPL Property	37%*(NCR_i/GCR_i)	20%*(NCR_i/GCR_i)

LOB	Technical net reserve risk factor	Recommended net reserve risk factor
Motor other	17.5%	12.5%
MAT	25%	17.5%
Assistance	25.5%	12.5%
Miscellaneous	23%	20%
NPL MAT	41%	25%

4.5 Other analyses

4.348. To get a further insight and consider other information available, CEIOPS reviewed additional exercises provided by CEIOPS or the industry as part of the final selection.

4.349. These additional exercises also suggest that factors proposed for QIS4 may not be appropriate at least for some lines of business.

Spain Analysis

4.350. Spain shared with CEIOPS an analysis based on the Spanish market. The analysis was:

- Carried out in respect of third party liability and motor third party liability and only for reserve risk.
- The analysis was gross
- Carried out consistently with the requirements of the standard formula.
- The results are aligned with the conclusions made in this analysis, considering that the calibration was based on only one member state.

Portugal Analysis

4.351. Portugal has performed an analysis based on the EU database collected by CEIOPS. The methodology is described in Annex 7.6.

4.352. This analysis was not performed for other LoBs because the assumptions underlying this particular methodology require triangles of a sufficiently high size, where the sum of each column is non-negative, and where a reasonable degree of proportionality between columns is observed. The data available for these other LoBs was not considered to fully satisfy these requirements.

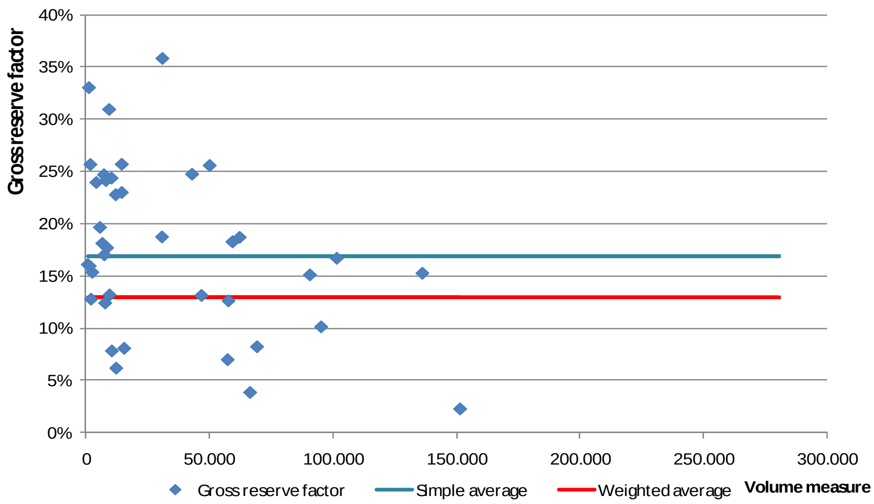
4.353. The analysis was made for Sickness, Workers' Compensation, Motor Other Classes and Motor Third-Party Liability.

4.354. Portugal proposes the following gross reserve factors:

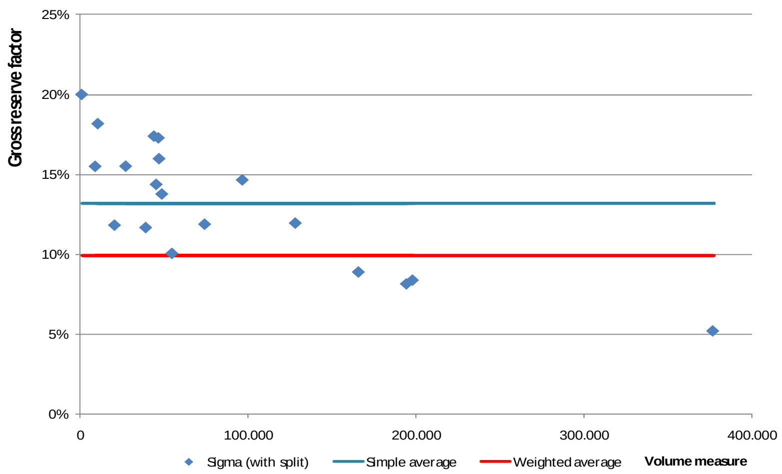
LOB	Gross Reserve Factor	
	Poisson Method (Simple average)	Poisson Method (Weighted average)
Motor, Other Classes	16.9%	12.9%
Motor, TPL	13.2%	10.0%

4.355. The following graphs show the adjustment obtained with the application of the methodology for these 4 lines of business. Each point represents the gross reserve factor calculated and the volume measure of each undertaking.

Motor Others



Motor TPL



QIS4 factor benchmarking analysis

4.356. CEIOPS did some additional analysis based on the information provided as a result of the QIS 4 results.

4.357. In order to calculate the non-life premium and reserve risk module (and the non-life part of the health underwriting risk module) QIS4 participants where requested to provide a time series of net loss ratios per line of business. (Cf. TS.XIII.B.30 of the QIS4 Technical Specifications.) All in all, about 3400 time series of European insurance and reinsurance undertakings were collected in this way.

4.358. CEIOPS carried out an analysis of the QIS4 database for the purpose of the calibration of the premium risk factors σ (prem, lob) as defined in *CEIOPS' Advice on the non-life underwriting risk module*. A detailed description of this analysis is included in section 7.3 of the Annex.

4.359. It is important to note for the purpose of making conclusions, that this analysis suffers from some shortcomings:

- The standard deviations are derived from time series of loss ratios. Conceptually, premium risk covers the volatility of claims and expenses. Loss ratios only reflect the volatility of claims. In order to estimate the volatility of claims and expenses, either combined ratios instead of loss ratios need to be studied or the loss ratios (or the resulting standard deviation) need to be scaled up to take the extra volatility of expenses into account. As this was not possible so far, the results are likely to underestimate the real risk.
- The distribution of loss ratios is likely to be skewed. In this case, the estimator is biased and underestimates the real standard deviation.
- The time series provided in QIS4 may not reflect the risk of the undertaking. The time series may be distorted, for example because of smoothing of held reserves, portfolio transfers, change of reinsurance programme or catastrophic losses.

- For some EU countries, most QIS4 responses where provided by the larger medium undertakings, and therefore any results will not be very representative of the smaller undertakings.
- Because of these shortcomings, the results of the analysis should rather be considered as lower boundaries of the final net standard deviations.

GDV analysis

4.360. GDV shared with CEIOPS analysis and data based on the German market. We understand the analysis was:

- Carried out in respect of six lines of business for reserve and premium risk.
- The analysis was gross.
- Carried out consistently with the requirements of the standard formula.
- CEIOPS was unable to incorporate the data into the analysis due to time constraints for premium risk. However the results were compared as a benchmark.
- However German data is included for reserve risk.

AON Benfield Analysis

4.361. CEIOPS also made reference to The Insurance Risk Study, Fourth edition 2009.

4.362. We understand from discussions with AON that the analysis was carried out for premium risk. The underlying assumptions and methodologies used are not totally consistent with the underlying assumptions of the standard formula.

4.363. Nevertheless we can draw some broad conclusions from the analysis for example including evidence of diversification by size of portfolio and the general magnitude of the underlying systemic volatility of the classes.

5. Catastrophe risk calibration

5.1. CEIOPS points out that the calibration in this advice is being considered to be in line with 99.5% VaR and a one year time horizon. QIS5 will give an indication of the overall impact of the proposed calibrations, not limited to the SCR but including technical provisions and own funds.

5.2. In line with the advice presented in CEIOPS-DOC-41/09, CEIOPS needs to provide undertakings with a set of factors per event to estimate a capital charge for the standard formula catastrophe risk sub module. This is called the "Factor Method".

5.3. CEIOPS has revised the calibration provided during QIS4.

5.4. A factor is required for the following events:

Events	Lines of business affected
Storm	Fire and property; Motor, other classes
Flood	Fire and property; Motor, other classes
Earthquake	Fire and property; Motor, other classes
Hail	Fire and property; Motor, other classes
Mayor fires, explosions	Fire and property
Major MAT disaster	MAT
Major motor vehicle liability disasters	Motor vehicle liability
Major third party liability disaster	Third party liability
Miscellaneous	Miscellaneous
NPL Property	NPL Property
NPL MAT	NPL MAT
NPL Casualty	NPL Casualty
Major claim	Credit and Suretyship

5.5. Estimating a factor by event across all EU countries and for valid all undertakings has resulted in a very difficult task:

- Lack of data. CEIOPS required 1 in 200 year loss equivalents by lob. Only data from a limited number of markets was available.

- Due to the nature of catastrophe business it is extremely difficult to come up with a single factor that represents a 1 in 200 year loss for all undertakings, across all countries in the EU and by LoB.
- The risk profile of undertakings is very different across countries and within a LoB.
- Some countries provide pooling arrangements to cover catastrophe risk. This was not taken into account in selecting the final factor.
- Different countries and undertakings cover different risks and therefore have different risk profiles. We have not been able to select a factor taking this into account.

5.6. For some of these reasons listed above, the factor method has been characterised for its lack of risk sensitivity if compared to other methods such as Standardised scenario or a Partial internal model.

5.7. More importantly, the factor method is unlikely to represent a 1 in 200 year loss for every undertaking, as required by the Level 1 text.

5.8. However, CEIOPS considers that a factor is necessary under the standard formula, in particular when a standardised scenario is not appropriate and when the use of a Partial internal model is not proportionate. Examples when a factor could be used are:

- When the risk profile of the undertaking is not well represented by the standardised scenario.
- The undertaking writes Miscellaneous Catastrophe business.
- The undertaking writes material Non proportional reinsurance
- The undertaking writes material business outside the EEA

5.9. CEIOPS acknowledges that possible further analysis could be performed to further improve such method. For example we could provide a factor for property by country or region rather than one factor for all EU. However this is very time consuming and requires further consultation.

5.10. CEIOPS would like to highlight that this is an area where both the undertaking and supervisor will need to assess whether indeed the capital estimated is sufficient to cover a 1 in 200 year loss and that possible supervisory measures may need to be applied.

5.1 Calibration

5.11. Compared to QIS4, CEIOPS has tried to improve the calibration of the factor method by introducing the following changes:

- The factor has been calibrated gross of reinsurance. This allows undertakings to apply their respective reinsurance programme in order to estimate the net amount.
- The factor has been calibrated by peril for the property line of business, in order to introduce further segmentation at a LoB level.

5.12. CEIOPS carried out two main analysis and used some external benchmarking information obtained through consultation.

Analysis 1

This analysis was the result of a CEIOPS exercise.

Methodology

5.13. The analysis was performed by the FSA using data based on firms regulated in the UK.

5.14. We collated empirical loss ratio distributions for the respective LoBs for various firms.

5.15. The LoB for which we had most data was Property, where we had many different undertaking distributions. The data was increasingly scanty for other classes of business.

5.16. We were provided with empirical distributions consisting of approximately 5-6 points. LogNormal distributions were fitted using the 75^{th} and 99.5th percentiles. If the fit was not reasonable then the data was discarded.

5.17. If the fit was reasonable, we generated an aggregate distribution by simulating correlated samples from each of the distributions using a Normal (Gaussian) copula. The same correlation coefficient was chosen across all data sets.

5.18. After 25,000 simulations we deducted the mean from the 99.5th percentile (except in the case where the data was purely cat related) to remove attritional claims.

Analysis 2

5.19. It involved the calibration of the factor-based non-life CAT sub-module based on German data

Results

LoB	**Gross loss in % of gross premium**	**Risks covered**
Fire and property (storm)	250%	Storm and hail
Fire and property (earthquake)	155%	Earthquake
Fire and property (flood)	140%	Flooding rivers
Fire and property (fire)	215%[8]	Fire and explosion
Motor, other classes	30%	Hail, flood, storm, lightning
Motor vehicle liability	25%	Large single accident
Third party liability	80%	Large single liability claim
MAT	95%	Large single MAT claim

Methodology natural catastrophes

5.20. The factors are derived from the CAT models that were used for German exposure in QIS4. The models were developed by a GDV working group in cooperation with BaFin and are based on data from reinsurers and claims data collected by GDV.

5.21. The models produce average gross claims as follows:

- property/storm: 1.15‰ of sum insured
- property/earthquake: 0.93 ‰ of sum insured
- property/flood: 0.84‰ of sum insured
- other motor: 65 euro per risk

5.22. We think that for the German market the risk-sensitivity of the approach for the natural catastrophes in property insurance can be improved by applying the volume measure 'sum insured' instead of 'premiums'.

5.23. The model takes into account a discount for basis claims. (The discount amounts to the average annual claims relating to the risk modelled.) If such a discount is not allowed, the factors need to be increased.

Methodology man-made catastrophes

5.24. The factors were derived from CAT models for severity risks which were developed by a GDV working group in cooperation with BaFin. The models were calibrated on claims data collected by GDV.

[8] This value is still under discussion with our industry. The overall range of the value is confirmed though.

5.25. For each risk, the model follows a generalised Pareto distribution as follows:

$$\text{gross claim} = u + (M - u) \cdot \frac{\left(\frac{200 \cdot c}{t}\right)^{1/a} - 1}{\left(\frac{200}{t}\right)^{1/a} - 1},$$

where

M	one in 200 year market loss
u	threshold claim size
c	market share of undertaking
t	recurrence period (corresponding to threshold claim size)
a	Pareto exponent

5.26. Depending on the risk, the parameters are chosen as follows:

LoB	**u**	**t**	**a**	**M**
MTPL	5 million	0.18	2.9	70 million
TPL	2.5 million	0.12	2.1	500 million
MAT	2.5 million	0.14	2.6	50 million
Household	1.25 million	0.22	3.1	20 million
Fire	20 million	0.12	4.8	500 million

5.27. The gross claim depends (in a non-linear way) on the market share of the undertaking. For the derivation of the risk factors a medium size market share (depending on the LoB) was chosen.

External benchmarks

5.28. We have worked closely with some major market participants (a large broker, catastrophe modelling agency, other industry data) and have compared our results to the information provided by them.

5.29. The information provided by the catastrophe modelling agency and the major broker focussed on the property line of business. Where they were able to provide more detailed results, down to peril and region.

5.30. They applied their models to their best estimates of industry insured exposures to generate industry insured losses. This was done for each territory and peril where an appropriate model was available.

5.31. Where a particular territory was not covered by a model, the territory was judgementally classified by its main peril and assigned a rating (High, Medium or Low) depending on the estimated level of risk that territory posed. The closest match was then used from territories where models do exist as proxies for the non-modelled territories.

5.32. The output from the modelling exercise was a list of simulated events which could be used to calculate a 1 in 200 year loss estimate.

5.33. These 1 in 200 losses estimates were then compared with industry premium amounts to produce loss ratios.

5.2 Results

5.34. A summary of the results carried out by CEIOPS:

	Net	Gross				
	QIS4	UK	Germany	Netherlands	Benchmarking	
Property	75%	150%		130%	95%	
Property - Windstorm			250%		100%	
Property - Earthquake			155%		85%	
Property - Flood			140%		85%	
Property – Fire			215%			
Credit & Surety ship	60%	150%				145%
MAT	50%	100%	95%			104%
Third Party Liability	15%	85%	80%			91%
Miscellaneous	25%	35%				39%
Motor, other classes	8%	30%	30%			
Motor vehicle liability	15%	50%	25%			219%
Non Proportional - Casualty	50%	85%				
Non Proportional - MAT	150%	150%				
Non Proportional - Property	150%	150%				

5.35. CEIOPS proposes the following factors for the Factor method:

Events	Lines of business affected	Factor
Storm	Fire and property; Motor, other classes	**175%**
Flood	Fire and property; Motor, other classes	**113%**
Earthquake	Fire and property; Motor, other classes	**120%**
Hail	Motor, other classes	**30%**
Major fires, explosions	Fire and property	**175%**

Major MAT disaster	MAT	**100%**
Major motor vehicle liability disasters	Motor vehicle liability	**40%**
Major third party liability disaster	Third party liability	**85%**
Miscellaneous	Miscellaneous	**40%**
NPL Property	NPL Property	**250%**
NPL MAT	NPL MAT	**250%**
NPL Casualty	NPL Casualty	**250%**
Major claim	Credit and Suretyship	**150%**

5.36. Furthermore, CEIOPS proposes the following factors for captives:

Line of business	Factor
Motor vehicle liability	225%
Motor, other classes	540%
MAT	920%
Fire and other damage	920%
Third Party Liability	450%
Miscellaneous	920%

6. CEIOPS' advice

Results

6.1. Based on the assumptions contained in the explanatory text, CEIOPS has calibrated the sub-module according to 99.5% VaR and a one-year time horizon.

6.2. The non-life factors for premium and reserve risk sub-module by lob are as follows:

LOB	Net premium factor	Net reserve factor
Motor Third Party Liability	$11.5\%*(NCR_i/GCR_i)$	9.5%
Motor Other	$8.5\%*(NCR_i/GCR_i)$	12.5%
MAT	$23\%*(NCR_i/GCR_i)$	17.5%
Fire and Other damage	$15\%*(NCR_i/GCR_i)$	12%
Third Party Liability	$17\%*(NCR_i/GCR_i)$	16%
Credit and suretyship	$28\%*(NCR_i/GCR_i)$	25%
Legal expenses	$8\%*(NCR_i/GCR_i)$	9%
Assistance	$5\%*(NCR_i/GCR_i)$	12.5%
Miscellaneous	$15.5\%*(NCR_i/GCR_i)$	20%
NPL Property	$20\%*(NCR_i/GCR_i)$	25.5%
NPL Casualty	$18.5\%*(NCR_i/GCR_i)$	25%
NPL MAT	$16.5\%*(NCR_i/GCR_i)$	25%

6.3. For premium risk, undertakings need to determine an adjustment factor based on their own historic experience. In the table above NCR_i and GCR_i refer to net combined ratio and gross combined ratio respectively for each lob. Reserve risk factors have already been adjusted for reinsurance and no further adjustment is needed.

for reinsurance as follows:

- The ratio of the net combined ratio at financial year end and the gross combined ratio at financial year end can be viewed as a transformation factor for performing gross-net transitions by accident year.
- This ratio is exact in the case of quota-share reinsurance and should be viewed as a convenient approximation for surplus and non-proportional reinsurance.
- Basing the ratio on the most recent 3 financial years, will create some stability of the ratio.
- At the same time the ratio will be responsive to changes in reinsurance programs in a 3-year moving average way.
- The inputs for determining the net-gross ratio should be purified of any catastrophe effect on premiums, losses and costs. ie both gross and net claims should exclude any catastrophe claims, and catastrophe reinsurance premiums should not be deducted from gross premiums when determining net premiums. When deciding which claims should be considered as catastrophe claims, undertakings should refer to the report of the Catastrophe Task Force.

6.5. The net-gross ratio, by line of business, is determined in three steps:

$$\text{gross combined ratio} = \frac{\text{gross loss}}{\text{gross earned premium}} + \frac{\text{gross costs}}{\text{gross written premium}}$$

$$\text{net combined ratio} = \frac{\text{net loss}}{\text{net earned premium}} + \frac{\text{net costs}}{\text{net written premium}}$$

$$\text{net-gross ratio} = \frac{\text{net combined ratio}}{\text{gross combined ratio}}$$

with the following definitions of the terms:

gross losses	total best estimate ultimate claims for the last three accident years gross of reinsurance, net of salvage and subrogation, but gross of ALAE (allocated loss adjustment expenses). The ultimate claims amounts are as booked as at the end of each accident year, without allowing for any subsequent development. These figures should not include any catastrophe claims.
gross earned premium	total ultimate premium earned over the last three accident years gross of reinsurance
gross costs	total expenses (ULAE – unallocated loss adjustment expenses - and other company expenses appropriately allocated to the LoB) excluding ALAE paid over the last three financial years.

gross written premium	total ultimate premium written over the last three financial years
net losses	total best estimate ultimate claims for the last three accident years net of reinsurance of reinsurance, net of salvage and subrogation, but gross of ALAE. The ultimate claims amounts are as booked as at the end of each accident year, without allowing for any subsequent development (to be consistent with the definition of gross losses). These figures should not include any catastrophe claims and similarly there should be no allowance for the reinsurance recoveries associated with those claims.
net earned premium	total ultimate premium earned over the last three accident years net of reinsurance. The net earned premium should include the cost of the catastrophe reinsurance protections, ie these should not be deducted from the associated gross figures.
net costs	total expenses (ULAE and other company expenses appropriately allocated to the LoB) excluding ALAE paid over the last three financial years, but including outwards reinsurance commissions. The outwards reinsurance commissions should not include any of the costs of the catastrophe protections.
net written premium	total ultimate premium written over the last three financial years net of reinsurance. The net written premium should include the cost of the catastrophe reinsurance protections, i.e. these should not be deducted from the associated gross figures.

Catastrophe risk calibration

6.6. In line with the advice presented in CEIOPS-DOC-41/09, CEIOPS needs to provide undertakings with a set of factors per event to estimate a capital charge for the standard formula catastrophe risk sub module. This is called the “Factor Method”.

6.7. CEIOPS has revised the calibration provided during QIS4.

6.8. A factor is required for the following events:

Events	Lines of business affected
Storm	Fire and other damage; Motor other
Flood	Fire and other damage; Motor other
Earthquake	Fire and other damage; Motor other

Hail	Fire and other damage; Motor other
Major fires, explosions	Fire and other damage
Major MAT disaster	MAT
Major motor vehicle liability disasters	Motor Third Party Liability
Major third party liability disaster	Third Party Liability
Miscellaneous	Miscellaneous
NPL Property	NPL Property
NPL Casualty	NPL Casualty
NPL MAT	NPL MAT
Major claim	Credit and Suretyship

6.9. Estimating a factor by event across all EU countries and for valid all undertakings has resulted in a very difficult task:

- Lack of data. CEIOPS required 1 in 200 year loss equivalents by lob. Only data from a limited number of markets was available.
- Due to the nature of catastrophe business it is extremely difficult to come up with a single factor that represents a 1 in 200 year loss for all undertakings, across all countries in the EU and by LoB.
- The risk profile of undertakings is very different across countries and within a LoB.
- Some countries provide pooling arrangements to cover catastrophe risk. This was not taken into account in selecting the final factor.
- Different countries and undertakings cover different risks and therefore have different risk profiles. We have not been able to select a factor taking this into account.

6.10. However, CEIOPS considers that a factor is necessary under the standard formula, in particular when a standardised scenario is not appropriate and when the use of a Partial internal model is not proportionate. Examples when a factor could be used are:

- When the risk profile of the undertaking is not well represented by the standardised scenario.
- The undertaking writes Miscellaneous Catastrophe business.
- The undertaking writes material Non proportional reinsurance
- The undertaking writes material business outside the EEA

undertaking and supervisor will need to assess whether indeed the capital estimated is sufficient to cover a 1 in 200 year loss and that possible supervisory measures may need to be applied.

6.12. Compared to QIS4, CEIOPS has tried to improve the calibration of the factor method by introducing the following changes:

6.13. The factor has been calibrated gross of reinsurance. This allows undertakings to apply their respective reinsurance programme in order to estimate the net amount.

6.14. The factor has been calibrated by peril for the property line of business, in order to introduce further segmentation at a LoB level.

6.15. CEIOPS carried out two main analysis and used some external benchmarking information obtained through consultation.

Results

6.16. CEIOPS proposes the following factors for the Factor method:

Events	Lines of business affected	Factor
Storm	Fire and property; Motor, other classes	**175%**
Flood	Fire and property; Motor, other classes	**113%**
Earthquake	Fire and property; Motor, other classes	**120%**
Hail	Motor, other classes	**30%**
Major fires, explosions	Fire and property	**175%**
Major MAT disaster	MAT	**100%**
Major motor vehicle liability disasters	Motor vehicle liability	**40%**
Major third party liability disaster	Third party liability	**85%**
Miscellaneous	Miscellaneous	**40%**
NPL Property	NPL Property	**250%**
NPL MAT	NPL MAT	**250%**
NPL Casualty	NPL Casualty	**250%**

Major claim	Credit and Suretyship	**150%**

6.17. Furthermore, CEIOPS proposes the following factors for captives:

Line of business	Factor
Motor Third Party Liability	225%
Motor Other	540%
MAT	920%
Fire and other damage	920%
Third Party Liability	450%
Miscellaneous	920%

7. Annexes

7.1. Data Request for Premium and Reserve risk calibration

Subject : **Data requirements for non-life premium and reserve risk calibration**

Introduction

As part of the SCR work, DE and UK will be working together on the non life calibration exercise for October 2009.

For the purposes of the calibration we require data at EU level. This document summarises the ideal data requirements. Please note, that we are aware that some data may not be available, but please send as much data as possible from the list below, stating clearly what this includes. As we are carrying out a number of methods, even if you cannot provide all information mentioned above, some information may still be useful.

Premium Risk Analyses

We will use various models and parameterisation techniques to quantify appropriate levels of premium risk by LoB, by comparing company and accident year information. In particular we will be looking at:

- historic levels of paid claims reported in the first year during which the policies are earned, along with the reserves posted after the first development year for that accident year, with
- the premium earned in these years respectively

7.1. Useful Data Requirements: **Either one of the following (or ideally both)**

a. Gross data, gross of reinsurance

1. Earned Premium vector, by accident year. This figure will be include acquisition costs (see column (a) on the template)

2. The ultimate losses vector: posted after the first year, by accident year. This is = paid (see column (e) on the template) + case estimates (outstanding) + IBNR (all posted after one year). (See column (g) on the template). These figures would ideally be:

   - Net of the effect of CAT events:
   - They should include ALAE (ie. Allocated claims handling expenses)
   - Should allow for receivables for salvage and subrogation.

3. Expense vector information:

- Relevant ULAE reserve (see column (c) on the template)

- Relevant paid expenses (all expenses excluding allocated Claims handling costs) (see column (d) on the template).

b. Net data, net of reinsurance: Same as gross but net of R/I

Template

Company

Line of Bunsiness

All figures in 000's

Non Standard

	Earned Premiums			
	Gross of acquisition costs (a)	Net of acquisition costs (b)	ULAE (c)	Paid Expenses (d)
1999	10.4	8.3	1.04	1.00
2000	0.8	0.6	0.08	0.61
2001	6.4	5.1	0.64	1.00
2002	3.2	2.6	0.32	1.00
2003	20.1	16.1	2.01	2.00
2004	6.0	4.8	0.60	2.00
2005	10.9	8.7	1.09	2.00
2006	20.0	16.0	2.00	2.00
2007	6.3	5.1	0.63	2.00
2008	4.7	3.8	0.47	2.00

Paid Development Data (Cumulative)

	1 (e)	2	3	4	5	6	7	8	9	10
1999	1.3	9.0	10.0	10.0	10.0	11.0	11.0	11.0	11.0	11.0
2000	0.2	3.0	3.0	4.0	5.0	5.0	5.0	5.0	5.0	
2001	0.6	4.0	4.0	4.0	4.0	4.0	4.0	4.0		
2002	0.4	2.0	3.0	4.0	4.0	4.0	4.0			
2003	2.5	4.0	7.0	7.0	7.0	7.0				
2004	0.3	2.0	6.0	6.0	7.0					
2005	0.7	6.0	10.0	10.0						
2006	1.0	2.0	3.0							
2007	0.3	1.0								
2008	0.6									

Incurred Development Data (Cumulative)

	1 (f)	2	3	4	5	6	7	8	9	10
1999	3.0	11	11	11	11	11	11	11	11	11
2000	4.0	5	4	4	5	5	5	5	5	
2001	2.0	4	4	4	4	4	4	4		
2002	4.0	4	7	7	7	7	7			
2003	4.0	7	6	7	7	7				
2004	6.0	6	10	10	10					
2005	2.0	8	11	11						
2006	1.0	5	5							
2007	0.0	2								
2008	1.0									

Incurred But Not Reported Data

	1 (g)	2	3	4	5	6	7	8	9	10
1999	9.7	2	1	1	1	-	-	-	-	-
2000	3.8	1	2	1	-	-	-	-	-	
2001	3.4	-	-	-	-	-	-	1		
2002	6.6	5	4	3	3	1	1			
2003	3.5	3	-	-	1	-				
2004	9.7	8	4	2	-					
2005	10.3	5	3	1						
2006	4.0	2	2							
2007	3.0	1								
2008	4.0									

Reserve risk analyses

7.2. We will use various models and parameterisation techniques to quantify appropriate levels of reserving risk by LoB, by performing analyses in two separate ways.

a. Analysing by company how opening reserves compare against the amounts paid in the subsequent calendar year along with the associated closing reserves.

b. Implementing one year reserving risk approaches directly from the triangles of either paid and/or incurred data.

7.3. Triangles: Gross of Reinsurance

Compulsory

- Paid triangles by accident year (see triangle including column (e) in the template)
    - Where possible with the effect of CAT events removed.
    - These figures should include ALAE, but exclude ULAE.
    - This should allow for receivables for salvage and subrogation.

Optional

- Incurred triangles by accident year (see triangle including column (f) in the template);
    - This figure is to exclude IBNR and be the sum of paid claims and case estimates.
    - Where possible with the effect of CAT events removed.
    - These figures should include ALAE, but exclude ULAE.
    - This should allow for receivables for salvage and subrogation.
- Triangle of IBNR by accident year (see triangle including column (g) in the template)
    - Where possible with the effect of CAT events removed.
    - These figures should include ALAE, but exclude ULAE.
    - This should allow for receivables for salvage and subrogation.

Or combination of the above

7.4. Triangles: Net of Reinsurance

Compulsory

- Paid triangles by accident year (see triangles in the following below)
    - Where possible with the effect of CAT events removed.
    - These figures should include ALAE, but exclude ULAE.
    - This should allow for receivables for salvage and subrogation.

Optional

- Incurred triangles by accident year (see triangles in the following template below);
    - This figure is to exclude IBNR and be the sum of paid claims and case estimates.
    - Where possible with the effect of CAT events removed.
    - These figures should include ALAE, but exclude ULAE.
    - This should allow for receivables for salvage and subrogation.
- Triangle of IBNR by accident year (see triangle in the following template)
    - Where possible with the effect of CAT events removed.
    - These figures should include ALAE, but exclude ULAE.
    - This should allow for receivables for salvage and subrogation.

Or combination of the above

Reserve Risk Template

Company

Line of Bunsiness

All figures in 000's

Paid Development Data (Cumulative)

	1	2	3	4	5	6	7	8	9	10
1999	1	9	10	10	10	11	11	11	11	11
2000	0	3	3	4	5	5	5	5	5	
2001	1	4	4	4	4	4	4	4		
2002	0	2	3	4	4	4	4			
2003	3	4	7	7	7	7				
2004	0	2	6	6	7					
2005	1	6	10	10						
2006	1	2	3							
2007	0	1								
2008	1									

Incurred Development Data (Cumulative)

	1	2	3	4	5	6	7	8	9	10
1999	3	11	11	11	11	11	11	11	11	11
2000	4	5	4	4	5	5	5	5	5	
2001	2	4	4	4	4	4	4	4		
2002	4	4	7	7	7	7	7			
2003	4	7	6	7	7	7				
2004	6	6	10	10	10					
2005	2	8	11	11						
2006	1	5	5							
2007	0	2								
2008	1									

Incurred But Not Reported Data

	1	2	3	4	5	6	7	8	9	10
1999	10	2	1	1	1	-	-	-	-	-
2000	4	1	2	1	-	-	-	-	-	
2001	3	-	-	-	-	-	-	1		
2002	7	5	4	3	3	1	1			
2003	3	3	-	-	1	-				
2004	10	8	4	2	-					
2005	10	5	3	1						
2006	4	2	2							
2007	3	1								
2008	4									

General considerations in case you have issues of confidentiality

We understand that some of you have confidentiality issues. Therefore please note as follows:

Since our analyses for premium risk involves investigating the relationship between claims and premiums it is important that the relationship between claims and premiums is maintained.

Therefore if member states wish to disguise the data they may wish to standardize the data as follows:

a. We have defined 3 size ranges as below: (Please note that this is supposed to reflect EU size ranges, as we are coming up with an EU calibration. If we were to look at individual countries this would obviously not apply).

Net Earned Premium						
Euros000's						
	Small		Medium		Large	
	Bottom	Top	Bottom	Top	Bottom	Top
Non SLT Health	-	50,000	50,000	200,000	200,000	+
Motor, vehicle liability	-	25,000	25,000	200,000	200,000	+
Motor, other classes	-	25,000	25,000	200,000	200,000	+
Marine, aviation, transport (MAT)	-	10,000	10,000	25,000	25,000	+
Fire and other property damage	-	100,000	100,000	500,000	500,000	+
Third-party liability	-	10,000	10,000	100,000	100,000	+
Credit and suretyship	-	10,000	10,000	50,000	50,000	+
Legal expenses	-	10,000	10,000	25,000	25,000	+
Assistance	-	10,000	10,000	25,000	25,000	+
Miscellaneous	-	40,000	40,000	100,000	100,000	+
Non-proportional reinsurance – property	-	10,000	10,000	25,000	25,000	+
Non-proportional reinsurance – casualty	-	10,000	10,000	25,000	25,000	+
Non-proportional reinsurance – MAT	-	10,000	10,000	25,000	25,000	+

b. The way participants should map undertakings to the size buckets, is by taking the average of the particular time series of (for example) premiums. The average would determine where the undertaking is allocated to in respect of the 3 sizes. We don't think taking the last year's volume measure would be correct, as could be biased.

c. **Data for the premium risk:** Once the data has been split into the 3 size categories the following should be carried out:

- the first premium entry is divided by itself so it becomes 1
- The remainder of the amounts (premium, paid, incurred, and IBNR) are also divided by this first premium amount. So all amounts are divided by the same number.

As an example
Non Standardised data

	Premiums
	Gross of acquisition costs (a)
1999	10
2000	1
2001	6
2002	3
2003	20
2004	6
2005	11
2006	20
2007	6
2008	5

Standardized data

	Earned premiums
	Gross of acquisitio n costs (a)
1999	1.00
2000	0.07
2001	0.61
2002	0.31
2003	1.93
2004	0.58
2005	1.04
2006	1.92
2007	0.61
2008	0.45

And the same for all other amounts (paid, etc…).

d. **Data for the reserve risk**: Once the data has been split into these 3 size categories the following should be carried out:

- the triangle cells are divided by the first cell in the premium column (non standardised), as follows:

Non standard

	1 (e)	2	3	4	5	6	7	8	9	10
1999	1.3	9.0	10.0	10.0	10.0	11.0	11.0	11.0	11.0	11.0
2000	0.2	3.0	3.0	4.0	5.0	5.0	5.0	5.0	5.0	
2001	0.6	4.0	4.0	4.0	4.0	4.0	4.0	4.0		
2002	0.4	2.0	3.0	4.0	4.0	4.0	4.0			
2003	2.5	4.0	7.0	7.0	7.0	7.0				
2004	0.3	2.0	6.0	6.0	7.0					
2005	0.7	6.0	10.0	10.0						
2006	1.0	2.0	3.0							
2007	0.3	1.0								
2008	0.6									

Standardized

	1	2	3	4	5	6	7	8	9	10
1999	0.12	0.87	0.96	0.96	0.96	1.06	1.06	1.06	1.06	1.06
2000	0.02	0.29	0.29	0.38	0.48	0.48	0.48	0.48	0.48	-
2001	0.06	0.38	0.38	0.38	0.38	0.38	0.38	0.38	-	-
2002	0.03	0.19	0.29	0.38	0.38	0.38	0.38	-	-	-
2003	0.24	0.38	0.67	0.67	0.67	0.67	-	-	-	-
2004	0.03	0.19	0.58	0.58	0.67	-	-	-	-	-
2005	0.07	0.58	0.96	0.96	-	-	-	-	-	-
2006	0.10	0.19	0.29	-	-	-	-	-	-	-
2007	0.03	0.10	-	-	-	-	-	-	-	-
2008	0.05	-	-	-	-	-	-	-	-	-

7.2 Data availability for the Non-life Calibration

7.5. Below we present a table that shows the availability of data for premium and reserve risk respectively for CP71 and the revised set of data set used for the current analysis.

CP71 Version1		
Premium risk	**Gross**	**Net**
Motor, vehicle liability	Poland, Lux, UK, Slovenia, Portugal	Poland, Lux, Germany
Motor, other classes	Poland, Lux, UK, Slovenia, Portugal	Poland, Lux, Germany
Marine, aviation, transport (MAT)	Poland, Lux	Poland, Lux, Germany, UK
Fire and other property damage	Poland, Lux, UK	Poland, Lux, Germany, UK
Third-party liability	Poland, Lux, UK,	Poland, Lux, Germany, UK,
Credit and suretyship	Poland, UK	Poland, Germany, UK
Legal expenses	Poland	Poland, Germany
Assistance	Poland	Poland, Germany
Miscellaneous	Poland, UK	Poland, Germany, UK
Non-proportional reinsurance - property	UK	UK
Non-proportional reinsurance - casualty	UK	UK
Non-proportional reinsurance - MAT	UK	UK

CP71 Version2		
Premium risk	**Gross**	**Net**
Motor, vehicle liability	Poland, Lux, Slovenia, Portugal, Slovakia, Iceland, Italy, Lithuania, Denmark, Sweden, Hungary, Finland	Poland, Lux, Germany, Iceland, denmark
Motor, other classes	Poland, Lux, Slovenia, Portugal, Slovakia, Iceland, Lithuania, Finland, Denmark, Sweden	Poland, Lux, Germany, denmark
Marine, aviation, transport (MAT)	Poland, Lux, Slovakia, Iceland, Denmark, Sweden	Poland, Lux, Germany, UK, Iceland, Denmark
Fire and other property damage	Poland, Lux, UK, Slovakia, Iceland, Finland, Denmark, Slovenia	Poland, Lux, Germany, UK, Iceland, Denmark
Third-party liability	Poland, Lithuania, UK, Slovakia, Iceland, Denmark, Slovenia, Lux	Poland, Lux, Germany, UK, Iceland
Credit and suretyship	Poland, UK, Slovakia, Denmark, Sweden, Slovenia	Poland, Germany, UK
Legal expenses	Poland, Slovakia, Finland, UK	Poland, Germany
Assistance	Poland, Slovakia, Denmark, UK	Poland, Germany, Denmark
Miscellaneous	Poland, UK, Denmark	Poland, Germany, UK
Non-proportional reinsurance - property	UK	UK
Non-proportional reinsurance - casualty	UK	UK
Non-proportional reinsurance - MAT	UK	UK

CP71 Version1		
RESERVE RISK	**Gross**	**Net**
Motor, third-party liability	Poland, Lux, Germany, Slovenia, Portugal, UK	Poland, Lux, UK
Motor, other classes	Poland, Lux, UK, Slovenia, Portugal	Poland, Lux
Marine, aviation, transport (MAT)	Poland, Lux,	Poland, Lux, UK,
Fire and other property damage	Poland, Lux, UK,	Poland, Lux, UK
Third-party liability	Poland, Lux, Germany, UK	Poland, Lux, UK
Credit and suretyship	Poland, UK,	Poland, UK
Legal expenses	Poland, Germany	Poland
Assistance	Poland, UK	Poland
Miscellaneous	Poland,UK	Poland,UK,
Non-proportional reinsurance – property	UK	UK
Non-proportional reinsurance – casualty	UK	UK
Non-proportional reinsurance – MAT	UK	UK

CP71 Version2		
RESERVE RISK	**Gross**	**Net**
Motor, vehicle liability	Poland, Lux, Slovenia, Portugal, Slovakia, Iceland, Italy, Lithuania, Denmark, Sweden, Hungary, Germany, Finland	Poland, Lux, Iceland, Denmark
Motor, other classes	Poland, Lux, Slovenia, Portugal, Slovakia, Iceland, Lithuania, Finland, Denmark, Sweden	Poland, Lux, Denmark
Marine, aviation, transport (MAT)	Poland, Lux, Slovakia, Iceland, Denmark, Sweden	Poland, Lux, UK, Iceland, Denmark
Fire and other property damage	Poland, Lux, UK, Slovakia, Iceland, Finland, Denmark, Slovenia	Poland, Lux, UK, Iceland, Denmark
Third-party liability	Poland, Lux, Germany, UK, Slovakia, Iceland, Denmark, Slovenia	Poland, Lux, UK, Iceland
Credit and suretyship	Poland, UK, Slovakia, Denmark, sweden. Luxemburg, Slovenia	Poland, UK
Legal expenses	Poland, Germany, Slovakia, Finland, UK	Poland
Assistance	Poland, UK, Slovakia, Denmark	Poland
Miscellaneous	Poland,UK, Denmark	Poland,UK,
Non-proportional reinsurance - property	UK	UK
Non-proportional reinsurance - casualty	UK	UK
Non-proportional reinsurance - MAT	UK	UK

7.6. During CP71, CEIOPS based the final selection on the data set with most observations and based on the goodness of fit test.

7.7. For the current analysis, the analysis has been based on gross data, as this had the widest availability of data by member state and by type of undertaking.

7.8. Below we provide a few examples of how we proceeded with the analysis. Please note that this process was followed for all lines of business.

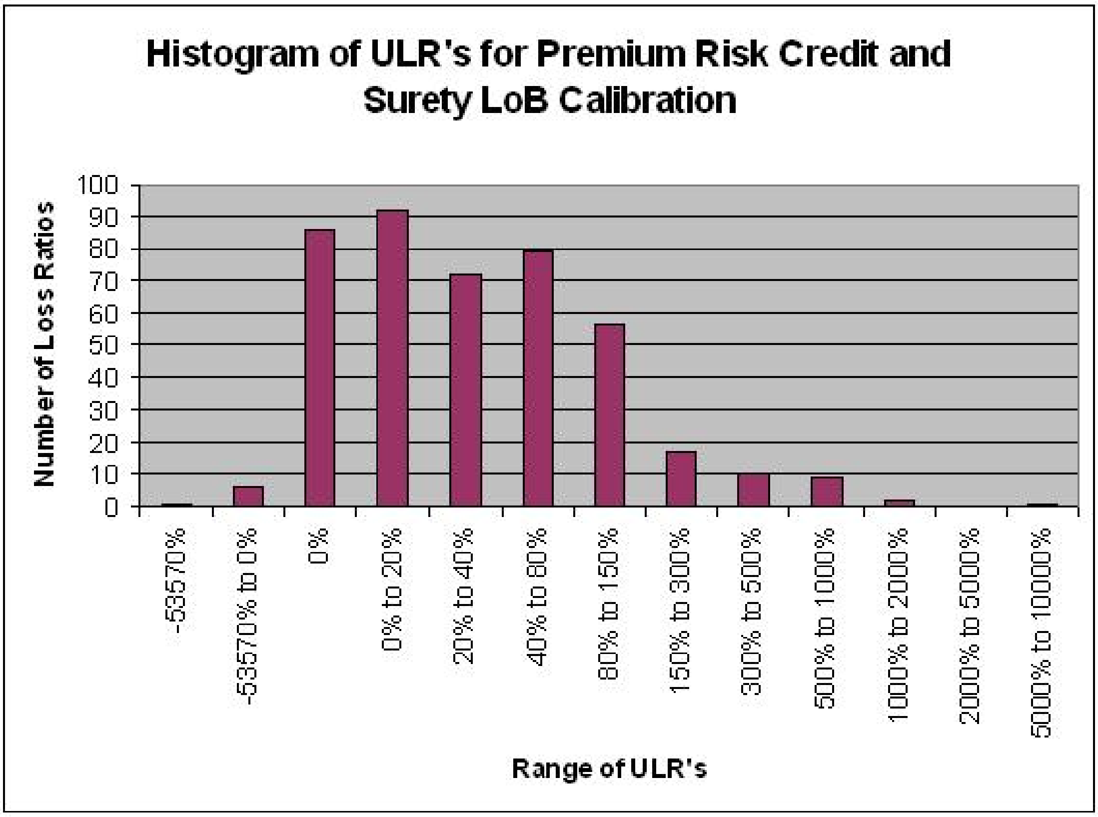
7.3 Examples of the process undertaken

Example 1. Gross Premium Risk for the Credit and Suretyship LoB

Data Cleaning

Raw Data

1. The ultimate loss ratios (posted after one year) coming from the raw data as supplied by the member states for the Credit and Suretyship LoB had a distribution as follows:



Band	Number of observations
-53570%	1
-53570% to 0%	6
0%	86
0% to 20%	92
20% to 40%	72
40% to 80%	79
80% to 150%	57
150% to 300%	17
300% to 500%	10
500% to 1000%	9
1000% to 2000%	2
2000% to 5000%	0
5000% to 10000%	1
Total	432

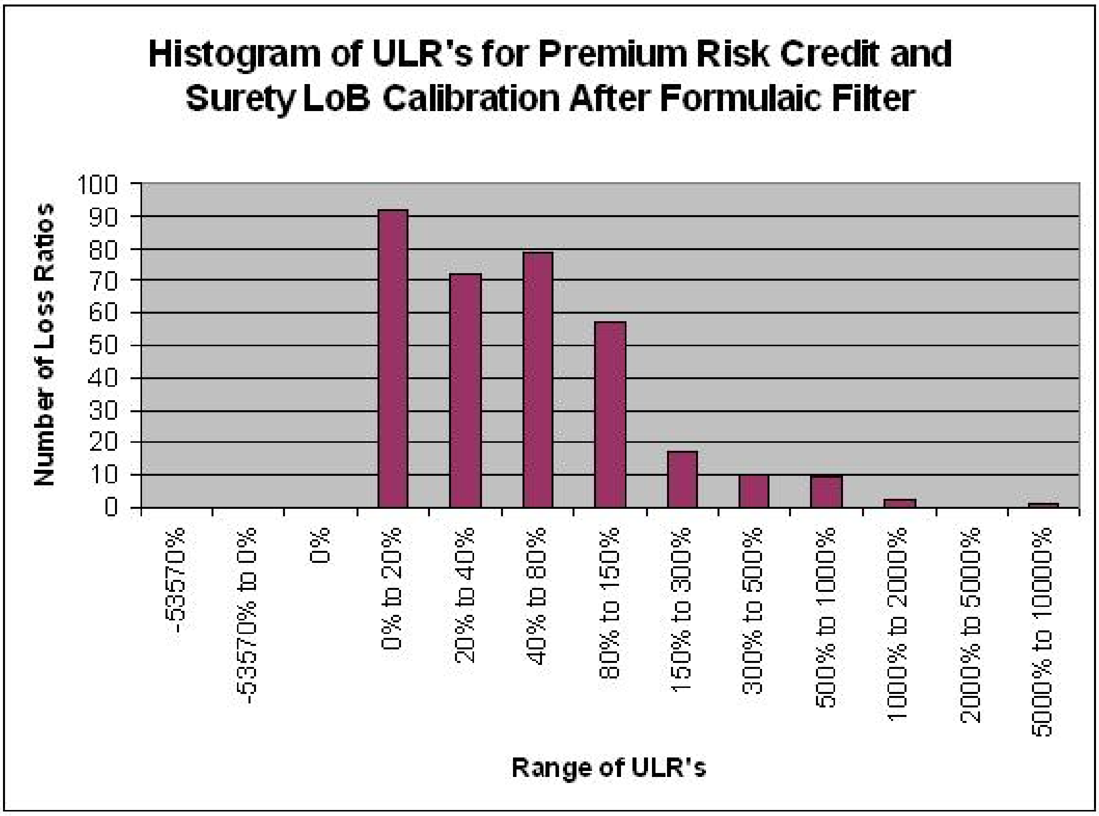
2. The member states that provided the data, excluding the FSA, confirmed that the data was net of the impact of catastrophe losses.

3. This raw dataset comprised of 74 companies.

Formulaic Filter

4. We then filter this data to remove negative loss ratios and zero loss ratios. Zero loss ratios are clearly feasible, however, they are not possible under the assumptions of the standard formula. Were we to include these loss ratios within the calibration data then there would be two effects:
    - Firstly we would not be able to include these observations anyway in the methods which use maximum likelihood for the LogNormal distribution since these observations are impossible.
    - Secondly, for those methods where we use least squares to fit the models, the inclusion of these zero loss ratios would materially increase the fitted parameters. This would be due to the fact that these zero observations lie outside the set of all other observations which would have the result of significantly increasing the observed dispersion of the data.

5. The ultimate loss ratios (posted after one year) coming from the raw data as supplied by the member states for the Credit and Suretyship LoB following the application of this formulaic filter had a distribution as follows:



Band	Number of observations

-53570%	0
-53570% to 0%	0
0%	0
0% to 20%	92
20% to 40%	72
40% to 80%	79
80% to 150%	57
150% to 300%	17
300% to 500%	10
500% to 1000%	9
1000% to 2000%	2
2000% to 5000%	0
5000% to 10000%	1
Total	339

6. As can be seen from the graph and table above, there are a significant number of very large loss ratios.

Manual Filter

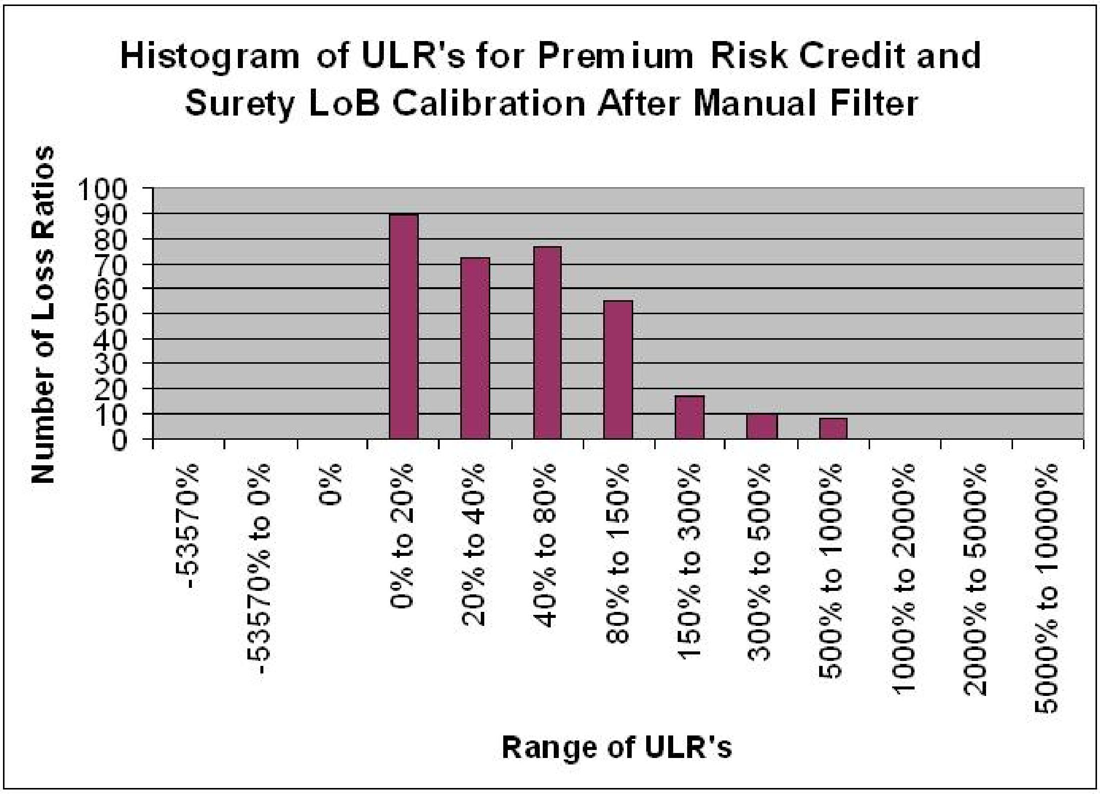
7. We then look through the data in detail, identifying problems in the observations, including:
    - Distortions due to mergers and acquisitions
    - Typographic mistakes
    - Apparent inconsistencies between different years and between premiums and claims for the same company

8. We then remove these identified distorting observations.

9. The FSA were informed of the impact of catastrophe losses on their claims for some companies and the data was adjusted to remove the impact.

10. The overall effect of these adjustments is to significantly reduce the volatility of data used for the calibration and as a result reduce the fitted parameters. This has been done to reduce the impact of the potential distortions mentioned above.

11. The ultimate loss ratios (posted after one year) coming from the raw data as supplied by the member states for the Credit and Suretyship LoB following the application of the formulaic filter and manual filter had a distribution as follows:



Band	Number of observations
-53570%	0
-53570% to 0%	0
0%	0
0% to 20%	89
20% to 40%	72
40% to 80%	77
80% to 150%	55
150% to 300%	17
300% to 500%	10
500% to 1000%	8
1000% to 2000%	0
2000% to 5000%	0
5000% to 10000%	0
Total	328

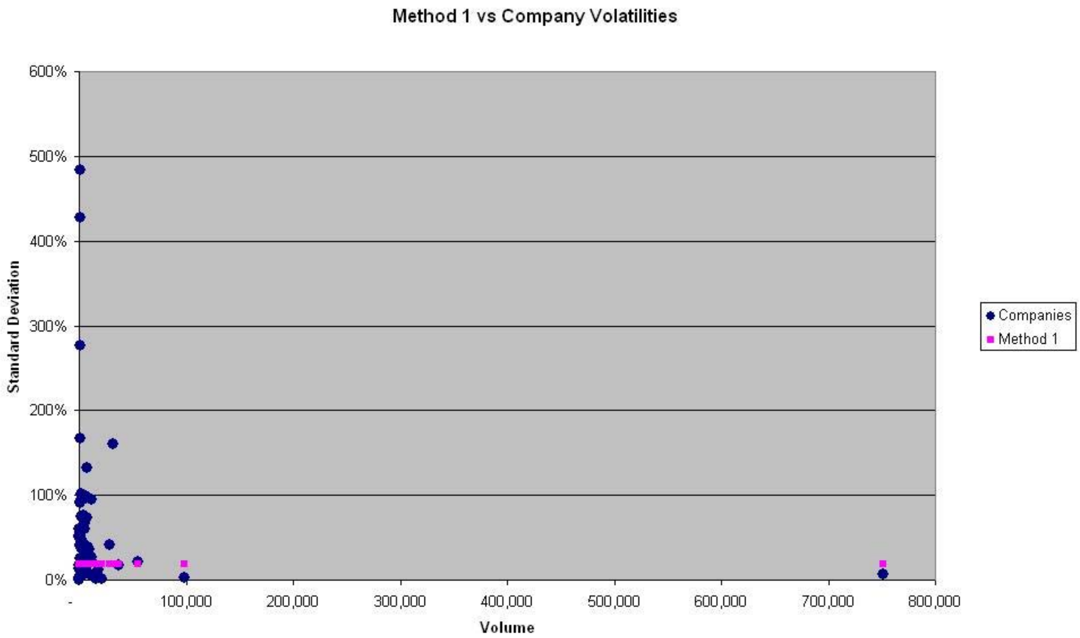
12. The graph and table above summarise the data used to calibrate the Gross Premium Risk for the Credit and Suretyship LoB.

13. This dataset comprised 60 companies each with Earned Premium amounts and Ultimate Losses posted after one year for various accident years.

Analysis

Method 1:

The graph below shows the standard deviations of ultimate loss ratio (as posted after one year) by company.



Graph Explanation

- Each blue dot represents a single company standard deviation as fitted via Method 1.
- The company specific fitted standard deviation is based upon the assumption that the standard deviation is proportional to the square root of the volume measure (premium).
- The pink dots represent the fitted premium risk factor for this LoB from this Method (ie 17%).

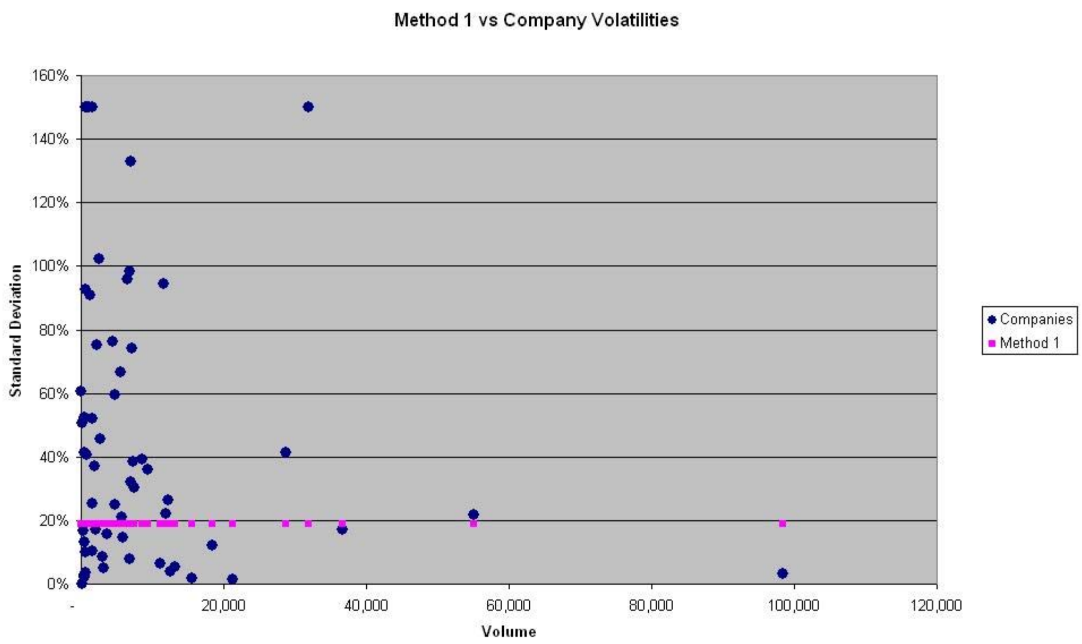
Commentary

The graph above is difficult to appraise since:

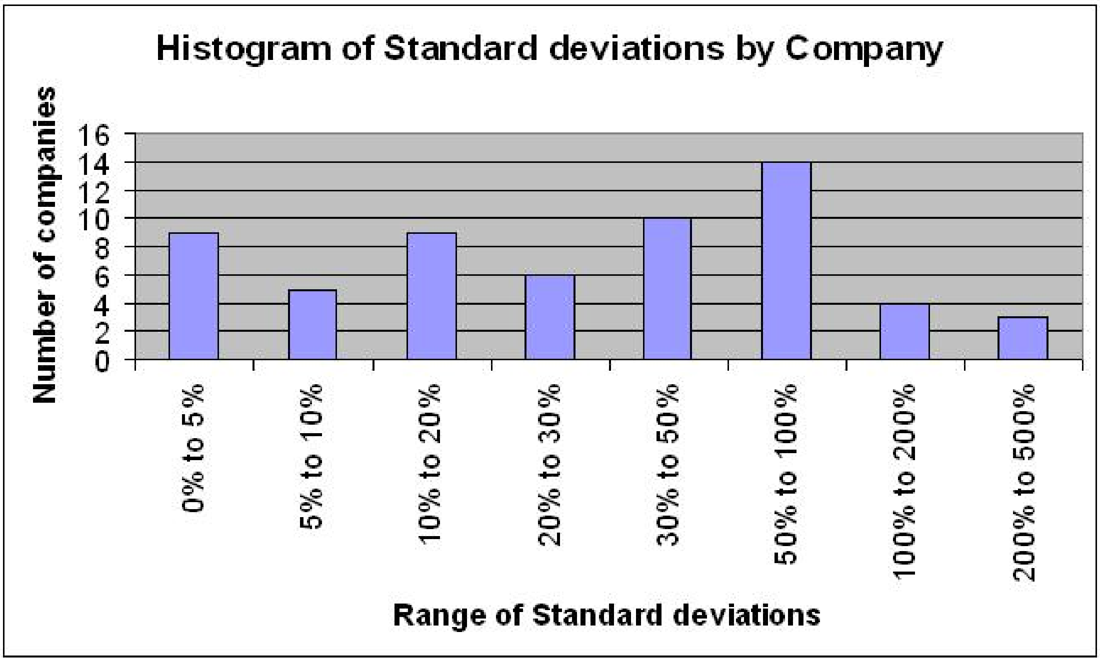
- There is a very large observation of premium that dominates the view of the x axis.
- There are some very large observed loss ratios that dominate the view of the y axis.

This results in the majority of the points lying in the bottom right of the graph.

If we cap the loss ratios at 150% and remove the one observation to the far right of the graph we get the following graph:



Another representation of the same information contained above is included in the histogram and table below:



Range of Standard Deviations	Number of Companies
0% to 5%	9
5% to 10%	5
10% to 20%	9
20% to 30%	6
30% to 50%	10
50% to 100%	14
100% to 200%	4
200% to 500%	3
Total	60

This shows that there are a significant number of companies whose standard deviations (approximately 60%) are above the fitted result of 19% (which is the undiscounted value corresponding to 17% discounted).

This is caused by the fitting algorithm placing more weight to the standard deviations from companies with larger volumes. These companies are the companies with, in general, lower volatilities – as shown in the graph above.

In general this Method is likely to underestimate the overall parameters due to the fact that implicitly the Method calculates company specific means, which it then uses to derive the estimations of company specific standard deviations. Given that some of the companies concerned will be writing similar business in similar markets, this approach will over-fit to the data and thus underestimate the underlying volatility.

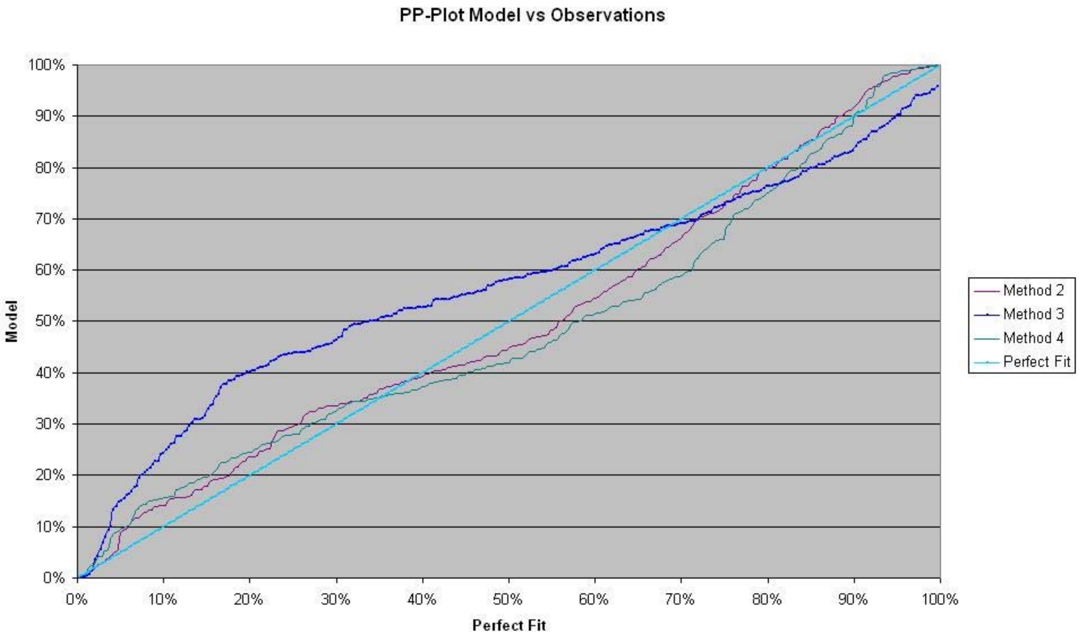
Methods 2, 3 and 4:

The table below gives the fitted (discounted) results for these methods.

Method	Small	Medium	Large	Fitted
Method 2	134%	64%	44%	18%
Method 3	12428%	5923%	4058%	1661%
Method 4	73%	73%	73%	73%

Clearly the results for Method 3 look very strange. We shall explore the reasons for this below.

The graph below shows the p-p plots of the observed posted claims after one year.



Graph Explanation

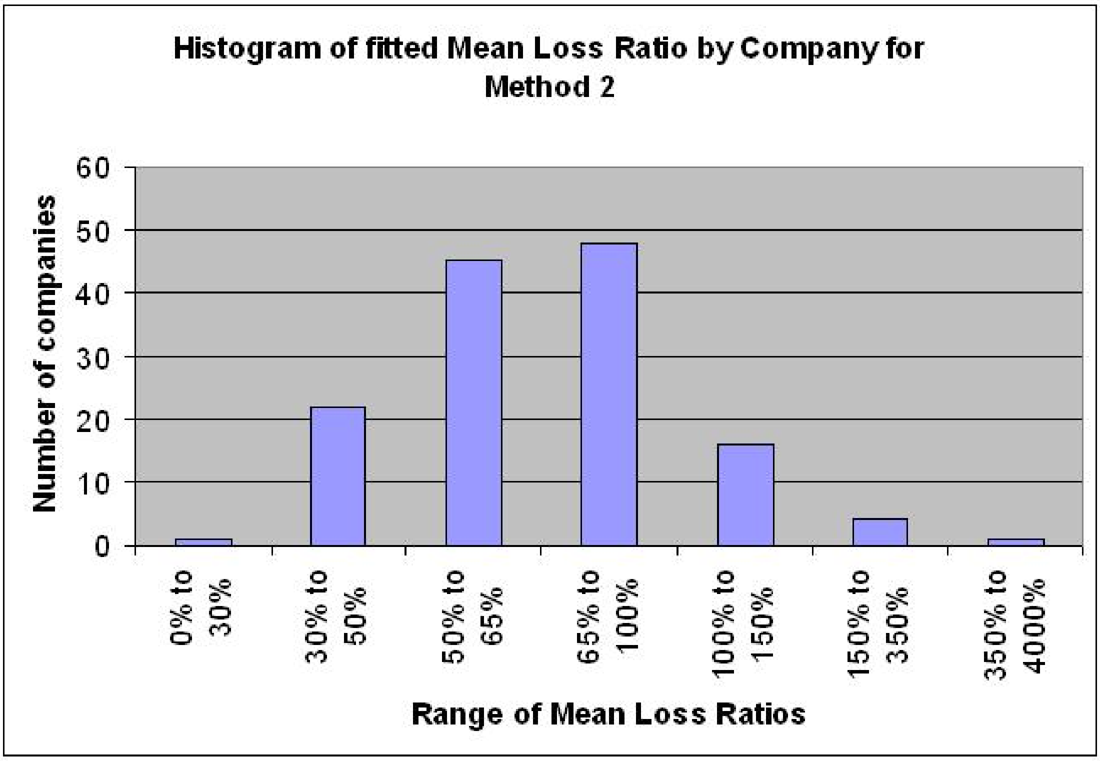
- Each line represents the goodness of fit plot for each of the respective methods.
- Each point on each curve is derived from an observation of ultimate loss posted after one year. This is converted to a probability of observing an loss ratio of at least that size given the specific model assumptions for that method. This is then plotted against the expected probability of observing an loss ratio of at least that size, assuming that each observation is independent of the other observations.
- If our model was perfectly accurate, our parameterisation appropriate and we had an infinite number of observations the p-p plot would lie on the "Perfect Fit" light blue curve above.
- The theory says that the better the fit, the closer the p-p plot will lie to the "Perfect Fit" line.
- The best fit appears to be Method 2.
- The worst fit appears to be Method 3, which is giving us the strangest results.

Commentary

The difference between the model assumptions behind Methods 2 and 4 and those behind Method 3 is that Methods 2 and 4 assume that different companies have different mean loss ratios. Method 3 assumes that all companies have the same loss ratio across the market.

Method 2

The graph and table below describe the fitted mean loss ratios by company for Method 2:



Range of fitted Mean Loss Ratio for Method 2	Number of Companies
0% to 30%	1
30% to 50%	22
50% to 65%	45
65% to 100%	48
100% to 150%	16
150% to 350%	4
350% to 4000%	1

As we can see from this split above the fitted mean loss ratios by company are very different.

In general this Method is likely to underestimate the overall parameters due to the fact that the Method calculates company specific means, which it then uses to derive the estimations of the market wide volatility parameter. Given that some of the companies concerned will be writing similar business in similar markets, this approach will over-fit to the data and thus underestimate the underlying volatility.

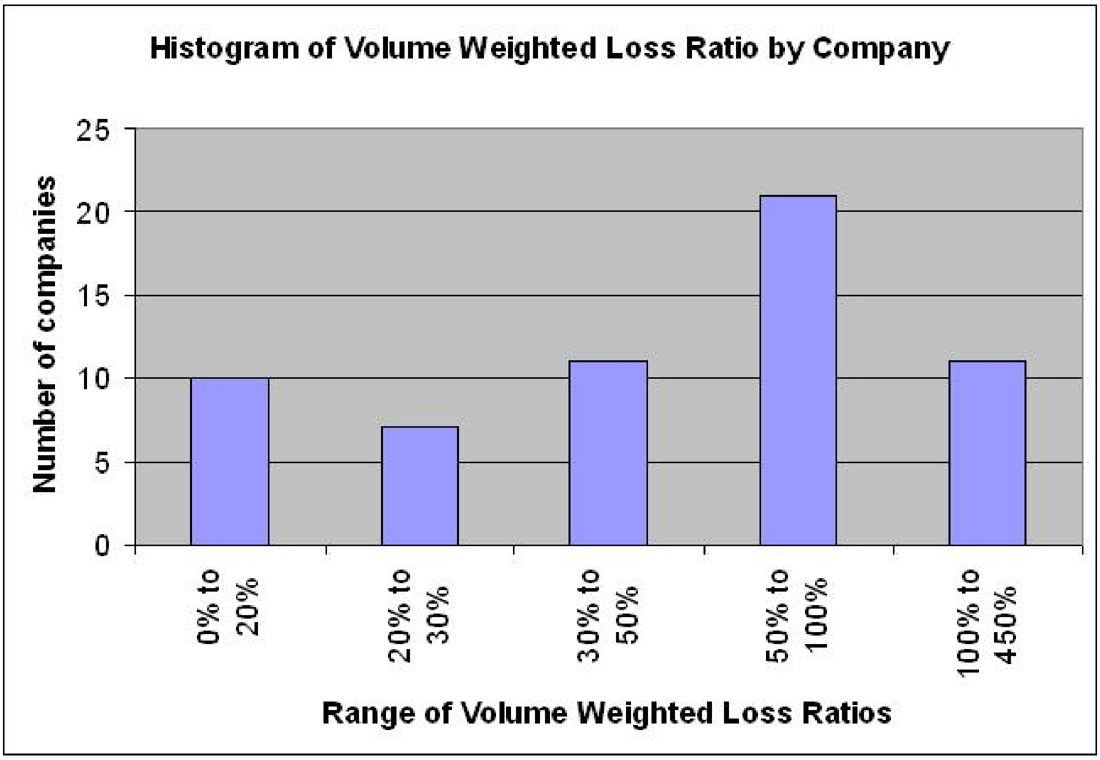
Method 3

This method fits a mean loss ratio across the whole market rather than by company. Other than this, Method 3 has the same assumptions as Method 2.

Method 3 should be preferred where there is little scope within a LoB for deviations in expected loss ratio between companies and avoids the risk of over-fitting of the parameters inherent in Method 2 (which would have the effect of underestimating the standard deviation).

Method 3 will always give larger results than Method 2, for this reason.

For this LoB we can see very different average loss ratios by company.



Range ofVolume Weighted Loss Ratio for Method 4	**Number of Companies**
0% to 20%	10
20% to 30%	7
30% to 50%	11
50% to 100%	21
100% to 450%	11
Total	60

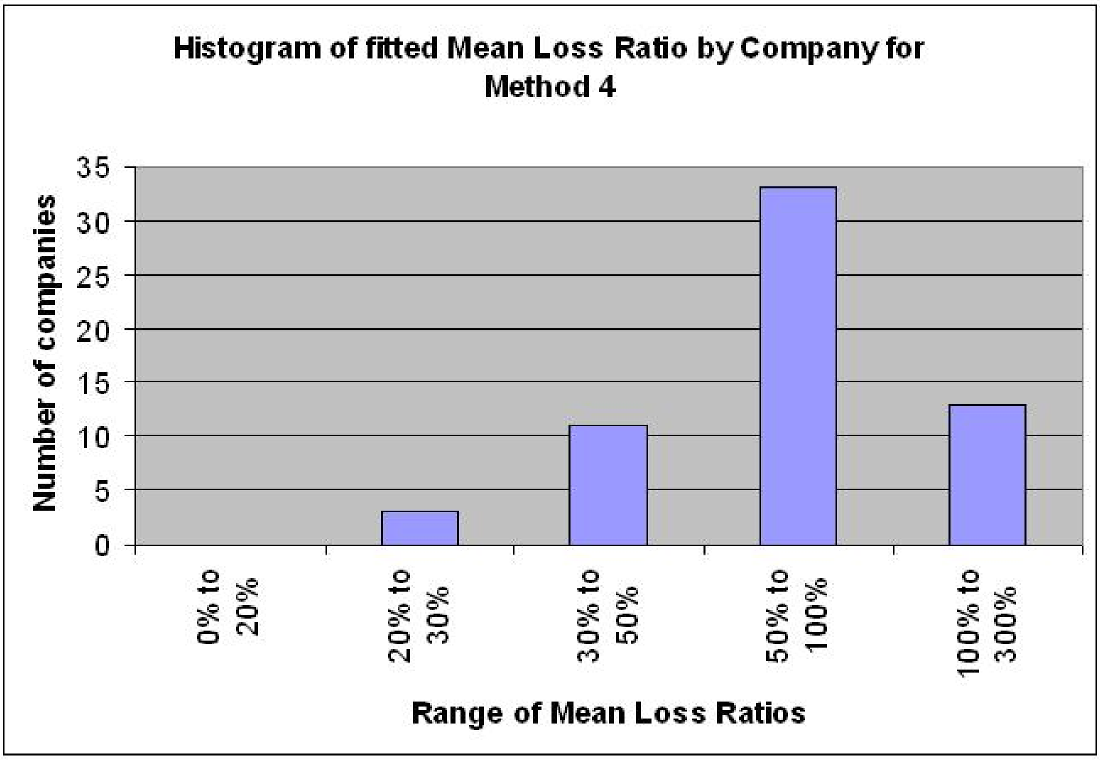
The graphs and table above show how very different the loss ratios are by company. They indeed seem to cluster around very different bases and as a result this method does not fit the data very well.

As a result it would seem unlikely that this model is appropriate for consideration for this LoB.

In general this Method is likely to overestimate the overall parameters due to the fact that the Method does not calculate company specific means, but rather calculates a market wide mean which it then uses to derive the estimations of the market wide volatility parameters. Given that some of the companies concerned will be writing very different books of business in different markets, this approach will under-fit to the data and thus overestimate the underlying volatility.

Method 4

The graph and table below describe the fitted mean loss ratios by company for Method 4:



Range of fitted Mean Loss Ratio for Method 4	**Number of Companies**
0% to 20%	0
20% to 30%	3
30% to 50%	11
50% to 100%	33
100% to 300%	13
Total	60

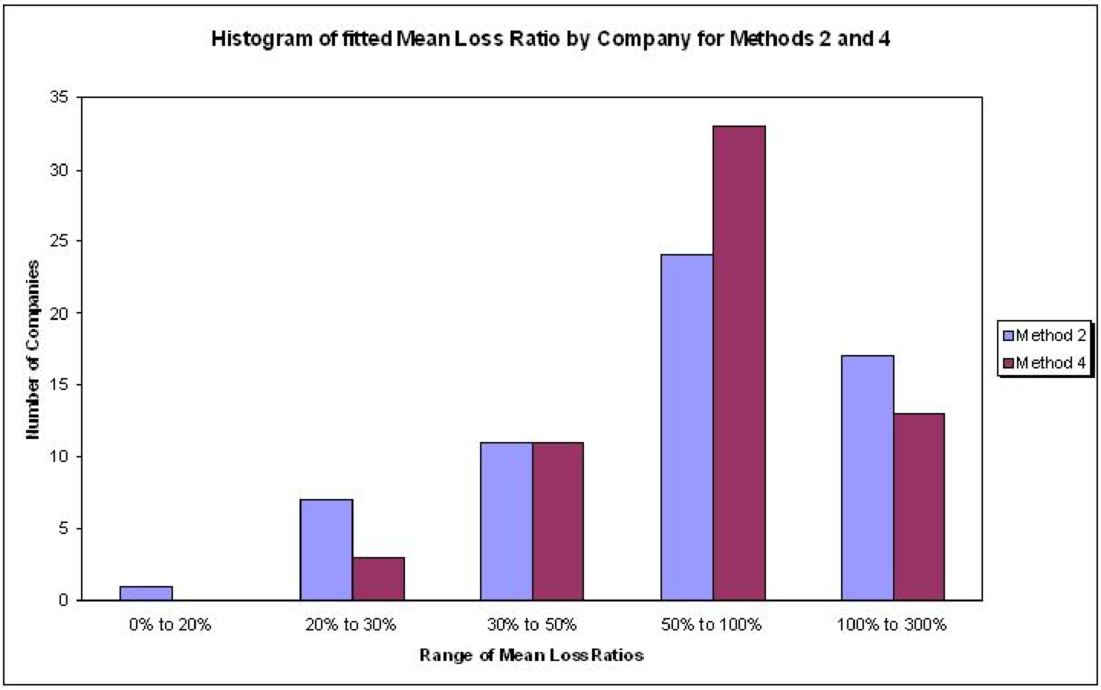
As we can see from this split above the fitted mean loss ratios by company are very different.

This method assumes that there is a constant standard deviation of loss ratio for each firm and that this does not change with the size of the firm. There is some evidence (see the graphs above for Method 1) that the variability does change with size of Volume measure as we would expect. However, the counter position is that the p-p plot for Method 4 is only slightly worse than that for Method 2.

In general this Method is likely to underestimate the overall parameters due to the fact that the Method calculates company specific means, which it then uses to derive the estimations of the market wide volatility parameters. Given that some of the companies concerned will be writing similar business in similar markets, this approach will over-fit to the data and thus underestimate the underlying volatility.

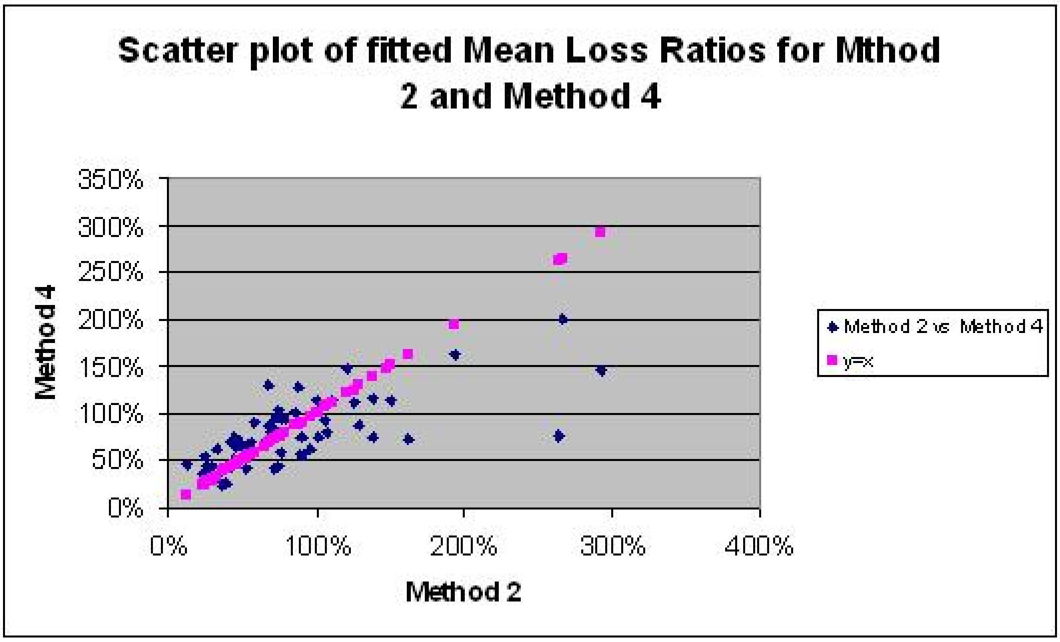
Method 2 versus Method 4

The fitted means from Method 4 are smaller than those coming from Method 2 – see the graph and table below:



Range of Volume Weighted Loss Ratio for Method 4	Number of Companies for Method 2	Number of Companies for Method 4
0% to 20%	1	0
20% to 30%	7	3
30% to 50%	11	11
50% to 100%	24	33
100% to 300%	17	13
Total	60	60

However, this is not conclusive as we can see in the graph below where we compare the fitted means for Method 2 and Method 4.



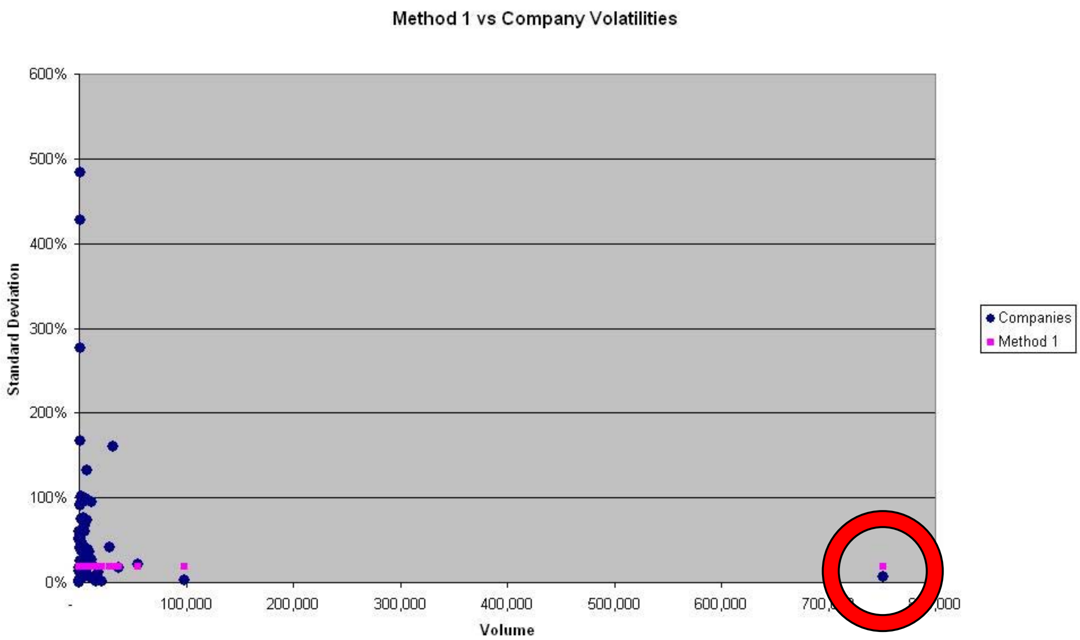
Each point on the graph above represents a single company, with the x-coordinate being the fitted mean for Method 2 and the y-coordinate being the fitted mean for Method 4.

As we can see there are many points both above and below the y=x line showing that the fitted means for Method 2 are not necessarily larger than those for Method 4, but it also shows that Method 2 does have the larger outliers in terms of fitted means.

Analysis of Outliers

There are two main outliers we will describe them in the sections below:

First Outlier

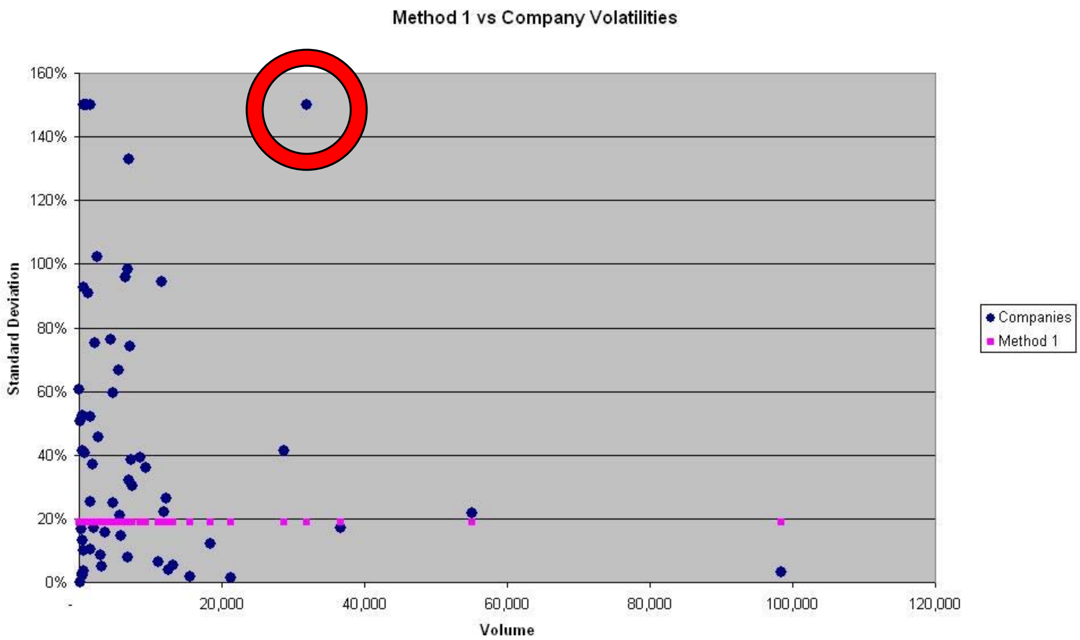


The first outlier as ringed in red in the graph above has a much larger volume than any other company. However, this company was not provided for the analysis as raw data, but rather as disguised data, which stated that the company in question was categorised as small. The company's premium income grew very dramatically over the period in question resulting in the anomaly that when the normalised data was converted into Euro observations the company data (although small in the precise terms of the disguised data) appears to have earned very large premium volumes on average. We suspect that this is not an accurate representation of the company in question.

The impact of this data point on the analysis is material. It does not affect the parameterisation of the respective models materially, however, it does have a material impact upon the "Fitted" results, since it materially affects the weighting between the different standard deviations by size. Since the volume measure for this company is so large and it has a low standard deviation this has the effect of significantly reducing the resultant "Fitted" parameters.

Since there are some doubts about the data and this does affect the results considerably we remove this company from the analysis. The effect of removing this data from the analysis will be to increase the "Fitted" parameters.

Second Outlier



The second outlier as ringed in red in the graph above has very large loss ratio volatility for a company of its apparent size. This seems to lie well outside the apparent curve above of loss ratio volatility against volume.

A closer examination of the company data concerned shows that in absolute terms the claims have remained relatively constant, but that the earned premiums seem to have undergone a couple of step changes in terms of their absolute size. This suggests that there is some mismatch between the claims and the premiums for this company.

The impact of this data point on the analysis is material. Having a large volatility for a large volume materially increases the model parameters for the models.

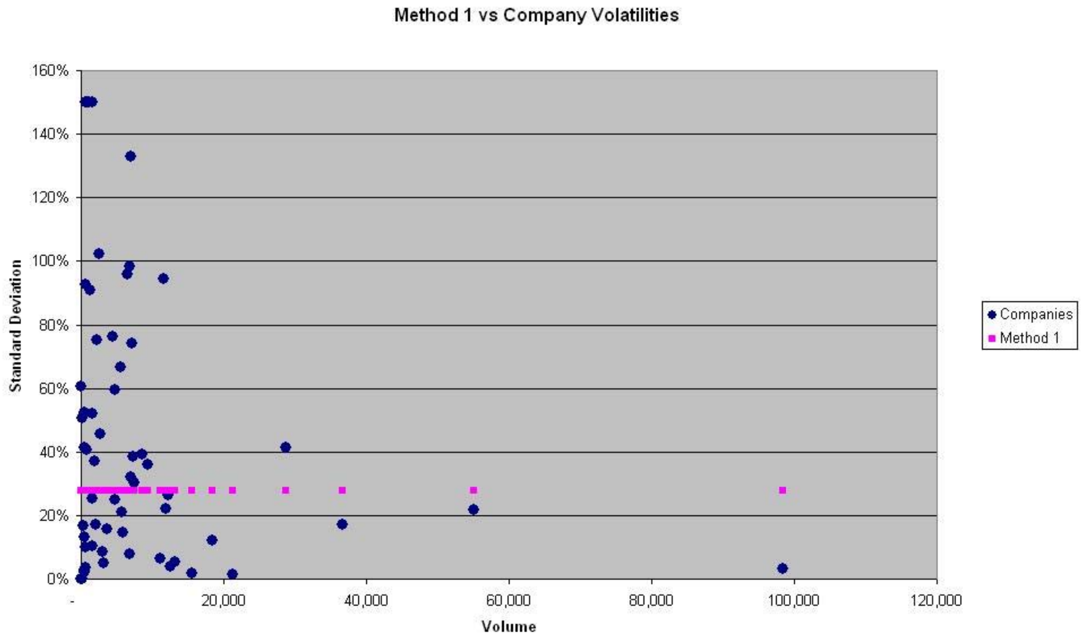
Since there are some doubts about the data and this does affect the results considerably we also remove this company from the analysis.

Results after removing the Outliers

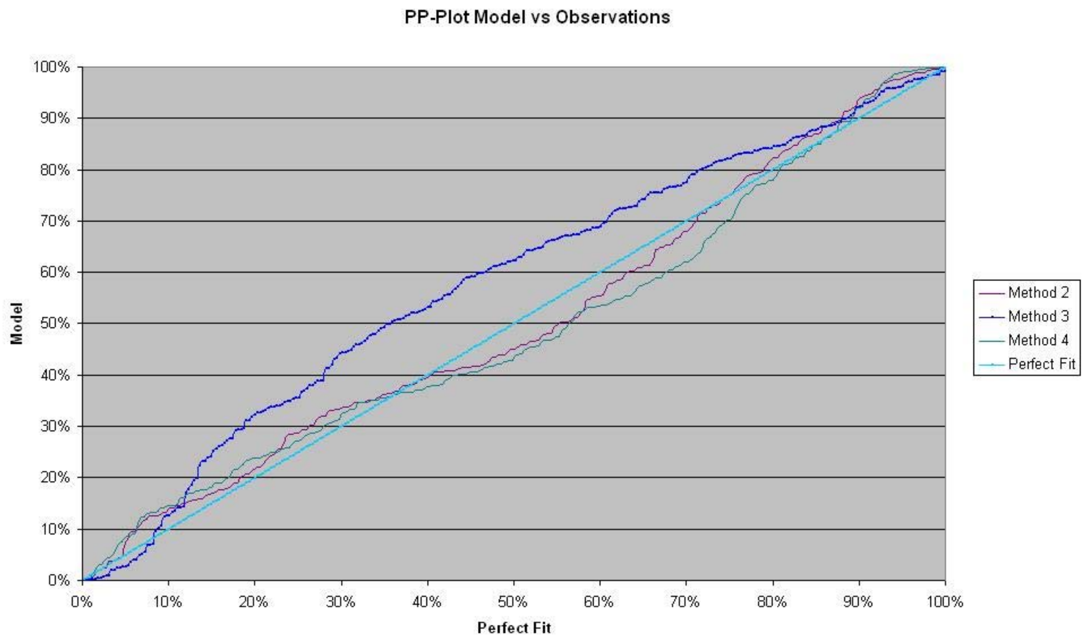
Method	Fitted with removal of Outliers	Fitted prior to removal of Outliers
Method 1	25%	17%
Method 2	31%	18%
Method 3	79%	1661%
Method 4	66%	73%

The adjusted results show an increase in Methods 1 and 2 due to the exclusion of the outliers, but a reduction in Methods 3 and 4.

- Method 1 increased due to the removal of the first outlier since this Method puts a significant weight upon the standard deviation of the largest company.
- Method 2 saw a reduction in the fitted parameter (due to the removal of the second outlier), but due to the new weighting due to the exclusion of the of the first outlier, it grew overall.
- Method 3 saw a massive reduction in the volatility. This was through the removal of the first outlier. The reason for this is a little convoluted.
    - The first outlier was a company that consistently had a very low loss ratio. Method 3 does not allow a company specific loss ratio, but rather a loss ratio that applies to all companies in the market.
    - As a result the model interprets deviations from the market mean as apparent volatility and tries to adjust the parameters so as to make this apparent volatility feasible.
    - The overall effect of this is to increase the fitted standard deviation.
    - The removal of the first outlier, removes this issue and thus drops the fitted standard deviation.
- Method 4 reduces due to the removal of the volatile second outlier and increases due to the removal of the low volatility first outlier. The impact of the removal of the second outlier is larger so as a result we get an overall reduction in the parameter.



The graph above shows the fitted volatilities as per Method 1 by company, after the removal of the two outliers. These show a reduction in volatility as the volume measure increases.



The graph above shows the p-p plot for Methods 2,3 and 4, after the removal of the two outliers.

This shows that Method 3 is still not a good fit. The fit for Model 2 has improved (slightly) and this method seems to best fit the data.

Conclusion

The table below gives us the fitted (discounted) parameters after the removal of the outliers.

Method	Fitted
Method 1	25%
Method 2	31%
Method 3	79%
Method 4	66%

Our analysis shows that Method 3 does not fit the data well, since companies appear to have different underlying mean loss ratios. We propose not considering this Method for this LoB.

Method 4 assumes that there is no diversification effect for writing larger volumes, but this is not borne out in the graphs. This Method also has a slightly worse p-p plot than Method 2.

As a result, we propose that an average of Methods 1 and 2 giving a factor of 28%.

Example 2. Gross Reserve Risk for the Motor TPL LoB

Raw Data

For the gross reserve risk analysis we have two set of data that we use for the analysis. Specifically:

- triangles of paid data by company and
- observations of movements over one year of blocks of reserves (for the same accident years) by company by calendar year.

Below is the commentary as to the data in relation to both of these parts of the data.

Paid Triangles

For this LoB we were provided with paid triangles for 457 companies. This data was only used for the Methods involving the Merz & Wuthrich formula. There are significant limitations to applicability of the formula and as a result a significant reduction in the number of companies was required so as to only include those where the data was appropriate.

Paid Triangles - Formulaic Filter

The limitations of the Merz & Wuthrich formula are such as to make it difficult to construct a useful formula for filtering the triangles. As a result we had to do the cleaning using the manual filter.

Paid Triangles - Manual Filter

We then look through the triangles in detail, identifying problems including:

- missing data from the triangles
- triangles where there were insufficient accident years for the earliest development year to be fully run-off
- potential typographical errors negative values
- Triangles whose historic development is such as to invalidate the chain ladder assumptions underlying the Merz-Wuthrich formula.

The triangles were then adjusted or removed from the analysis with two aims in mind

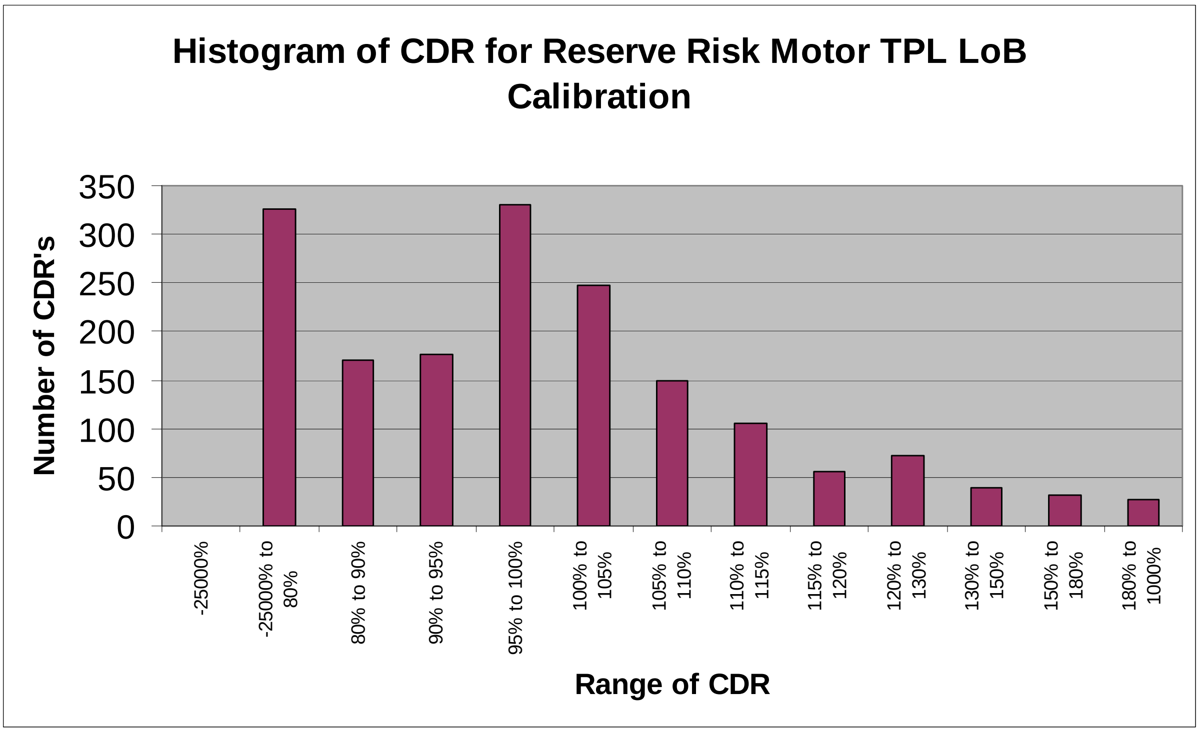
- To include as much data as possible so as to give as much credibility as possible to the resultant analyses.
- Ensure that the data used for the calibration is fit for purpose.

If there was missing or obviously erroneous data within the triangle then if it was possible to remove some of the older accident years (whilst leaving the triangle sufficiently large such that the oldest remaining accident year was nearly fully developed) then these were removed.

For this LoB we assumed that we needed at least 7 accident years of data for the triangle to reasonably be expected to be sufficiently run-off for the formula to derive sensible estimates. The process of cleaning and adjusting the data resulted in reducing the original 457 triangles down to 241 triangles.

Observations of Reserve Movements over one year

The claims development ratios (CDR – being the reserve posted after one year plus the claims paid within the year as a proportion of the initial reserve) coming from the raw data as supplied by the member states for the Motor TPL LoB had a distribution as follows:

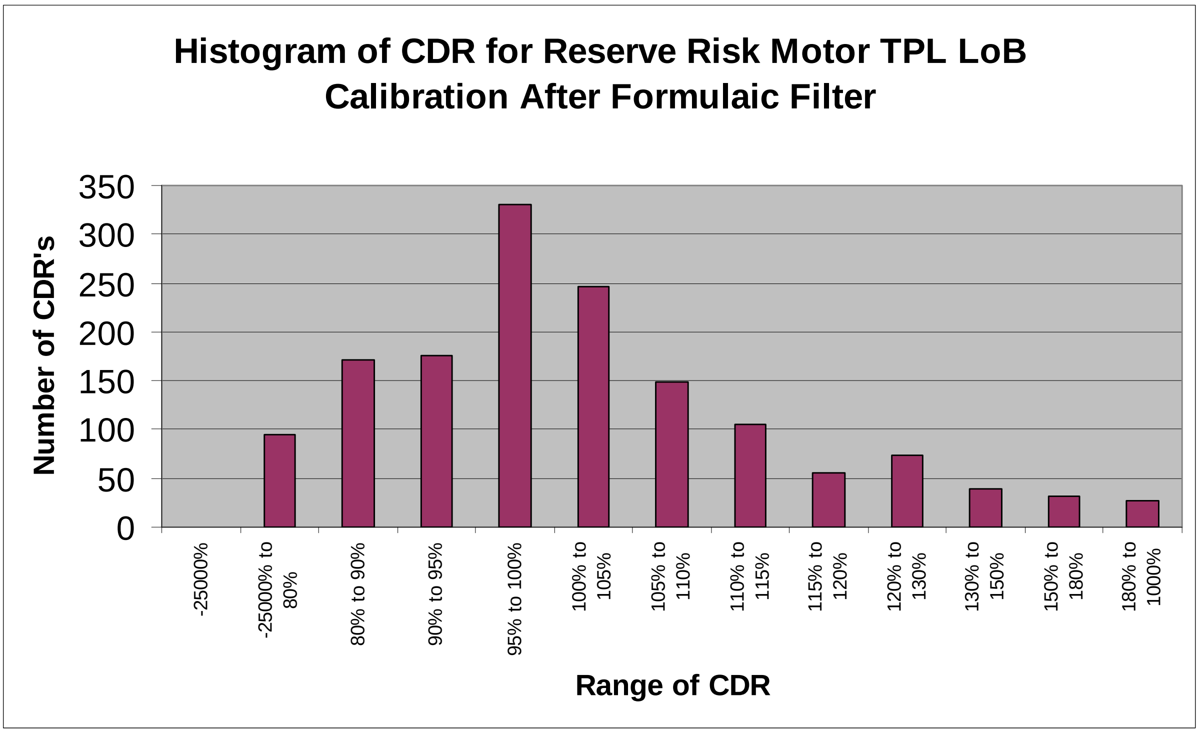


Band	Number of observations
-25000%	0
-25000% to 80%	326
80% to 90%	171
90% to 95%	176
95% to 100%	331
100% to 105%	247
105% to 110%	149
110% to 115%	105
115% to 120%	56
120% to 130%	73
130% to 150%	39
150% to 180%	31
180% to 1000%	27
Total	1731

Observations of Reserve Movements over one year - Formulaic Filter

We then filter this data to remove negative CDR's and zero CDR's.

The CDR (posted after one year) coming from the raw data as supplied by the member states for the Motor TPL LoB following the application of this formulaic filter had a distribution as follows:



Band	Number of observations
-25000%	0
-25000% to 80%	95
80% to 90%	171
90% to 95%	176
95% to 100%	331
100% to 105%	247
105% to 110%	149
110% to 115%	105
115% to 120%	56
120% to 130%	73
130% to 150%	39
150% to 180%	31
180% to 1000%	27
Total	1500

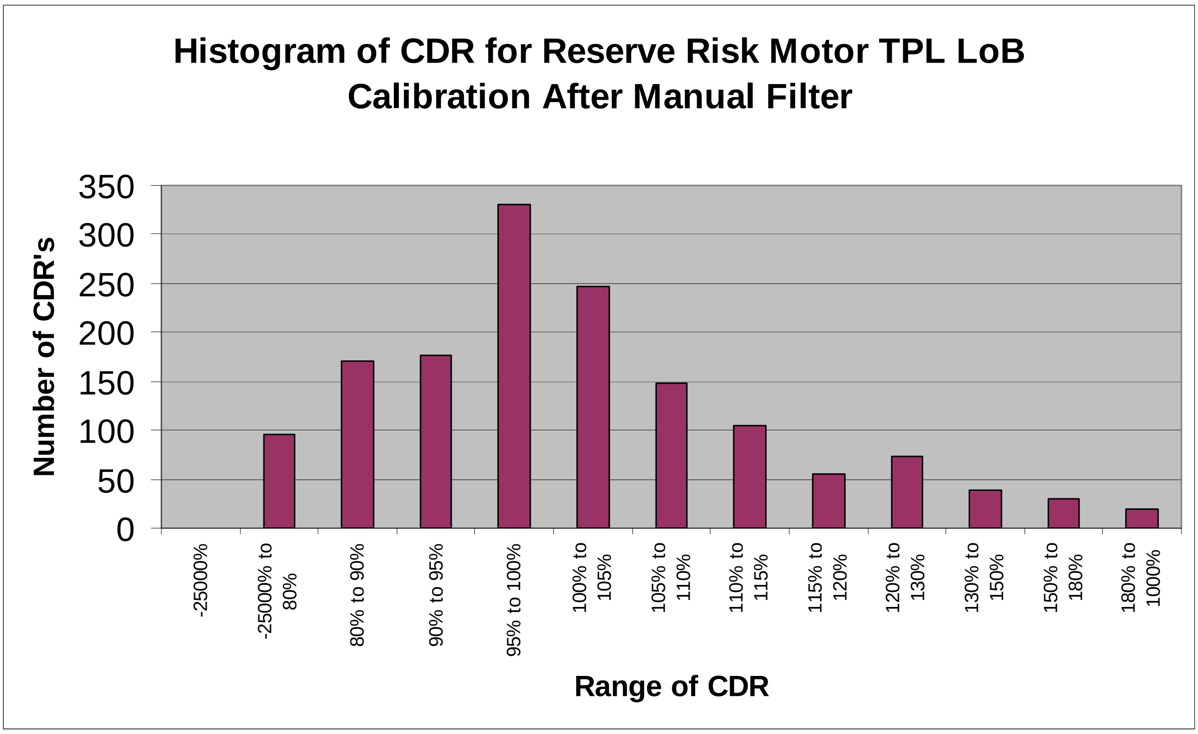
Observations of Reserve Movements over one year - Manual Filter

We then look through the data in detail, identifying problems in the observations, including:

- Distortions due to mergers and acquisitions
- Typographic mistakes
- Apparent inconsistencies between different years and between opening reserve and closing reserve for the same company

We then remove these identified distorting observations.

The CDR's (posted after one year) coming from the raw data as supplied by the member states for the Motor TPL LoB following the application of the formulaic filter and manual filter had a distribution as follows:



Band	Number of observations
-25000%	0
-25000% to 80%	95
80% to 90%	171
90% to 95%	176
95% to 100%	330
100% to 105%	247
105% to 110%	148
110% to 115%	105
115% to 120%	56
120% to 130%	73
130% to 150%	39
150% to 180%	30
180% to 1000%	19
Total	1489

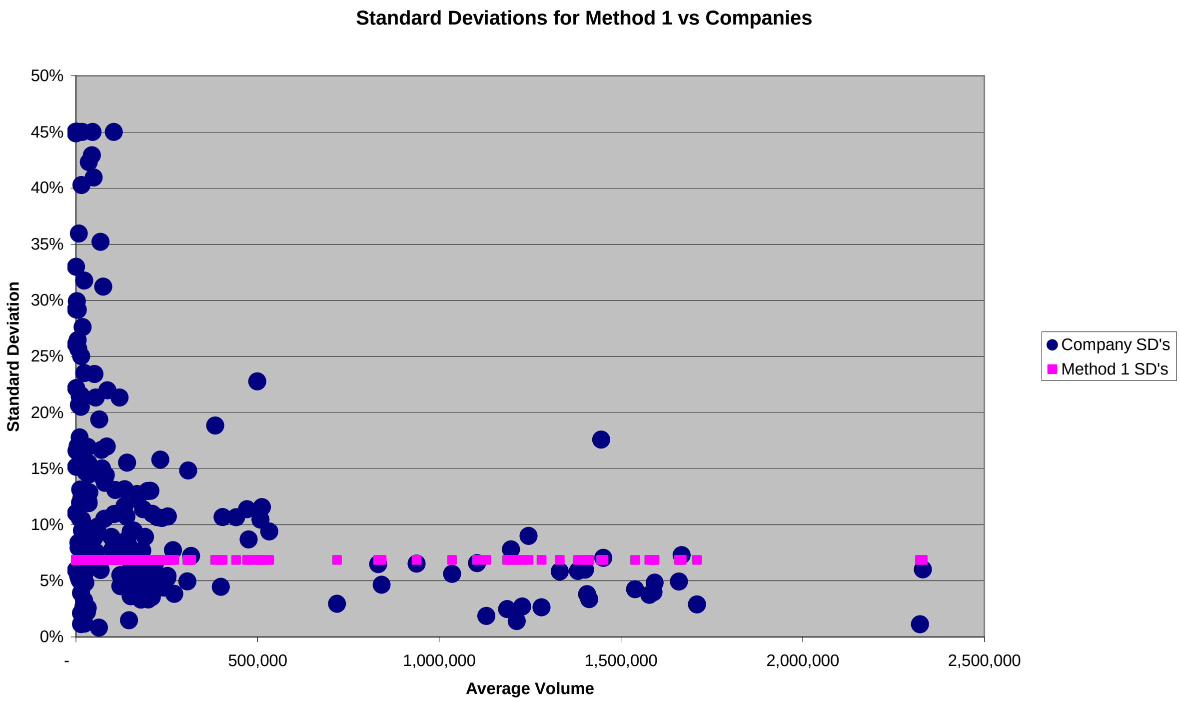
The graph and table above summarise the data used to calibrate the Gross Reserve Risk for the Motor TPL LoB.

This dataset comprised 216 companies each with Opening Reserve amounts and Closing Reserve + incremental Paid amounts after one year for various accident years.

Run analysis

Method 1:

The graph below shows the standard deviations of CDR (as posted after one year) by company.



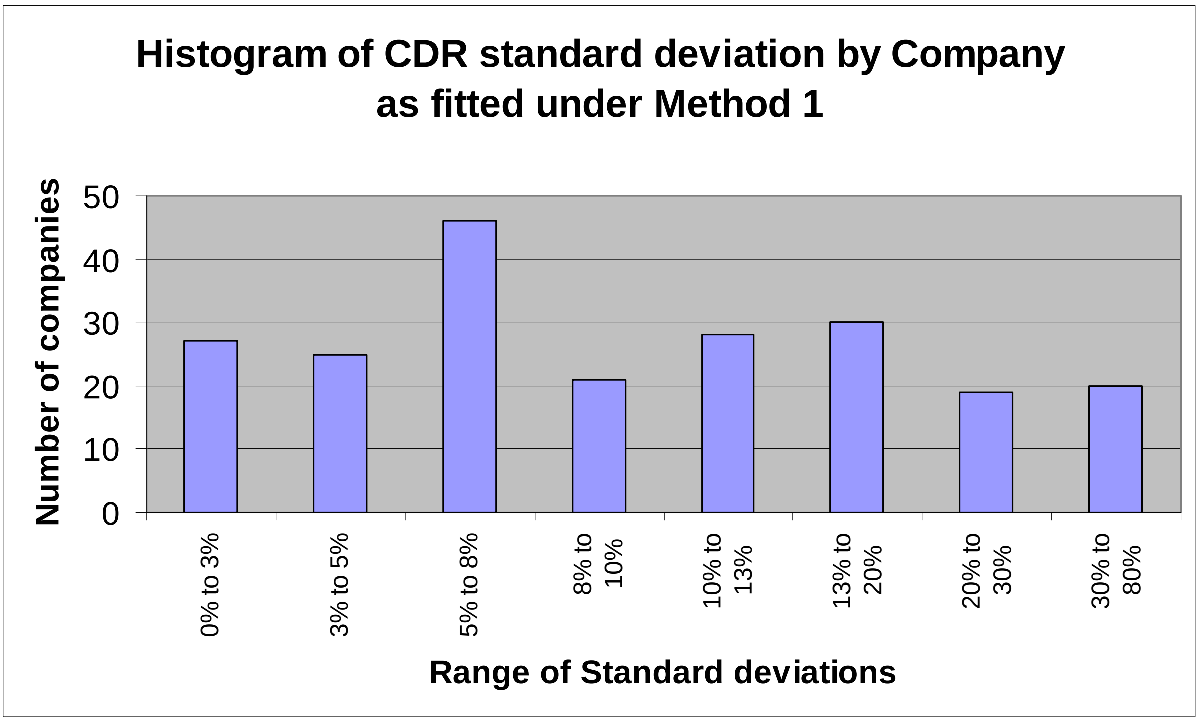
Graph Explanation

- Each blue dot represents a single company standard deviation as fitted via Method 1.
- The company specific fitted standard deviation is based upon the assumption that the standard deviation is proportional to the square root of the volume measure (opening reserve) and assuming that the opening reserve is unbiased.
- The pink dots represent the fitted reserve risk factor for this LoB from this Method (ie 7%).

Commentary

This graphs demonstrates a relationship between the size of the reserve and the size of the variability of the reserve. Specifically, it provides evidence that the larger the volume of reserves, the smaller the CDR volatility.

Another representation of the same information contained above is included in the histogram and table below:



Range of Standard Deviations	Number of Companies
0% to 3%	27
3% to 5%	25
5% to 8%	46
8% to 10%	21
10% to 13%	28
13% to 20%	30
20% to 30%	19
30% to 80%	20
Total	216

This shows that there are a significant number of companies whose standard deviations (approximately 65%) are above the fitted result of 7%.

This is caused by the fitting algorithm placing more weight to the standard deviations from companies with larger volumes. These companies are the companies with, in general, lower volatilities – as shown in the graph above.

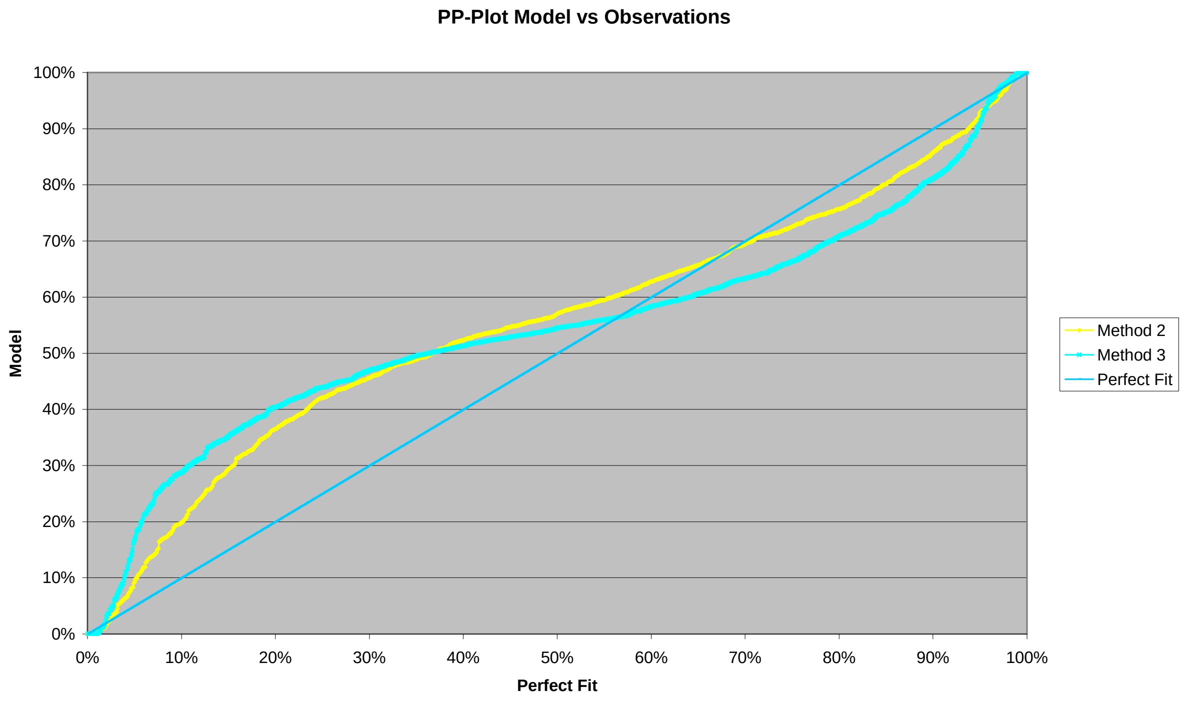
Methods 2 and 3:

The table below gives the fitted results for these methods.

Method	Small	Medium	Large	Fitted
Method 2	40%	19%	10%	7%
Method 3	25%	25%	25%	25%

The fitted results are very different between the two Methods. However, in some senses these Methods are fitting the data in a similar way, however, the approach used to produce the fitted result for Method 2 effectively places more weight on results for larger companies which have lower fitted standard deviations.

The graph below shows the p-p plots of the observed payment within the one year plus the posted reserve after one year.



Graph Explanation

- Each line represents the goodness of fit plot for each of the respective methods.
- Each point on each curve is derived from an observation of payments within the year plus the closing reserve after one year. This is converted to a probability of observing a CDR of at least that size given the specific model assumptions for that method. This is then plotted against the expected probability of observing a CDR of at least that size, assuming that each observation is independent of the other observations.
- If our model was perfectly accurate, our parameterisation appropriate and we had an infinite number of observations the p-p plot would lie on the “Perfect Fit” light blue curve above.
- The theory says that the better the fit, the closer the p-p plot will lie to the “Perfect Fit” line.
- There is little to distinguish these two Methods in terms of their fit to the data. Neither of these Methods are giving a particularly good fit to the data.

Commentary

- Methods 2 and 3 both assume that there is no inherent bias within the posted reserves for this LoB (as is implicit within the standard formula assumptions).

The p-p plots for both graphs lie almost completely above the Perfect Fit line (y=x). This tells us that both of the fitted models would predict there to be more observations below any respective value than are actually seen in the data. However this issue is less prevalent in the tail of the distribution. As a result when the standard formula uses the lognormal assumptions to quantify the marginal capital requirement, the parameters derived from these models should give capital requirements relatively consistent with the observed CDR's.

The similarity between the fits of these two Methods is demonstrated by:

- The similarity of the p-p plots for the two methods
- The similarity between the values of the fitted results for the Medium reference company.

However, since Method 2 initially fits a model that gives a different volatility by size, this Method contains an extra step in order to derive a figure across all companies. This Method takes a volume weighted approach which places more weight on the larger companies within the data who have lower volatilities according to the initial fitted model. As a result Method 2 gives a smaller Fitted parameter than Method 3 despite the model fits being very similar.

Methods 4, 5 and 6:

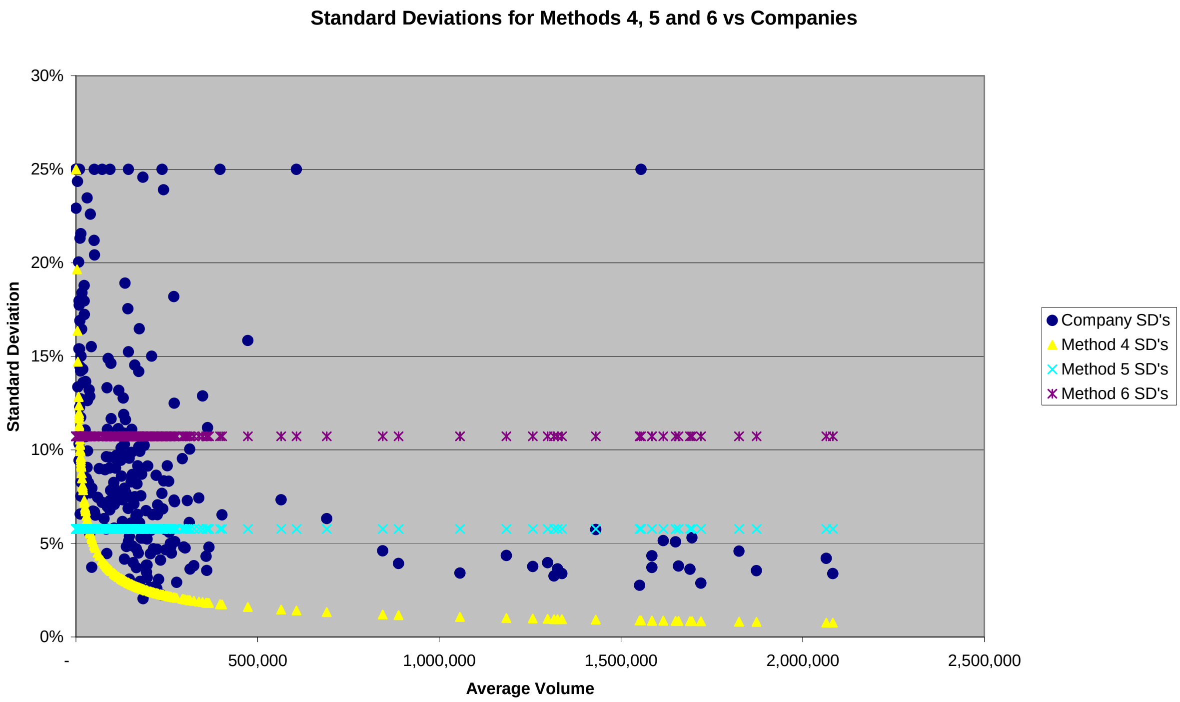
The table below gives the fitted results for these methods.

Method	Small	Medium	Large	Fitted
Method 4	9%	4%	2%	2%
Method 5	6%	6%	6%	6%
Method 6	11%	11%	11%	11%

These results are different from the results coming from Methods 1, 2 and 3. There are various reasons for the differences:

- Methods 4, 5 and 6 are fitted to different datasets than Methods 1, 2 and 3. For some LoB's there is some overlapping of the contributing companies, however this is not necessarily the case.
  - Methods 1, 2 and 3: The data used is observations of historic claims development over one year.
  - Methods 4, 5 and 6: The data used is paid triangles.
- The model assumptions behind Methods 4, 5 and 6 are significantly different to those behind Methods 1, 2 and 3.
  - Methods 1, 2 and 3: These Methods directly fit models to observed posted claims development.
  - Methods 4, 5 and 6: These Methods attempt to derive an estimate of the one year reserve volatility by company without reference to the company's posted reserves.

The graph below shows the fitted standard deviations by company and compares these to the results from Methods 4, 5 and 6.



Graph Explanation

- Each dark blue dot represents a single company standard deviation of the one year claims development (via the Merz-Wuthrich formula), against the volume weighted chain ladder opening reserve amount for that company.
- The yellow points represent, for each company used in the calibration, the predicted standard deviation of the one year claims development (for Method 4) against the volume weighted chain ladder opening reserve amount for that company.
- The light blue points represent, for each company used in the calibration, the predicted standard deviation of the one year claims development (for Method 5) against the volume weighted chain ladder opening reserve amount for that company.
- The red points represent, for each company used in the calibration, the predicted standard deviation of the one year claims development (for Method 6) against the volume weighted chain ladder opening reserve amount for that company

Commentary

Method 4 tends to underestimate the standard deviation at all but the very smallest volume of reserves as seen in the graph above that most of the dark blue points (about 92%) are above the yellow curve.

Method 5 has about 69% of the dark blue points above the curve and comes out with an estimate slightly larger than would be appropriate for the larger companies (the far right of the graph).

Method 6 has about 33% of the dark blue points above the curve and comes out with an estimate much larger than would be appropriate for the larger companies (the far right of the graph).

These Methods all use the Merz-Wuthrich formula applied to paid data. There are as a consequence of the limitations in the model various things we should consider when looking at these results:

- The Merz-Wuthrich formula assumes that the chain ladder is an appropriate model for the claims. Clearly, these assumptions are going to be most reasonable for shorter tailed classes and for later accident years. Where the chain ladder is not such an appropriate technique, the Merz approach is likely to over estimate the company specific standards deviation.
- This formula has been applied to the paid triangles. This will implicitly over estimate the resultant company specific standards deviation as a result of interpreting uncertainty in payment timing as uncertainty in future ultimate payments. This issue would be less if the triangles used were incurred claim amounts rather than paid amounts.
- The implementation of the Merz formula used does not allow for any future development of the triangle beyond the number of development years of the triangle. Ie the implemented approach does not allow for any tail factors or run-off of the tail. This will have the effect of understating the estimate the company specific standards deviations. This issue is especially relevant since we have performed the analyses on paid data for which the triangle has a longer tail than on incurred data which has a shorter tail.

It is not clear to what extent these factors offset each other and whether the resultant estimates are prudent or optimistic.

Conclusion

The table below gives us the fitted parameters:

Method	Fitted
Method 1	6%
Method 2	7%
Method 3	25%
Method 4	2%
Method 5	6%
Method 6	11%

Our analysis shows that:

- Methods 2 and 3 have bad p-p plots, though there is some credibility in the tail of the distributions.
- Method 4 understates the volatility.

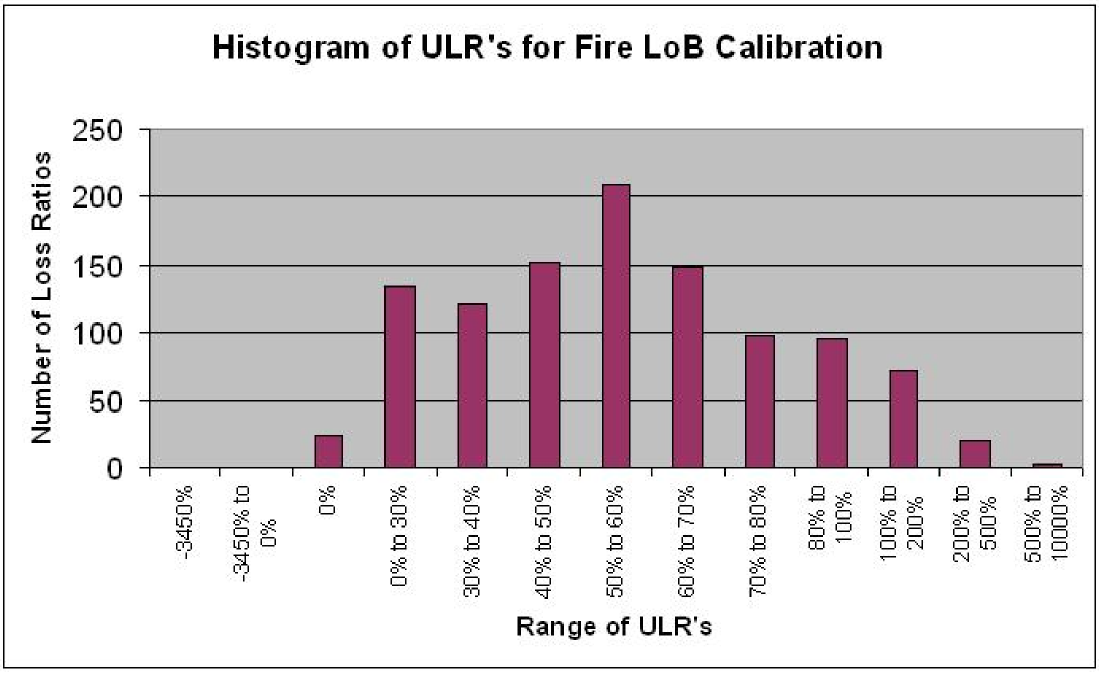
As a result, we propose that an average of Methods 1, 2, 3, 5 and 6 giving a factor of 10.8%.

Example 3. Gross Premium Risk for the Fire LoB calibration

Data Cleaning

Raw Data

The ultimate loss ratios (posted after one year) coming from the raw data as supplied by the member states for the Fire LoB had a distribution as follows:



Band	Number of observations
-3450%	1
-3450% to 0%	1
0%	24
0% to 30%	134
30% to 40%	121
40% to 50%	152
50% to 60%	209
60% to 70%	148
70% to 80%	98
80% to 100%	95
100% to 200%	72
200% to 500%	21
500% to 10000%	3
Total	1079

The member states that provided the data, excluding the FSA, confirmed that the data was net of the impact of catastrophe losses.

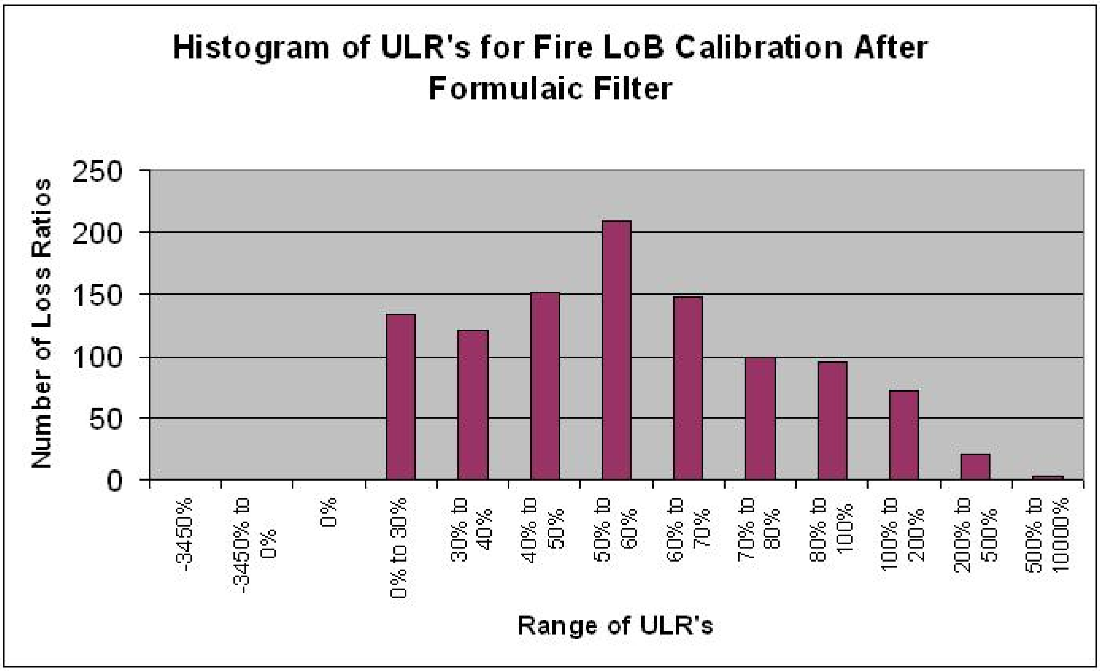
This raw dataset comprised of 150 companies.

Formulaic Filter

We then filter this data to remove negative loss ratios and zero loss ratios. Zero loss ratios are clearly feasible, however, they are not possible under the assumptions of the standard formula. Were we to include these loss ratios within the calibration data then there would be two effects:

- Firstly we would not be able to include these observations anyway in the methods which use maximum likelihood for the LogNormal distribution since these observations are impossible.
- Secondly, for those methods where we use least squares to fit the models, the inclusion of these zero loss ratios would materially increase the fitted parameters. This would be due to the fact that these zero observations lie outside the set of all other observations which would have the result of significantly increasing the observed dispersion of the data.

The ultimate loss ratios (posted after one year) coming from the raw data as supplied by the member states for the Fire LoB following the application of this formulaic filter had a distribution as follows:



Band	Number of observations
-3450%	0
-3450% to 0%	0
0%	0
0% to 30%	134
30% to 40%	121
40% to 50%	152
50% to 60%	209
60% to 70%	148
70% to 80%	98
80% to 100%	95
100% to 200%	72

200% to 500%	21
500% to 10000%	3
Total	1053

As can be seen from the graph and table above, there are a significant number of very large loss ratios.

Manual Filter

We then look through the data in detail, identifying problems in the observations, including:

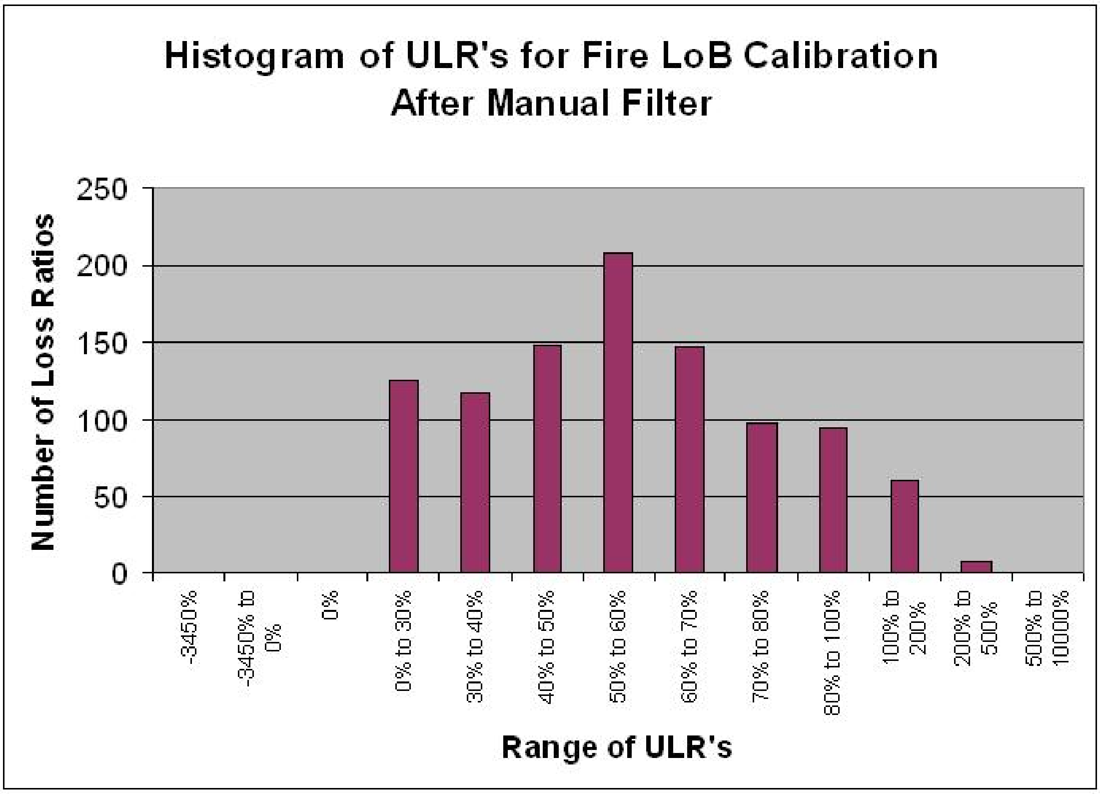
- Distortions due to mergers and acquisitions
- Typographic mistakes
- Apparent inconsistencies between different years and between premiums and claims for the same company

We then remove these identified distorting observations.

The FSA were informed of the impact of catastrophe losses on their claims for some companies and the data was adjusted to remove the impact.

The overall effect of these adjustments is to significantly reduce the volatility of data used for the calibration and as a result reduce the fitted parameters. This has been done to reduce the impact of the potential distortions mentioned above.

The ultimate loss ratios (posted after one year) coming from the raw data as supplied by the member states for the Fire LoB following the application of the formulaic filter and manual filter had a distribution as follows:



Band	Number of observations
-3450%	0
-3450% to 0%	0
0%	0
0% to 30%	126
30% to 40%	117
40% to 50%	148
50% to 60%	208
60% to 70%	147
70% to 80%	97
80% to 100%	94
100% to 200%	60
200% to 500%	8
500% to 10000%	0
Total	1005

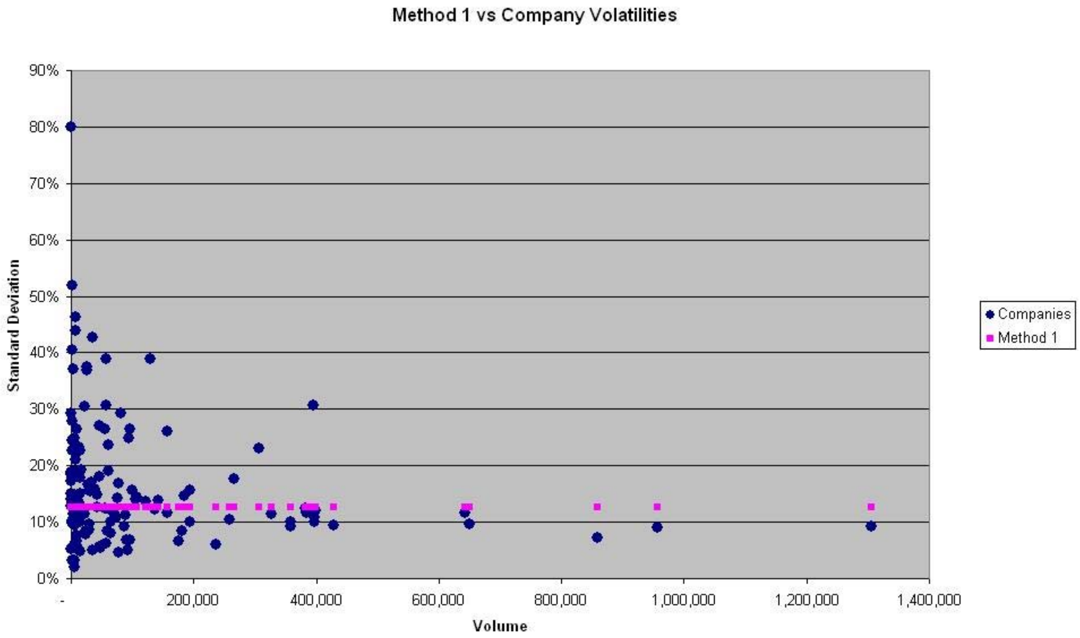
The graph and table above summarise the data used to calibrate the Gross Premium Risk for the Fire LoB.

This dataset comprised 138 companies each with Earned Premium amounts and Ultimate Losses posted after one year for various accident years.

Analysis

Method 1:

The graph below shows the standard deviations of ultimate loss ratio (as posted after one year) by company.



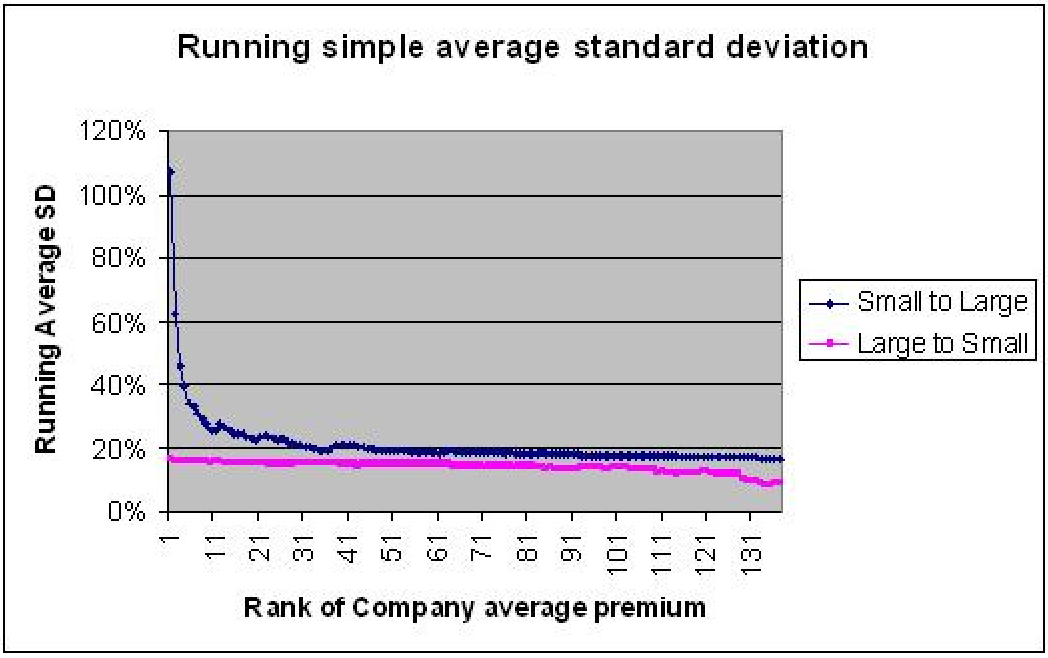
Graph Explanation

- Each blue dot represents a single company (undiscounted) standard deviation as fitted via Method 1.
- The company specific fitted (undiscounted) standard deviation is based upon the assumption that the standard deviation is proportional to the square root of the volume measure (premium).
- The pink dots represent the fitted (undiscounted) premium risk factor for this LoB from this Method (ie 17%).

Commentary

The graph gives a clear indication of diversification credit by volume.

To illustrate the diversification effect further, the following graph gives the running average company (undiscounted) standard deviation, as fitted via Method 1.



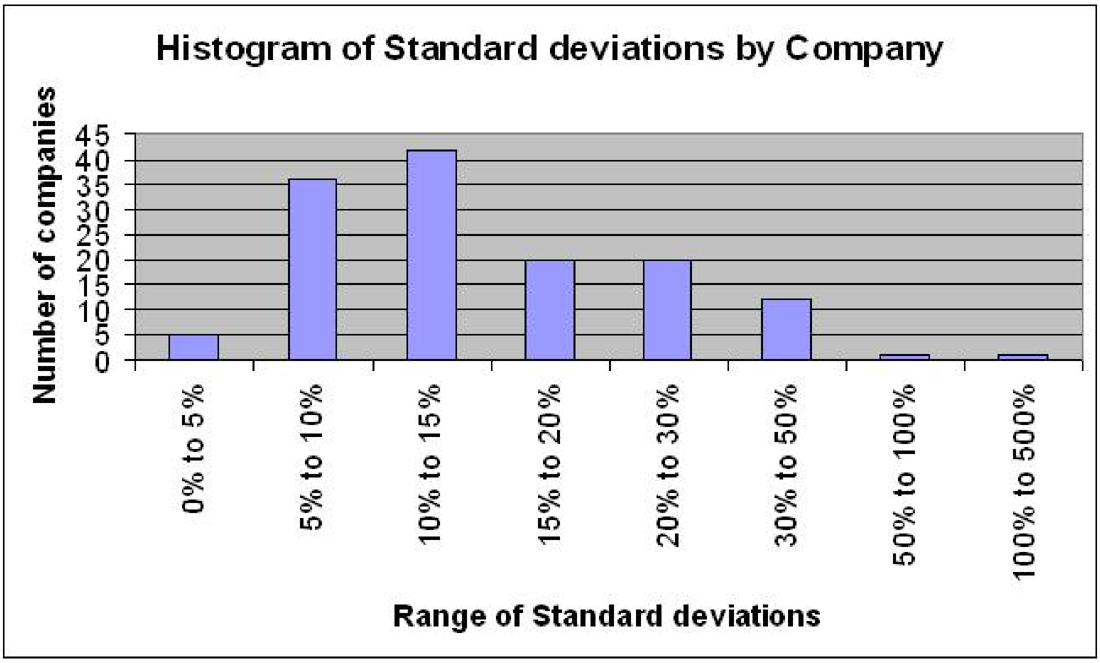
Graph Explanation

- Small to Large: Each point on the blue line represents the average company (undiscounted) standard deviation, as fitted via Method 1, for all companies with smaller average premium than the company with rank as per the x axis (larger rank means larger average premium).
- Large to Small: Each point on the pink line represents the average company (undiscounted) standard deviation, as fitted via Method 1, for all companies with larger average premium than the company with rank as per the x axis (larger rank means larger average premium).

Commentary

Both graphs show that the running average decreases as volume increases, further showing the diversification effect by volume.

Yet another representation of the same information contained above is included in the histogram and table below:



Range of Standard Deviations	Number of Companies
0% to 5%	5
5% to 10%	36
10% to 15%	42
15% to 20%	20
20% to 30%	20
30% to 50%	12
50% to 100%	1
100% to 500%	1
Total	137

This shows that there are a significant number of companies whose standard deviations (approximately 51%) are above the fitted result of 12.7% (which is the undiscounted value corresponding to 12.1% discounted).

This is caused by the fitting algorithm placing more weight to the standard deviations from companies with larger volumes. These companies are the companies with, in general, lower volatilities – as shown in the graphs above.

In general this Method is likely to underestimate the overall parameters due to the fact that implicitly the Method calculates company specific means, which it then uses to derive the estimations of company specific standard deviations. Given that some of the companies concerned will be writing similar business in similar markets, this approach will over-fit to the data and thus underestimate the underlying volatility.

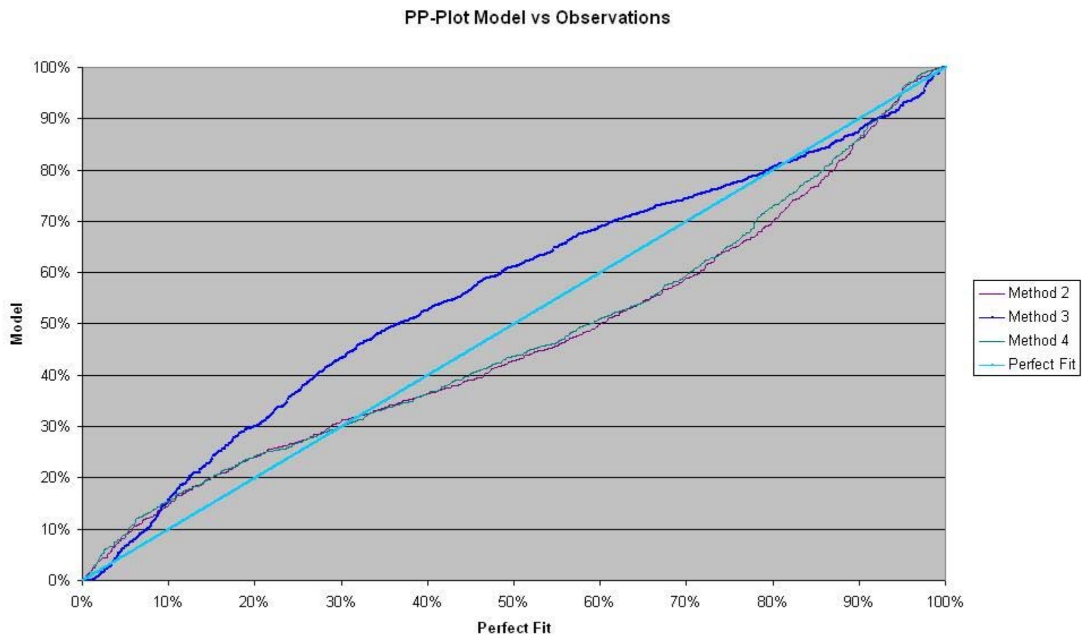
Methods 2, 3 and 4:

The table below gives the fitted (discounted) results for these methods.

Method	Small	Medium	Large	Fitted
Method 2	61%	26%	16%	11%
Method 3	96%	41%	25%	18%
Method 4	20%	20%	20%	20%

Clearly the results for Methods 2 and 3 look relatively large in comparison to those for Method 2. We shall explore the reasons for this below.

The graph below shows the p-p plots of the observed posted claims after one year.

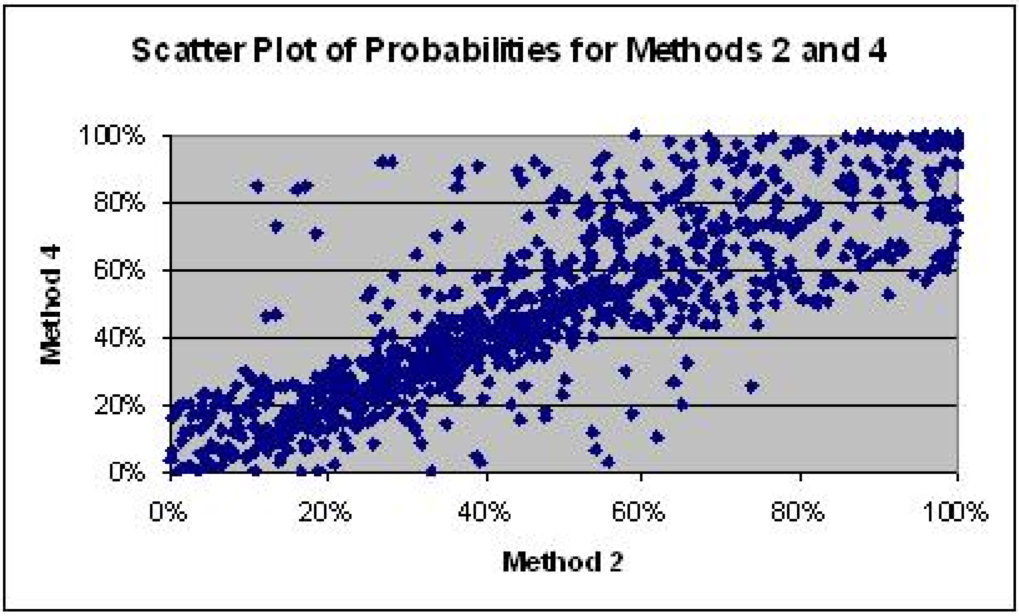


Graph Explanation

- Each line represents the goodness of fit plot for each of the respective methods.
- Each point on each curve is derived from an observation of ultimate loss posted after one year. This is converted to a probability of observing a loss ratio of at least that size given the specific model assumptions for that method. This is then plotted against the expected probability of observing a loss ratio of at least that size, assuming that each observation is independent of the other observations.
- If our model was perfectly accurate, our parameterisation appropriate and we had an infinite number of observations the p-p plot would lie on the "Perfect Fit" light blue curve above.
- The theory says that the better the fit, the closer the p-p plot will lie to the "Perfect Fit" line.
- Methods 2 and 4 have very similar plots.
- It is difficult to distinguish the fit between the 3 different Methods. Although Method 3 fits the observations differently from the other 2 Methods, it seems to deviate equally from the Perfect Fit.

Commentary

Although the p-p plots are very similar between methods 2 and 4, this does not mean that they have the same interpretation of the likelihood of specific observations. The graph below illustrates this point:



Graph Explanation

- Each point represents a single observation of a posted loss ratio after one year for a specific company, for a specific accident year.
- The x-axis value for that point represents the probability of observing a loss ratio of at least that size given the specific model assumptions for method 2.
- The y-axis value for that point represents the probability of observing a loss ratio of at least that size given the specific model assumptions for method 4.

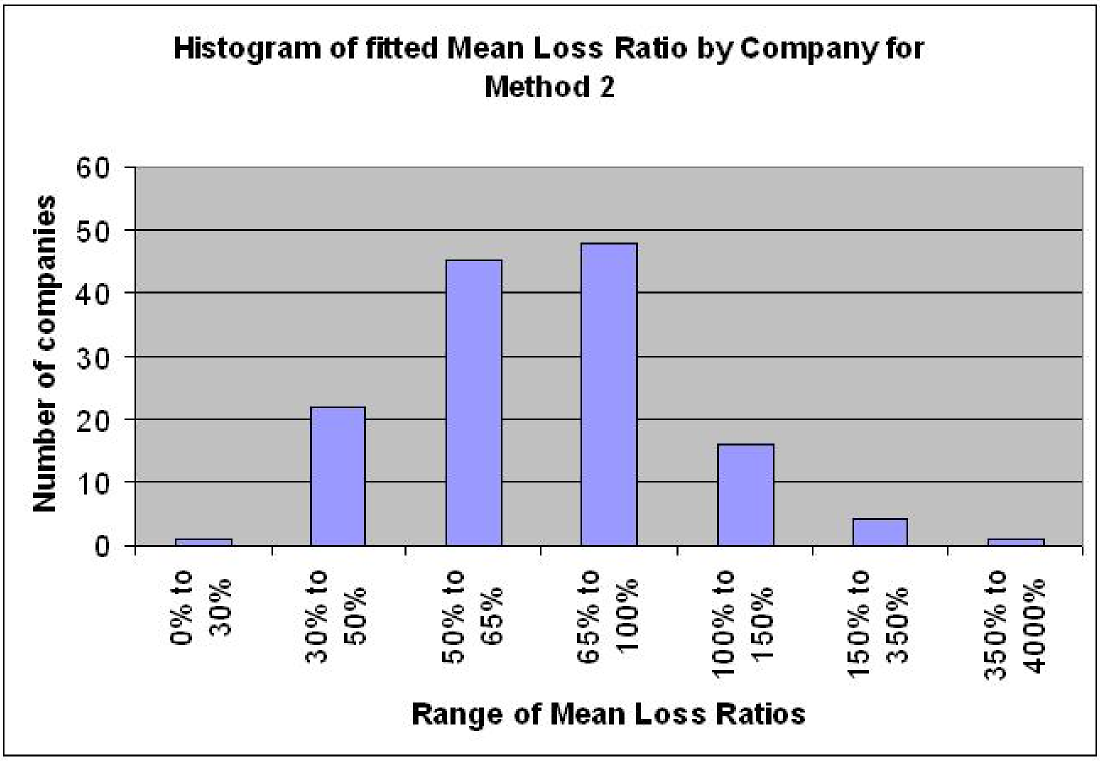
Commentary

If the models were interpreting the likelihood of each observation similarly then the graph would be a straight line along y=x. As a result we can conclude that it is pretty coincidental that the p-p plots appear to be so similar.

The difference between the model assumptions behind Methods 2 and 4 and those behind Method 3 is that Methods 2 and 4 assume that different companies have different mean loss ratios. Method 3 assumes that all companies have the same loss ratio across the market.

Method 2

The graph and table below describe the fitted mean loss ratios by company for Method 2:



Range of fitted Mean Loss Ratio for Method 2	Number of Companies
0% to 30%	1
30% to 50%	22
50% to 65%	45
65% to 100%	48
100% to 150%	16
150% to 350%	4
350% to 4000%	1
Total	137

As we can see from this split above the fitted mean loss ratios by company are very different.

In general this Method is likely to underestimate the overall parameters due to the fact that the Method calculates company specific means, which it then uses to derive the estimations of the market wide volatility parameters. Given that some of the companies concerned will be writing similar business in similar markets, this approach will over-fit to the data and thus underestimate the underlying volatility.

However, this Method only fits one variability structure across all the market, as opposed to Method 1 which fits volatility separately by company. Since, some companies will be writing different business in different markets, this can be regarded as under-fitting the volatility, as opposed to Method 1 which will be over-fitting the volatility.

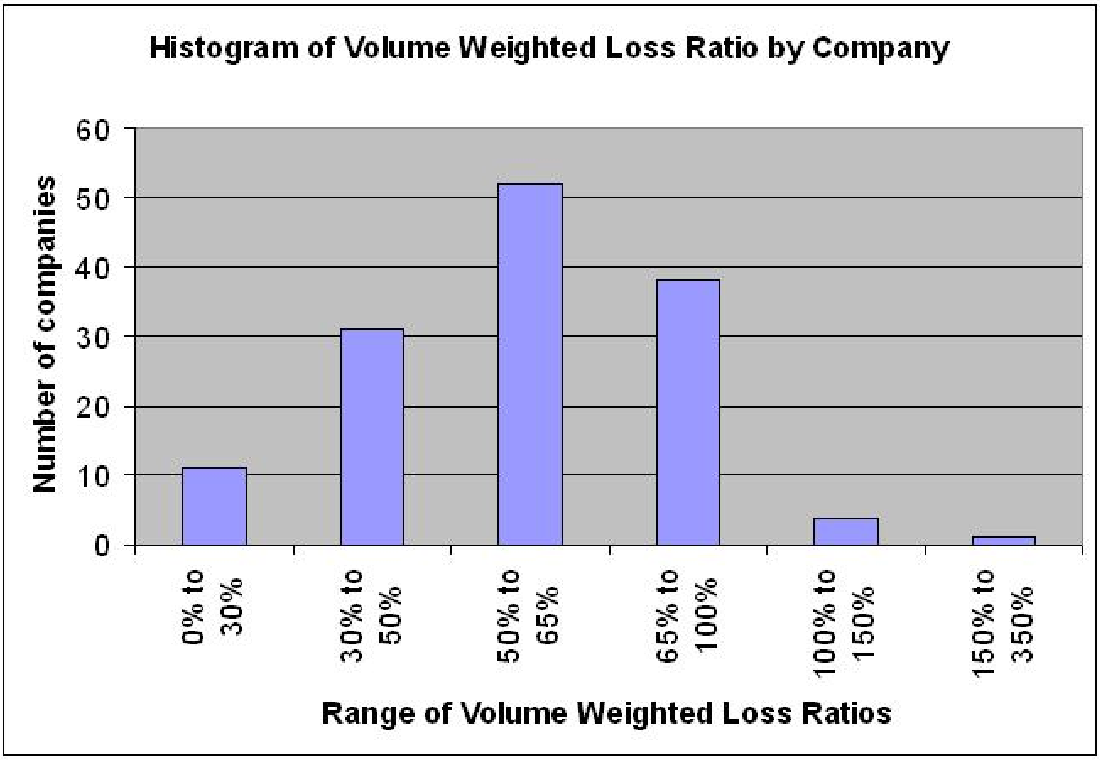
Method 3

This method fits a mean loss ratio across the whole market rather than by company. Other than this, Method 3 has the same assumptions as Method 2.

Method 3 should be preferred where there is little scope within a LoB for deviations in expected loss ratio between companies and avoids the risk of over-fitting of the parameters inherent in Method 2 (which would have the effect of underestimating the standard deviation).

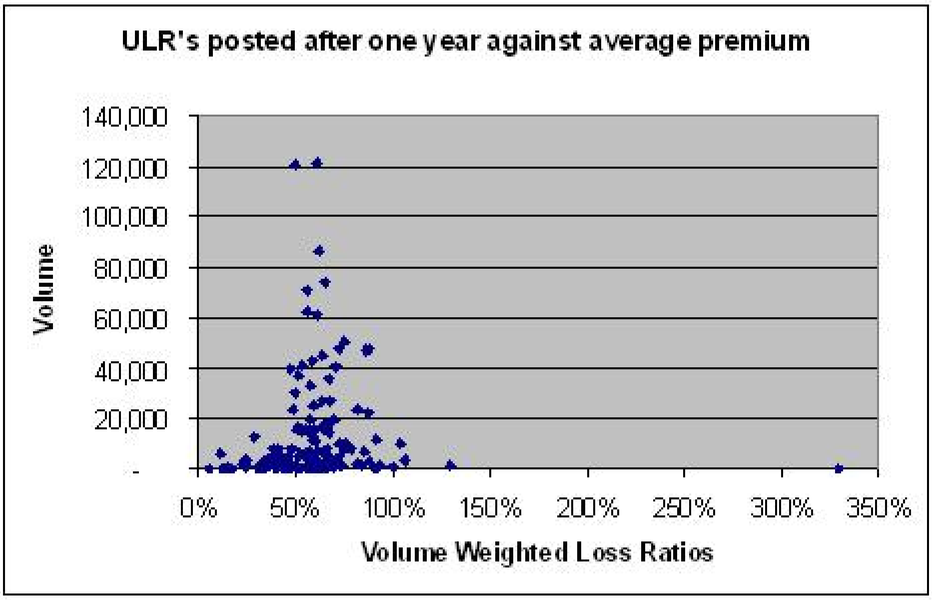
Method 3 will always give larger results than Method 2, for this reason.

For this LoB we can see very different average loss ratios by company.



Range of Volume Weighted Loss Ratio for Method 4	Number of Companies
0% to 30%	11
30% to 50%	31
50% to 65%	52
65% to 100%	38
100% to 150%	4
150% to 350%	1
Total	137

The graphs and table above show how very different the loss ratios are by company. However, the larger and the smaller loss ratios as fitted are for relatively small companies as shown in the graph below:



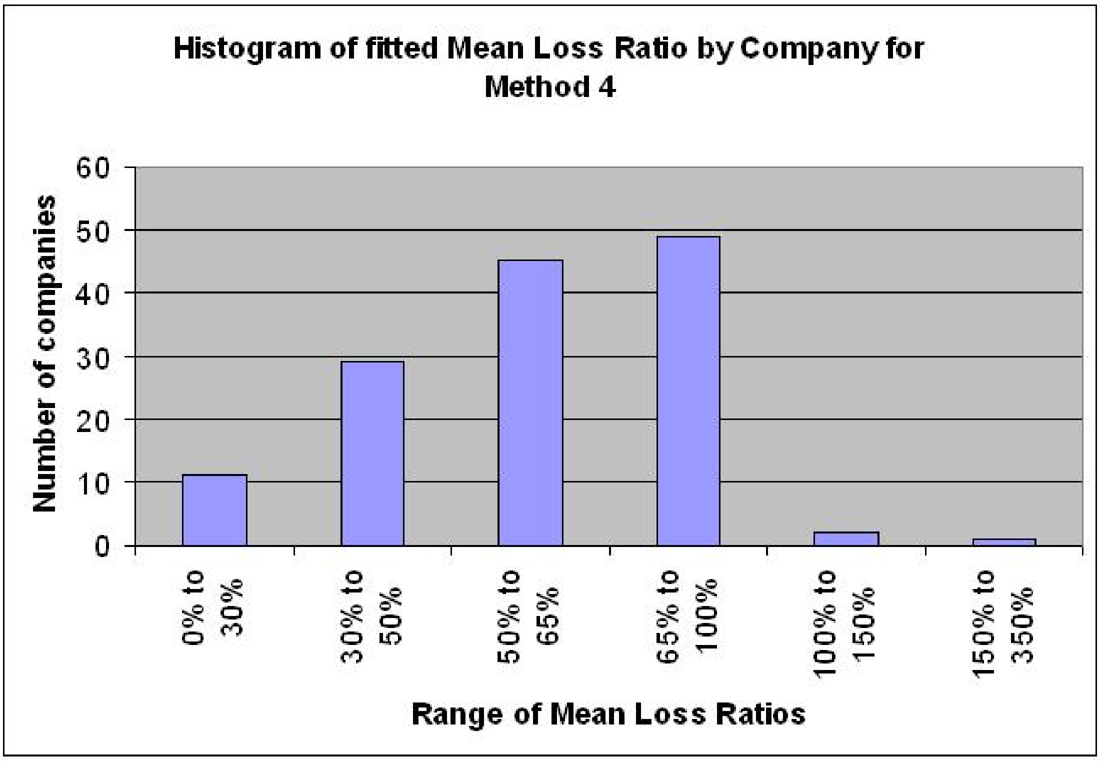
The graph above shows that smaller companies have relatively volatile Volume Weighted Loss Ratio's compared to the larger firms. This is consistent with what we would expect to see when there is a diversification credit for volume, since the larger volatility for lower volumes will result in larger parameter error for the loss ratio estimations.

Our conclusion from this is that although there is a wide spread of fitted loss ratios, there is insufficient evidence to rule out the possibility that the mean loss is not very company or market dependent. Indeed the graph above seems to show that the loss ratios are centred about a similar level for all firms. Method 3 fits the mean market loss ratio as about 60%.

In general this Method is likely to overestimate the overall parameters due to the fact that the Method does not calculate company specific means, but rather calculates a market wide mean which it then uses to derive the estimations of the market wide volatility parameters. Given that some of the companies concerned will be writing very different books of business in different markets, this approach will under-fit to the data and thus overestimate the underlying volatility.

Method 4

The graph and table below describe the fitted mean loss ratios by company for Method 4:



Range of fitted Mean Loss Ratio for Method 4	Number of Companies
0% to 30%	11
30% to 50%	29
50% to 65%	45
65% to 100%	49
100% to 150%	2
150% to 350%	1
Total	137

As we can see from this split above the fitted mean loss ratios by company are different.

This method assumes that there is a constant standard deviation of loss ratio for each firm and that this does not change with the size of the firm. There is some evidence (see the graphs above for Method 1) that the variability does change with size of Volume measure as we would expect. However, the counter position is that the p-p plot for Method 4 is very similar to that for Method 2.

In general this Method is likely to underestimate the overall parameters due to the fact that the Method calculates company specific means, which it then uses to derive the estimations of the market wide volatility parameters. Given that some of the companies concerned will be writing similar business in similar markets, this approach will over-fit to the data and thus underestimate the underlying volatility.

Conclusion

The table below gives us the fitted (discounted) parameters.

Method	Fitted
Method 1	12%
Method 2	11%
Method 3	18%
Method 4	20%

Our analyses have not been able to undermine any of the Models. We would expect Model 3 to under-fit and Model 1 to over fit, as explained above. Thus including both estimates in some way can be regarded as offsetting these issues.

As a result, we propose that an average of Methods 1,2, 3 and 4 giving a factor of 15%.

Example 4. Gross Reserve Risk for the TPL LoB calibration

Data Cleaning

Raw Data

For the gross reserve risk analysis we have two set of data that we use for the analysis. Specifically:

- triangles of paid data by company and
- observations of movements over one year of blocks of reserves (for the same accident years) by company by calendar year.

Below is the commentary as to the data in relation to both of these parts of the data.

Paid Triangles

For this LoB we were provided with paid triangles for 261 companies. This data was only used for the Methods involving the Merz & Wuthrich formula. There are significant limitations to applicability of the formula and as a result a significant reduction in the number of companies was required so as to only include those where the data was appropriate.

Paid Triangles - Formulaic Filter

The limitations of the Merz & Wuthrich formula are such as to make it difficult to construct a useful formula for filtering the triangles. As a result we had to do the cleaning using the manual filter.

Paid Triangles - Manual Filter

We then look through the triangles in detail, identifying problems including:

- missing data from the triangles
- triangles where there were insufficient accident years for the earliest development year to be fully run-off
- potential typographical errors such as negative values
- Triangles whose historic development is such as to invalidate the chain ladder assumptions underlying the Merz-Wuthrich formula.

The triangles were then adjusted or removed from the analysis with two aims in mind

- To include as much data as possible so as to give as much credibility as possible to the resultant analyses.
- Ensure that the data used for the calibration is fit for purpose.

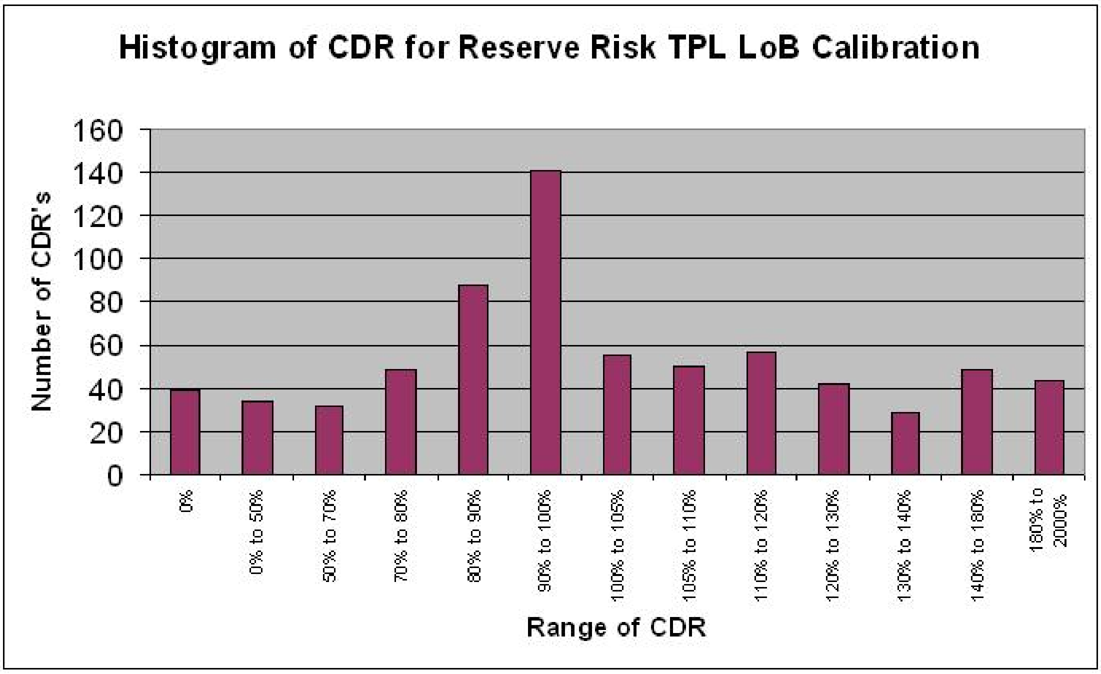
If there was missing or obviously erroneous data within the triangle then if it was possible to remove some of the older accident years (whilst leaving the triangle sufficiently large such that the oldest remaining accident year was nearly fully developed) then these were removed.

For this LoB we assumed that for the vast majority of the companies we needed at least 10 accident years of data for the triangle to reasonably be expected to be sufficiently run-off for the formula to derive sensible estimates, however, there were some exceptions where the held reserve was close to zero for the earliest accident year and the earliest years of the triangle had

not moved, over recent calendar years where we relaxed this restriction on a case by case basis.
The process of cleaning and adjusting the data resulted in reducing the original 261 triangles down to 159 triangles.

Observations of Reserve Movements over one year

The claims development ratios (CDR – being the reserve posted after one year plus the claims paid within the year as a proportion of the intial reserve) coming from the raw data as supplied by the member states for the TPL LoB had a distribution as follows:

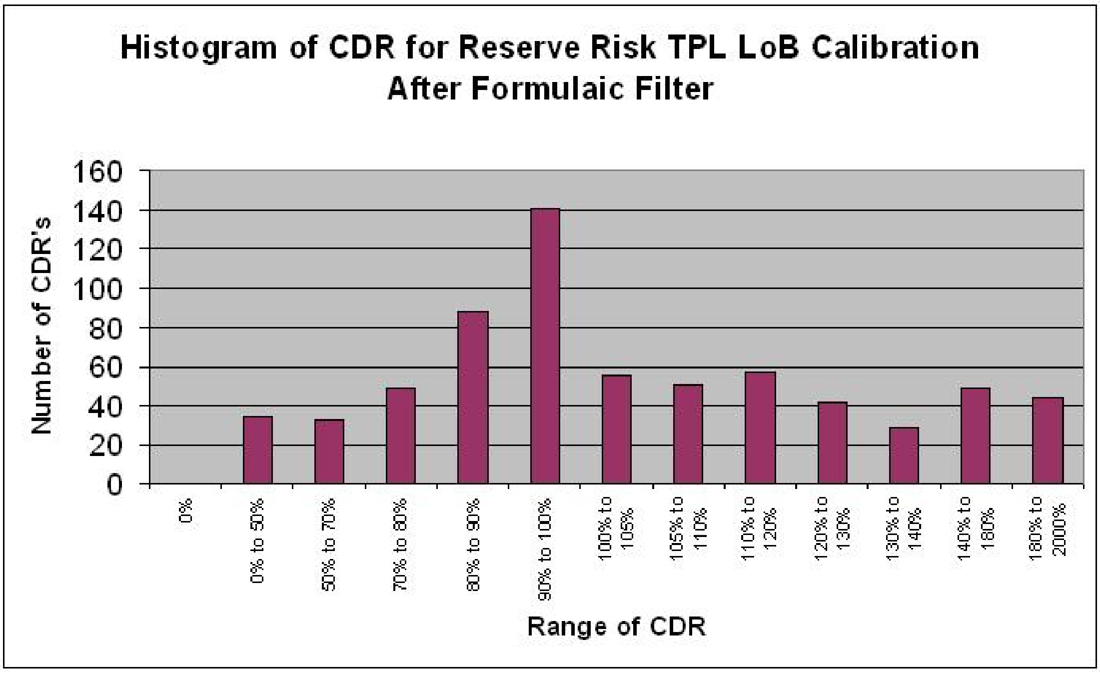


Band	Number of observations
0%	39
0% to 50%	34
50% to 70%	32
70% to 80%	49
80% to 90%	88
90% to 100%	141
100% to 105%	55
105% to 110%	50
110% to 120%	57
120% to 130%	42
130% to 140%	29
140% to 180%	49
180% to 2000%	44
Total	709

Observations of Reserve Movements over one year - Formulaic Filter

We then filter this data to remove negative CDR's and zero CDR's.

The CDR (posted after one year) coming from the raw data as supplied by the member states for the TPL LoB following the application of this formulaic filter had a distribution as follows:



Band	Number of observations
0%	0
0% to 50%	34
50% to 70%	32
70% to 80%	49
80% to 90%	88
90% to 100%	141
100% to 105%	55
105% to 110%	50
110% to 120%	57
120% to 130%	42
130% to 140%	29
140% to 180%	49
180% to 2000%	44
Total	670

This dataset after the application of the formulaic filter comprised 109 companies each with Opening Reserve amounts and Closing Reserve + incremental Paid amounts after one year for various accident years.

Observations of Reserve Movements over one year - Manual Filter

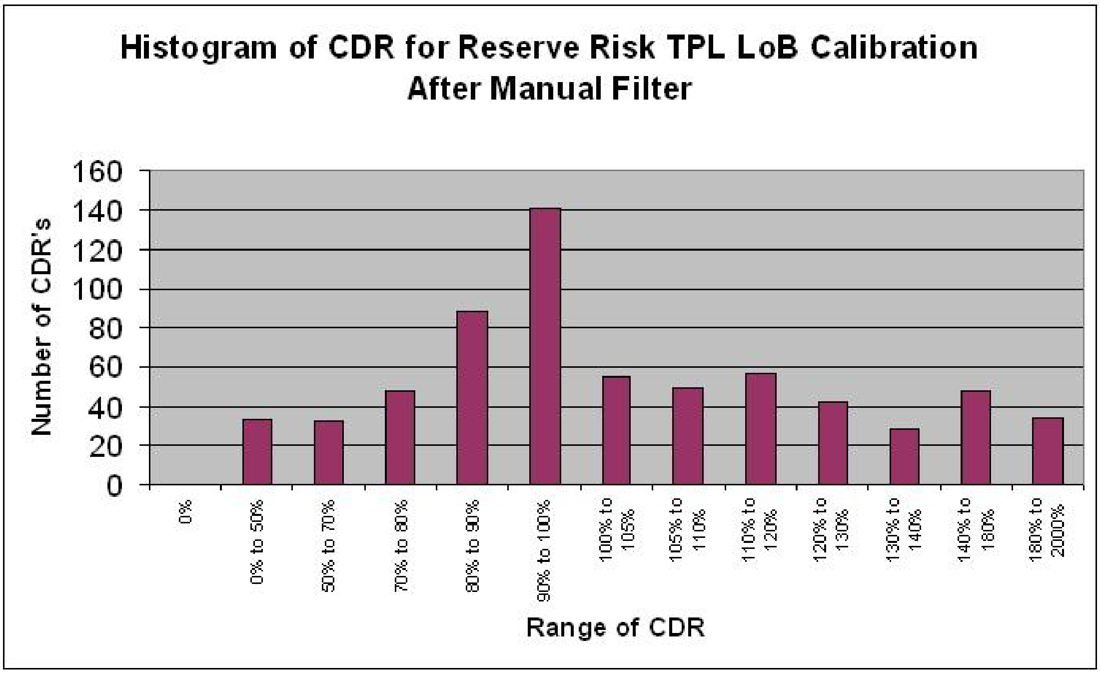
We then look through the data in detail, identifying problems in the observations, including:

- Distortions due to mergers and acquisitions
- Typographic mistakes

- Apparent inconsistencies between different years and between opening reserve and closing reserve for the same company

We then remove these identified distorting observations.

The CDR's (posted after one year) coming from the raw data as supplied by the member states for the TPL LoB following the application of the formulaic filter and manual filter had a distribution as follows:



Band	Number of observations
0%	0
0% to 50%	33
50% to 70%	32
70% to 80%	48
80% to 90%	88
90% to 100%	141
100% to 105%	55
105% to 110%	49
110% to 120%	57
120% to 130%	42
130% to 140%	29
140% to 180%	48
180% to 2000%	34
Total	656

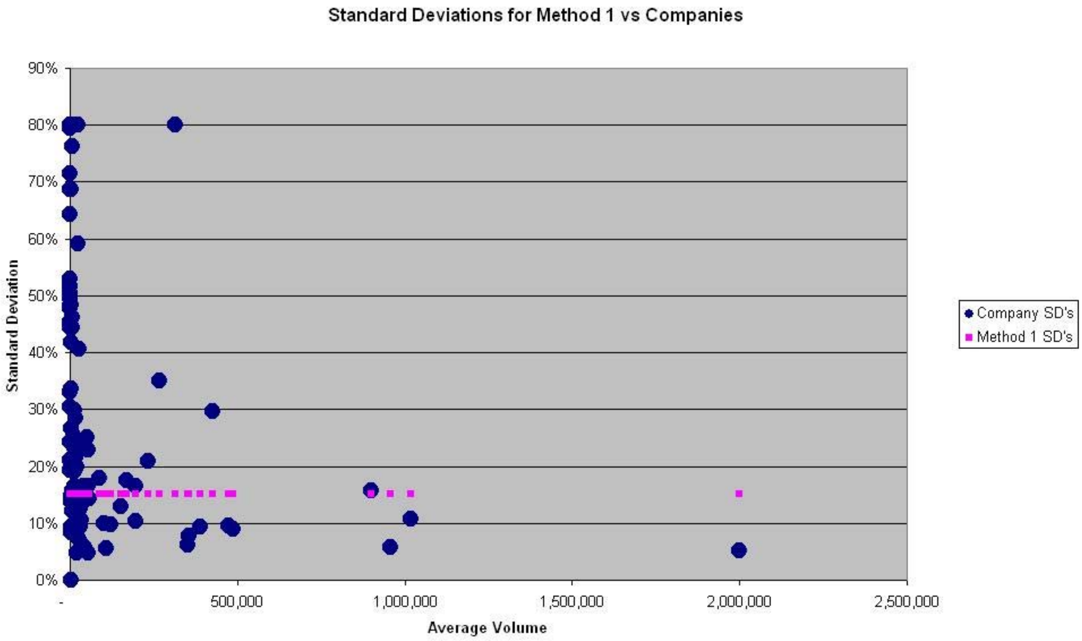
The graph and table above summarise the data used to calibrate the Gross Reserve Risk for the TPL LoB.

This dataset comprised 103 companies each with Opening Reserve amounts and Closing Reserve + incremental Paid amounts after one year for various accident years.

Run Analysis

Method 1:

The graph below shows the standard deviations of CDR (as posted after one year) by company.



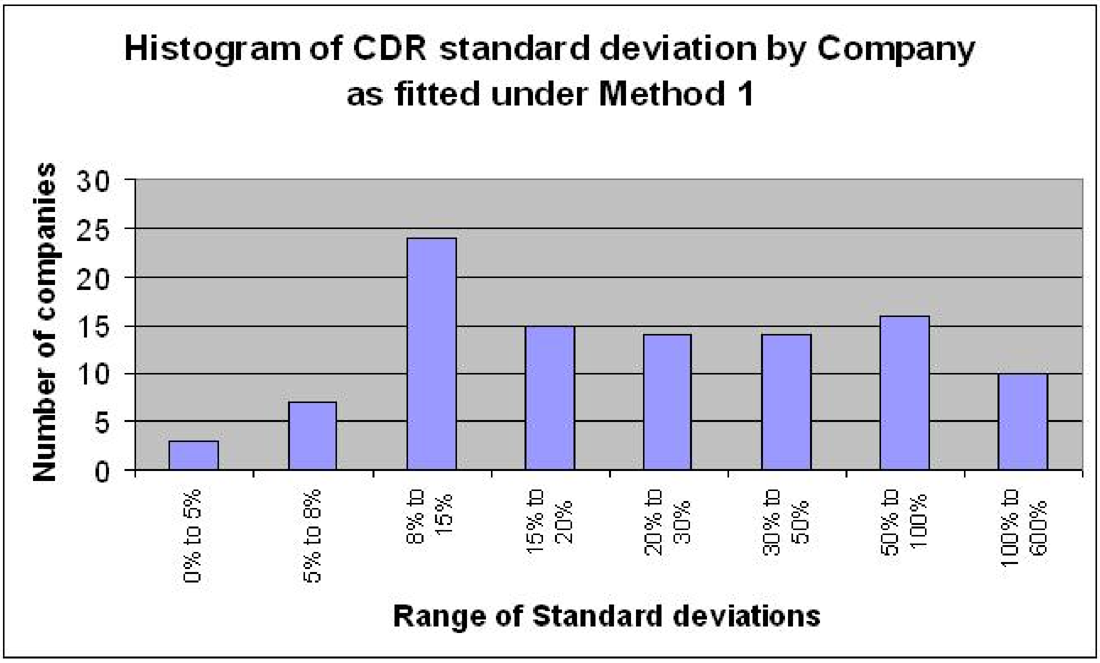
Graph Explanation

- Each blue dot represents a single company standard deviation as fitted via Method 1.
- The company specific fitted standard deviation is based upon the assumption that the standard deviation is proportional to the square root of the volume measure (opening reserve) and assuming that the opening reserve is unbiased.
- The pink dots represent the fitted reserve risk factor for this LoB from this Method (ie 15%).

Commentary

This graphs demonstrates a relationship between the size of the reserve and the size of the variability of the reserve. Specifically, it provides evidence that the larger the volume of reserves, the smaller the CDR volatility.

Another representation of the same information contained above is included in the histogram and table below:



Range of Standard Deviations	Number of Companies
0% to 5%	3
5% to 8%	7
8% to 15%	24
15% to 20%	15
20% to 30%	14
30% to 50%	14
50% to 100%	16
100% to 600%	10
Total	103

This shows that there are a significant number of companies whose standard deviations (approximately 61%) are above the fitted result of 15%.

This is caused by the fitting algorithm placing more weight to the standard deviations from companies with larger volumes. These companies are the companies with, in general, lower volatilities – as shown in the graph above.

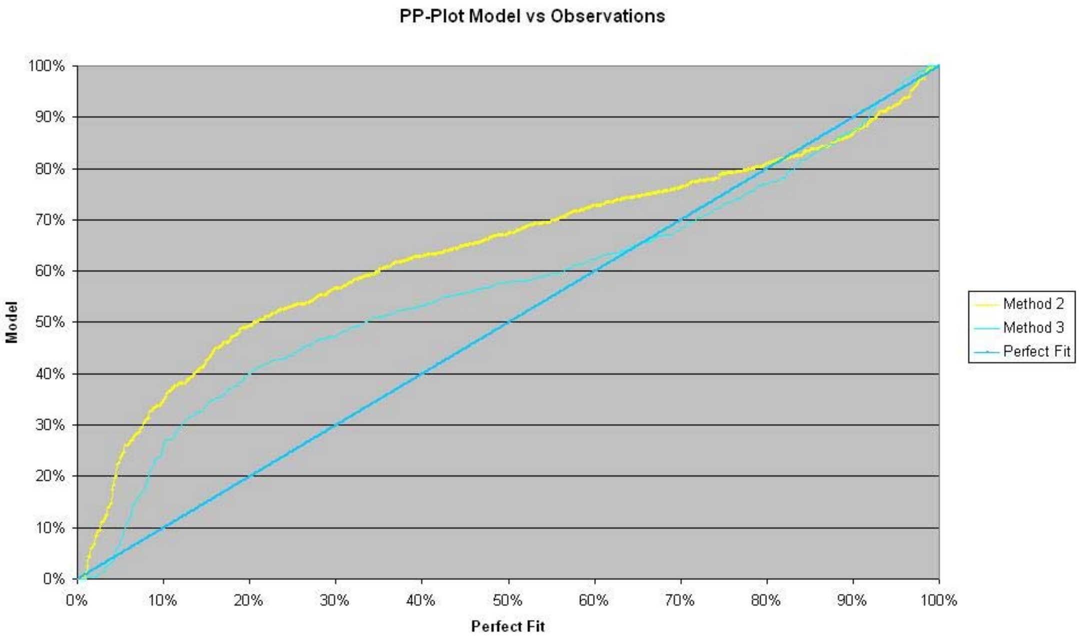
Methods 2 and 3:

The table below gives the fitted results for these methods.

Method	Small	Medium	Large	Fitted
Method 2	221%	74%	38%	17%
Method 3	43%	43%	43%	43%

The fitted results are very different between the two Methods. It should be noted that the approach used to produce the fitted result for Method 2 effectively places more weight on results for larger companies which have lower fitted standard deviations. The reason why the fitted factor for method 2 is beyond the factor for "Large" companies is due to there being many companies much larger than the definition of a large company (75th percentile of volume by company) whose weight and low volatilities are bringing the factor down.

The graph below shows the p-p plots of the observed payment within the one year plus the posted reserve after one year.



Graph Explanation

- Each line represents the goodness of fit plot for each of the respective methods.
- Each point on each curve is derived from an observation of payments within the year plus the closing reserve after one year. This is converted to a probability of observing a CDR of at least that size given the specific model assumptions for that method. This is then plotted against the expected probability of observing a CDR of at least that size, assuming that each observation is independent of the other observations.
- If our model was perfectly accurate, our parameterisation appropriate and we had an infinite number of observations the p-p plot would lie on the “Perfect Fit” light blue curve above.
- The theory says that the better the fit, the closer the p-p plot will lie to the “Perfect Fit” line.
- Method 3 is giving a much better fit to the data.

Commentary

Methods 2 and 3 both assume that there is no inherent bias within the posted reserves for this LoB (as is implicit within the standard formula assumptions). The mean observed CDR is:

- 111% using a simple average
- 100% using a volume weighted average

The p-p plots for both graphs lie mostly above the Perfect Fit line (y=x). This tells us that both of the fitted models would predict there to be more observations below any respective value than are actually seen in the data. However this issue is less prevalent in the tail of the distribution. As a result when the standard formula uses the lognormal assumptions to quantify the marginal capital requirement, the parameters derived from these models should give capital requirements relatively consistent with the observed CDR's.

Method 2 initially fits a model that gives a different volatility by size and this is borne out by the graphs of volatility by volume shown with Method 1. However, the relatively good p-p plot for Method 3 in comparison to Method 2 shows that this volatility structure is not the only interpretation of the data.

Methods 4, 5 and 6:

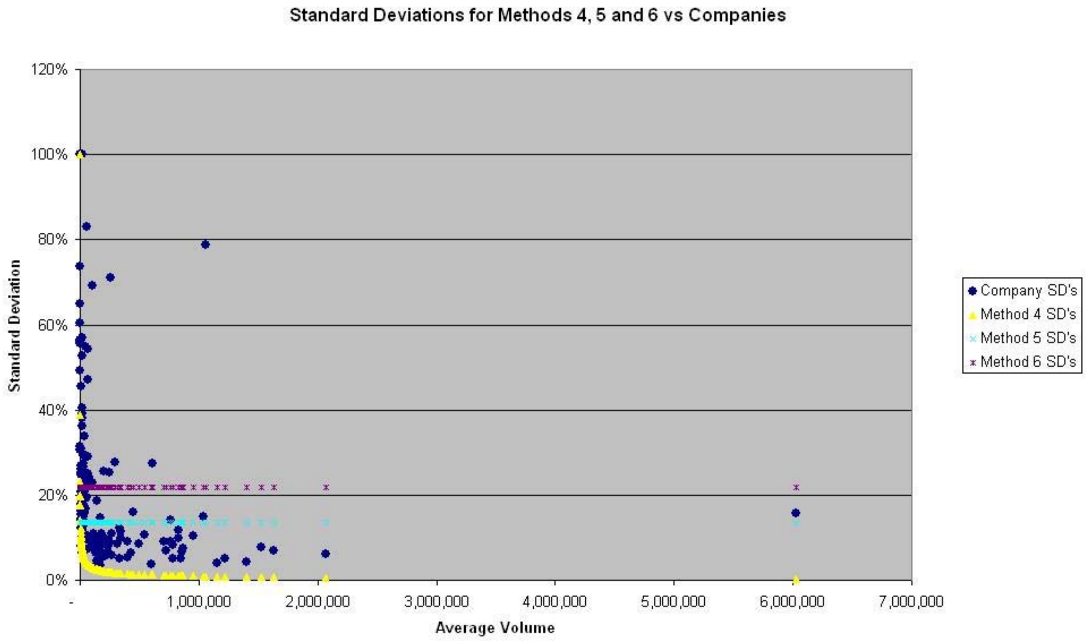
The table below gives the fitted results for these methods.

Method	Small	Medium	Large	Fitted
Method 4	25%	8%	4%	2%
Method 5	14%	14%	14%	14%
Method 6	22%	22%	22%	22%

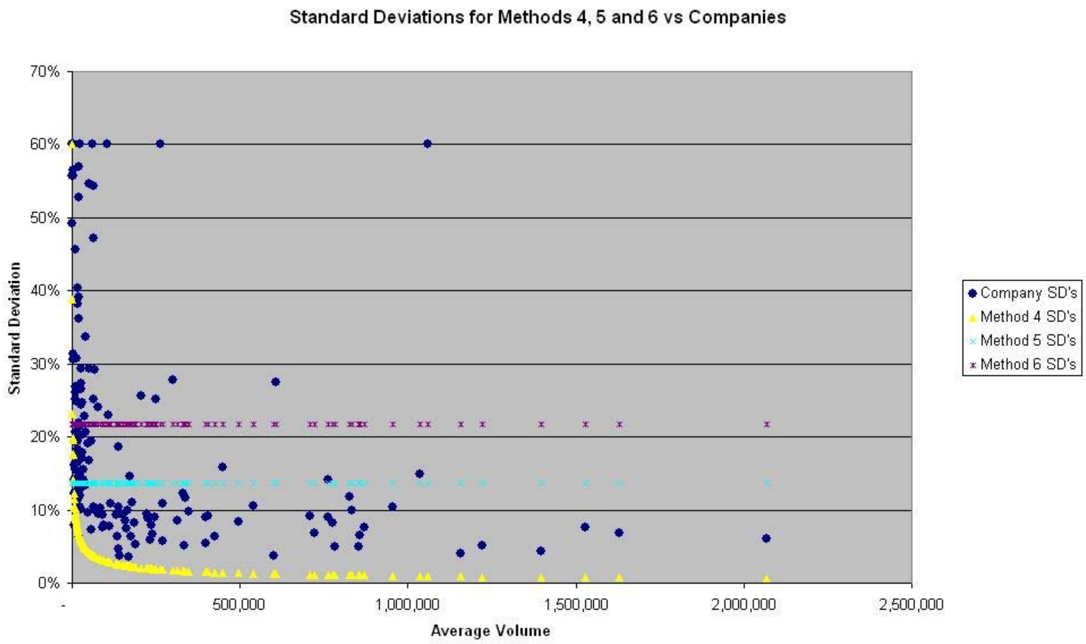
These results are different from the results coming from Methods 1, 2 and 3. There are various reasons for the differences:

- Methods 4,5 and 5 are fitted to different datasets than Methods 1, 2 and 3. For some LoB's there is some overlapping of the contributing companies, however this is not necessarily the case.
    - Methods 1, 2 and 3: The data used is observations of historic claims development over one year.
    - Methods 4, 5 and 6: The data used is paid triangles.
- The model assumptions behind Methods 4, 5 and 6 are significantly different to those behind Methods 1, 2 and 3.
    - Methods 1, 2 and 3: These Methods directly fit models to observed posted claims development.
    - Methods 4, 5 and 6: These Methods attempt to derive an estimate of the one year reserve volatility by company without reference to the company's posted reserves.

The graph below shows the fitted standard deviations by company and compares these to the results from Methods 4, 5 and 6.



In order to examine this graph more clearly the following graph has focussed on the standard deviations below 60% by capping the fitted standard deviations (for illustrative purposes only) and removed the large company with about 6bn of volume weighted chain ladder reserves.



Graph Explanation

- Each dark blue dot represents a single company standard deviation of the one year claims development (via the Merz-Wuthrich formula), against the volume weighted chain ladder opening reserve amount for that company.

- The yellow points represent, for each company used in the calibration, the predicted standard deviation of the one year claims development (for Method 4) against the volume weighted chain ladder opening reserve amount for that company.
- The light blue points represent, for each company used in the calibration, the predicted standard deviation of the one year claims development (for Method 5) against the volume weighted chain ladder opening reserve amount for that company.
- The red points represent, for each company used in the calibration, the predicted standard deviation of the one year claims development (for Method 6) against the volume weighted chain ladder opening reserve amount for that company

Commentary

Method 3 tends to underestimate the standard deviation at all but the very smallest volume of reserves as seen in the graph above that most of the dark blue points (about 98%) are above the yellow curve.

Method 4 has about 52% of the dark blue points above the curve and comes out with an estimate slightly larger than would be appropriate for the larger companies (the far right of the graph).

Method 5 has about 30% of the dark blue points above the curve and comes out with an estimate much larger than would be appropriate for the larger companies (the far right of the graph).

These Methods all use the Merz-Wuthrich formula applied to paid data. There are as a consequence of the limitations in the model various things we should consider when looking at these results:

- The Merz-Wuthrich formula assumes that the chain ladder is an appropriate model for the claims. Clearly, these assumptions are going to be most reasonable for shorter tailed classes and for later accident years. Where the chain ladder is not such an appropriate technique, the Merz approach is likely to over estimate the company specific standards deviation. This is especially relevant to this relatively long tailed class where an exposure based method such as the B-F is more likely to be more appropriate for more recent accident years than the chain ladder.
- This formula has been applied to the paid triangles. This will implicitly over estimate the resultant company specific standards deviation as a result of interpreting uncertainty in payment timing as uncertainty in future ultimate payments. This issue would be less if the triangles used were incurred claim amounts rather than paid amounts.
- The implementation of the Merz formula used does not allow for any future development of the triangle beyond the number of development years of the triangle. Ie the implemented approach does not allow for any tail factors or run-off of the tail. This will have the effect of understating the estimate the company specific standards deviations. This issue is especially relevant since we have performed the analyses on paid data for which the triangle has a longer tail than on incurred data which has a shorter tail. This is especially relevant for this relatively long tailed class.

It is not clear to what extent these factors offset each other and whether the resultant estimates are prudent or optimistic.

Conclusion

The table below gives us the fitted parameters:

Method	Fitted
Method 1	15%
Method 2	17%
Method 3	43%
Method 4	2%
Method 5	14%
Method 6	22%

Our analysis shows that:

- Method 3 has a relatively bad p-p plot.
- Method 4 understates the volatility.

As a result, we propose that an average of Methods 1, 3, 5 and 6 giving a factor of 23% would best represent an interpretation of this line of business.

7.4 Calibration by member state Portugal for reserve risk

1. Introduction

The present paper develops a proposal for a methodology to calibrate the non-life reserve risk in the SCR standard formula. It is built on the application of the well-know stochastic model based on the over-dispersed Poisson distribution developed by Renshaw and Verrall (1998)[9].

Such stochastic model delivers estimates for the mean values of the claims provision which are identical to those given by the widely used deterministic Chain Ladder method, but it also provides estimates of the variability of such amounts. The variability measure given by the model is the mean squared error of prediction (MSE) which can be decomposed in 2 components: process error (PE) and the estimation error (EE).

2. Design of the methodology

The reserve risk materialises when the amounts of the best estimate of claims provision prove to be insufficient. It could due to:

a) **Process Variance:** Volatility of the realisations relative to the expected values, *i.e.* the amounts paid will differ from their expected value due to statistically normal randomness of the process;
b) **Estimation Variance:** Uncertainty of the estimates, *i.e.* the model and/or parameters may prove to be wrong;

In mathematical terms, let P be the random variable corresponding to the total undiscounted amount outstanding for incurred but not settled claims (RBNS+IBNR), and $\hat{P}$ the estimate for its expected value. Thus, the 'total' error (mean squared error) is given by:

$$MSE(P) = E\left((P-\hat{P})^2\right) \qquad \textbf{(2.1)}$$

It can be shown that, in the framework of the model, this total error can be decomposed as follows:

$$MSE(P) = \underbrace{E\left((P-E(P))^2\right)}_{Process\ Variance} + \underbrace{\left((\hat{P}-E(P))^2\right)}_{Estimation\ Variance} \qquad \textbf{(2.2)}$$

(For simplicity, in the following the effect of discounting is ignored)

The best estimate of claims provision corresponds to *E(P)*, whose estimate is $\hat{P}$ given by the model and identical to the Chain Ladder result.

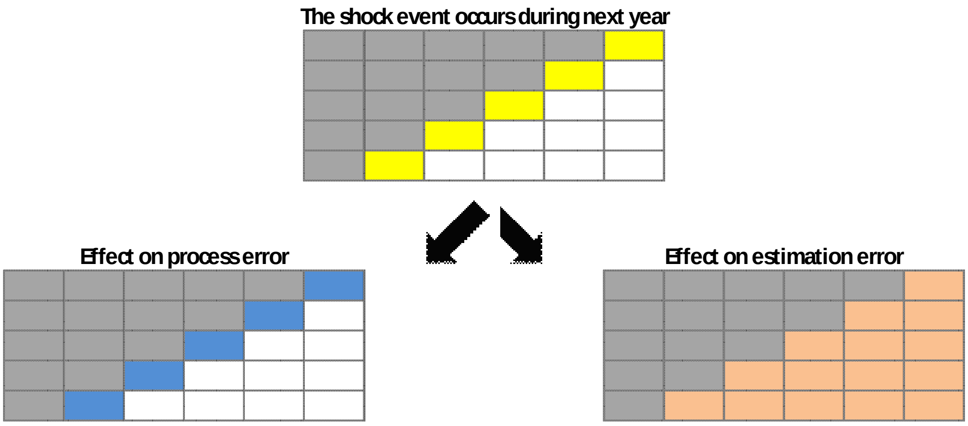
In order to calculate the reserve risk for SCR purposes, we need to derive a standard deviation measure for the random variable P that is consistent with the calculation of the *Value-at-Risk* at the confidence level of 99,5% and the one-year time horizon. The question lies on the interpretation of the one-year time horizon: this should be understood to reflect the period in which the 'shock' event is assumed to occur, but the effects of such event may indeed extend to the full term of the obligations. For SCR calculation purposes, the shock event is thus expected to occur during the next one-year time period and the effects would be as follows:

---

[9] Renshaw, A. E. and Verrall, R. J. (1998), *A stochastic model underlying the chain ladder technique*, British Actuarial Journal 4, pp. 903-923.

- By nature, a shock event related to the **process variance component** of the risk is assumed to only affect the cash flows at the same time that the event occurs, *i.e.* the effects are restricted to the same one-year time period. This is because this component measures the statistically normal variability of the stochastic process, and thus the occurrence of an extreme observation would not lead to the recalculation of the future estimates (as the model and parameters would be assumed to remain accurate);
- On the other hand, a shock event related to the **estimation variance component** of the risk will affect the future estimates, *i.e.* the effects will be propagated till the full term of the obligations. For instance, a shock event of this type could be the recognition that the model or parameters are wrong, thus leading to the need to recalculate not only the present but also the future estimates.

In summary, the proposal is to consider a measure of standard deviation for P that reflects both process and estimation error in the first diagonal of the run-off matrix and only estimation error in all the remaining diagonals. The figure below illustrates this.



The desired output of this calibration exercise is to calculate such measure of standard deviation and express it as a percentage of the relevant best estimate, $\hat{P}$.

Such standard deviation measure could then be plugged into the Non-life premium & reserve risk formula as the relevant parameter *σ(res,lob)*.

7.5 Analysis of QIS4 results for the calibration of health and non-life premium risk

7.9. In order to calculate the non-life premium and reserve risk module (and the non-life part of the health underwriting risk module), QIS4 participants provided time series of net loss ratios for their business per line of business. (Cf. TS.XIII.B.30 of the QIS4 Technical Specifications.) All in all, about 3400 time series of European insurance and reinsurance undertakings were collected in this way.

7.10. This annex analyses the QIS4 database for the purpose of the calibration of the premium risk factors $\sigma_{(prem,lob)}$ as defined in CEIOPS' Advice on the non-life underwriting risk module (CEIOPS-DOC-41-09).

Methodology

7.11. From each time series of net loss ratios a weighted standard deviation was derived as defined in TS.XIII.B.30 of the QIS4 Technical Specifications ($\sigma_{(U,prem,lob)}$). For each line of business, several figures were derived from the distribution of standard deviations:

- Median, percentile and minimum/maximum values for each national market as well as for the European market.
- A weighted average for each national market. The weights were the premiums that were used in the QIS4 calculation of the premium and reserve risk sub-module (cf. TS.XIII.B.23).
- A weighted average for the European market. This average was derived from the national weighted averages. The weight of each national figure was the number of undertakings that provided the time series for the national average.[10]

7.12. The analysis suffers from some shortcomings in the data and the standard deviation estimator:

- The standard deviations are derived from time series of loss ratios. Conceptually, premium risk covers the volatility of claims and expenses. Loss ratios only reflect the volatility of claims. In order to estimate the volatility of claims and expenses, either combined ratios instead of loss ratios need to be studied or the loss ratios (or the resulting standard deviation) need to be scaled up to take the extra volatility of expenses into account. As this was not possible so far, the results are likely to underestimate the real risk.
- The distribution of loss ratios is likely to be skewed. In this case, the estimator is biased and underestimates the real standard deviation.
- The time series provided in QIS4 may not reflect the risk of the undertaking. The time series may be distorted, for example because of

[10] Compared to the use of market premiums as weights for the national figure the use of QIS4 participants gives more weight to the smaller markets and thereby ensures that the estimate is not dominated by the large insurance markets.

portfolio transfers, change of reinsurance programme or catastrophic losses.

7.13. Because of these shortcomings, the results of the analysis should rather be considered as lower boundaries of the real standard deviations. Other sources of information and/or judgement needs to be applied to choose the final calibration of the risk factors.

European results

7.14. The following table shows the results for the European sample.

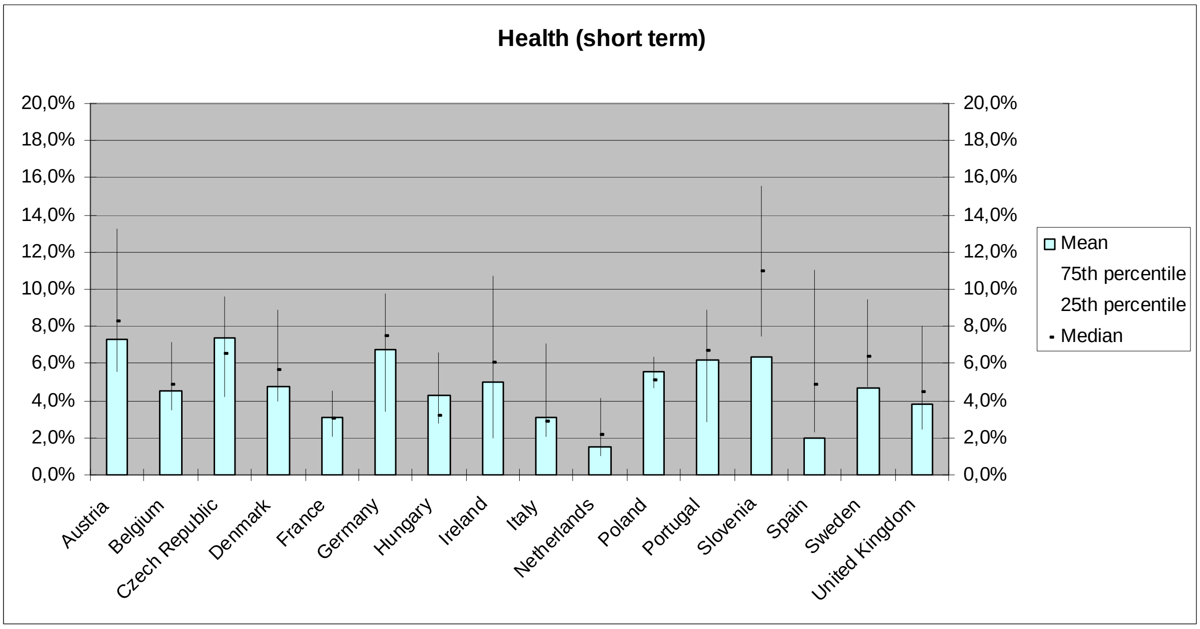
	Minimum	10th Percentile	25th Percentile	Median	75th Percentile	90th Percentile	Maximum	Mean	Standard deviation	Number of Firms with available data
Health (short-term)	0,0%	1,5%	2,4%	4,6%	8,3%	16,3%	372,4%	4,3%	28,0%	275
Health (other)	0,0%	2,3%	3,8%	6,8%	12,4%	22,0%	10240,9%	5,9%	565,1%	330
Workers' compensation	0,1%	4,1%	6,2%	10,6%	19,4%	47,3%	271,6%	9,3%	37,4%	108
Motor, third party liability	0,3%	3,7%	5,5%	8,4%	12,9%	19,7%	107,4%	7,9%	9,8%	373
Motor, other classes	0,0%	2,1%	3,0%	4,6%	7,4%	11,0%	167,1%	4,8%	10,6%	348
Marine, aviation and transport	0,3%	5,8%	8,7%	14,0%	23,8%	46,1%	425,8%	14,0%	44,5%	255
Fire and other damage to property	0,0%	2,8%	4,4%	6,6%	11,8%	20,6%	1041,6%	7,4%	51,7%	460
Third-party liability	0,0%	4,7%	7,2%	11,8%	18,8%	37,9%	320,9%	11,9%	31,7%	428
Credit and suretyship	0,7%	6,1%	10,1%	22,7%	60,9%	157,5%	28179,0%	16,8%	2423,9%	142
Legal expenses	0,0%	1,4%	2,8%	5,4%	10,6%	20,0%	172,3%	6,4%	22,0%	164
Assistance	0,0%	1,2%	2,9%	8,0%	16,6%	42,6%	239,8%	7,6%	38,6%	127
Miscellaneous non-life insurance	0,0%	2,3%	5,9%	13,1%	30,0%	66,2%	4979,8%	15,5%	366,4%	266
NP reins property	0,1%	7,6%	20,5%	32,2%	53,5%	110,8%	1129,7%	28,0%	155,1%	52
NP reins casualty	0,0%	3,0%	12,4%	18,9%	41,8%	54,1%	44192,0%	20,7%	6983,4%	41
NP reins MAT	6,8%	11,0%	15,9%	26,1%	43,1%	92,4%	355,7%	27,9%	69,2%	27

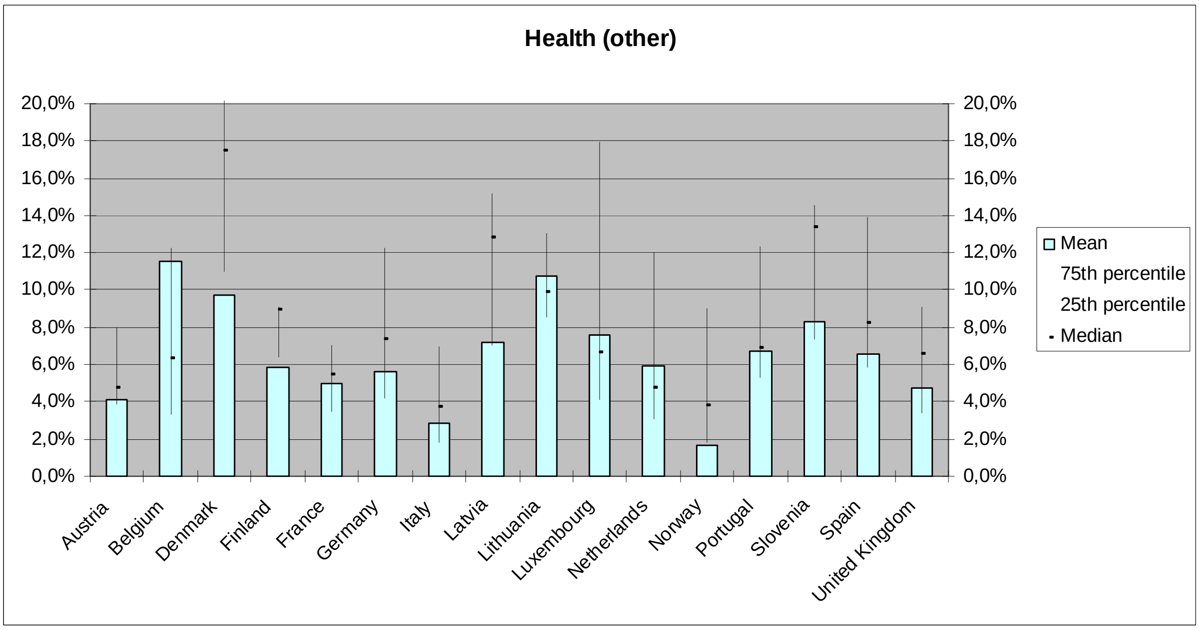
Comparison with QIS4 calibration

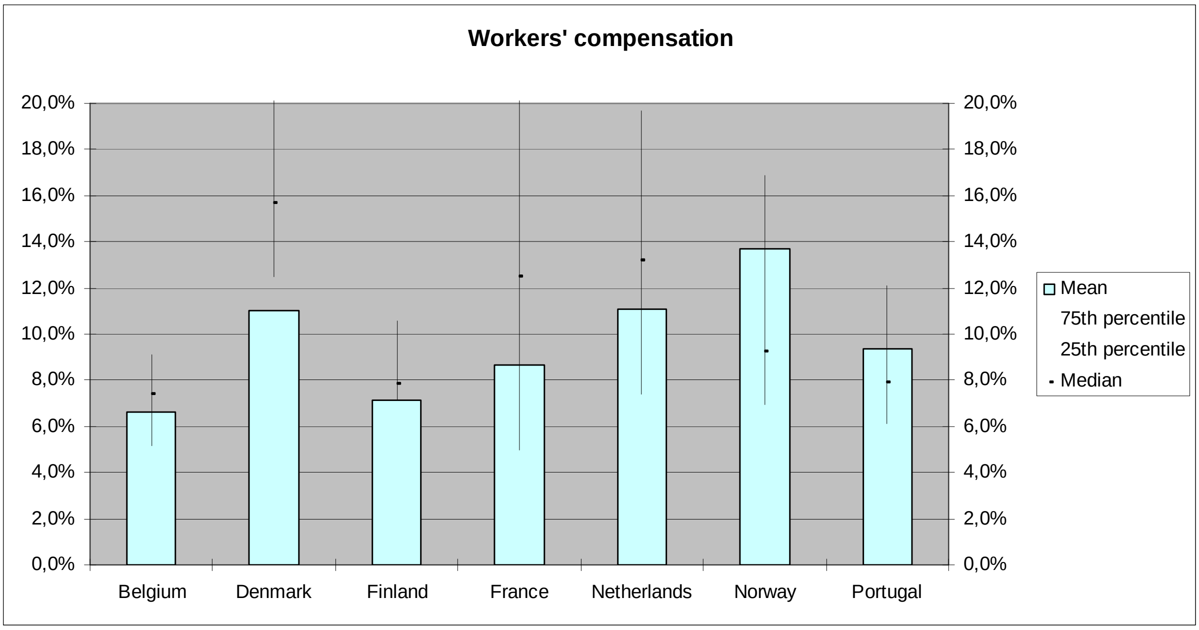
	Median	Mean	QIS4
Health (short-term)	4,6%	4,3%	3,0%
Health (other)	6,8%	5,9%	5,0%
Workers' compensation	10,6%	9,3%	7,0%
Motor, third party liability	8,4%	7,9%	9,0%
Motor, other classes	4,6%	4,8%	9,0%
Marine, aviation and transport	14,0%	14,0%	12.5%
Fire and other damage to property	6,6%	7,4%	10,0%
Third-party liability	11,8%	11,9%	12.5%
Credit and suretyship	22,7%	16,8%	15,0%
Legal expenses	5,4%	6,4%	5,0%
Assistance	8,0%	7,6%	7.5%
Miscellaneous non-life insurance	13,1%	15,5%	11,0%
NP reins property	32,2%	28,0%	15,0%
NP reins casualty	18,9%	20,7%	15,0%
NP reins MAT	26,1%	27,9%	15,0%

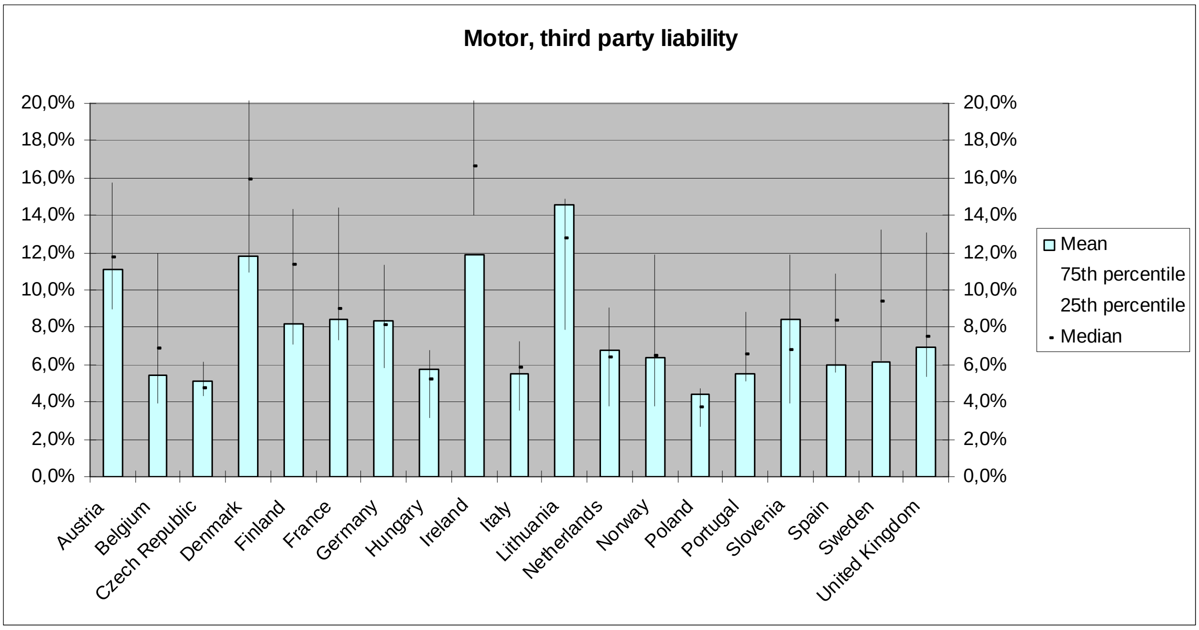
Country results

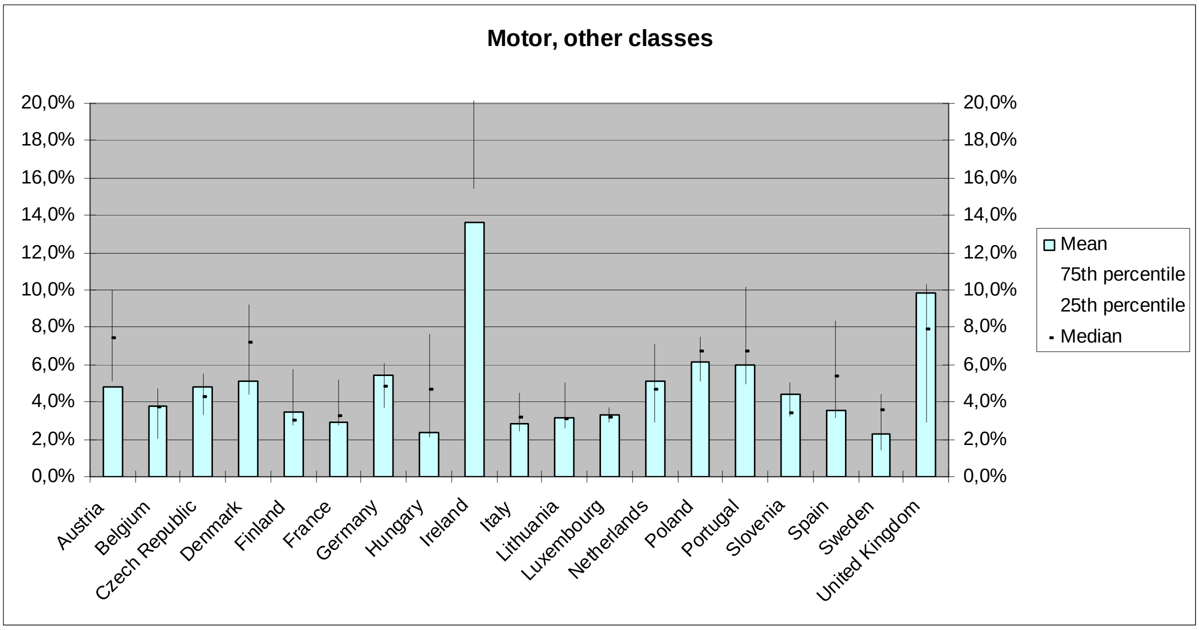
7.15. The following diagrams show the estimates for each national market. Only countries with at least five time series are shown in the diagrams.

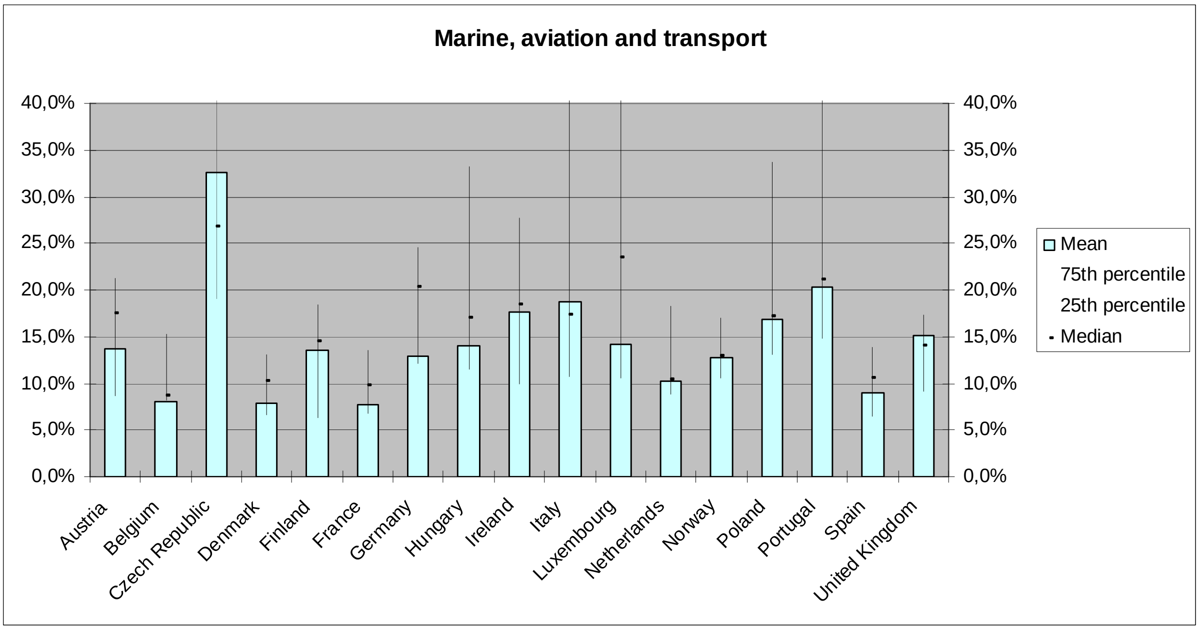

Health (short term)
20,0%
18,0%
16,0%
14,0%
12,0%
10,0%
8,0%
6,0%
4,0%
2,0%
0,0%
Mean
75th percentile
25th percentile
Median
Austria
Belgium
Czech Republic
Denmark
France
Germany
Hungary
Ireland
Italy
Netherlands
Poland
Portugal
Slovenia
Spain
Sweden
United Kingdom

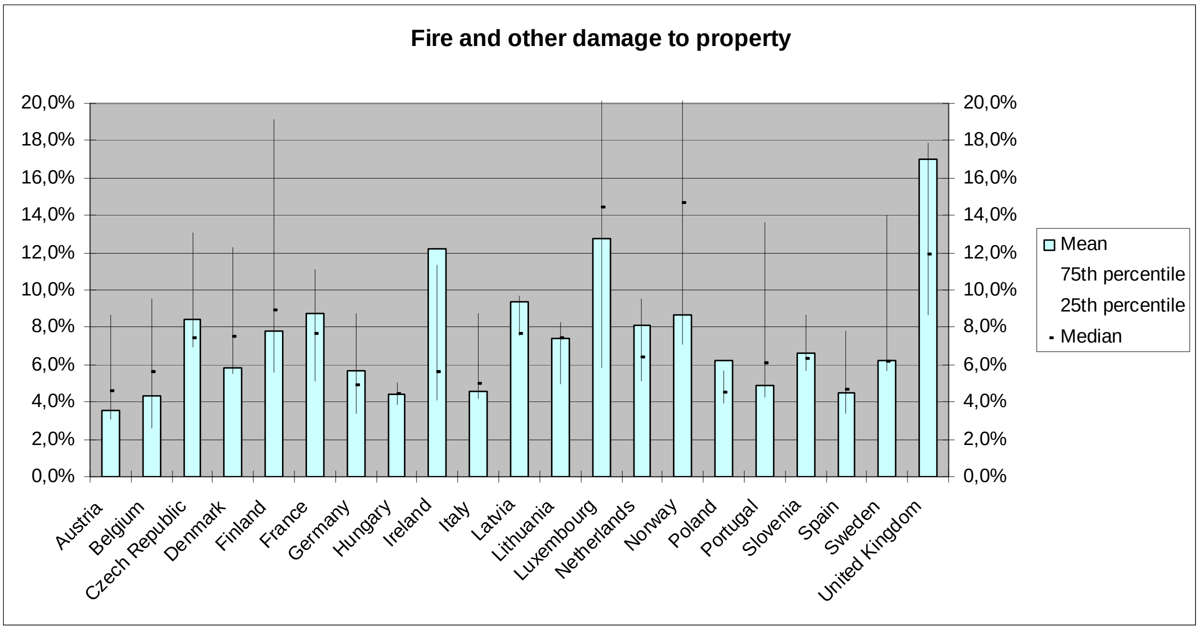

Health (other)
20,0%
18,0%
16,0%
14,0%
12,0%
10,0%
8,0%
6,0%
4,0%
2,0%
0,0%
Mean
75th percentile
25th percentile
Median
Austria
Belgium
Denmark
Finland
France
Germany
Italy
Latvia
Lithuania
Luxembourg
Netherlands
Norway
Portugal
Slovenia
Spain
United Kingdom

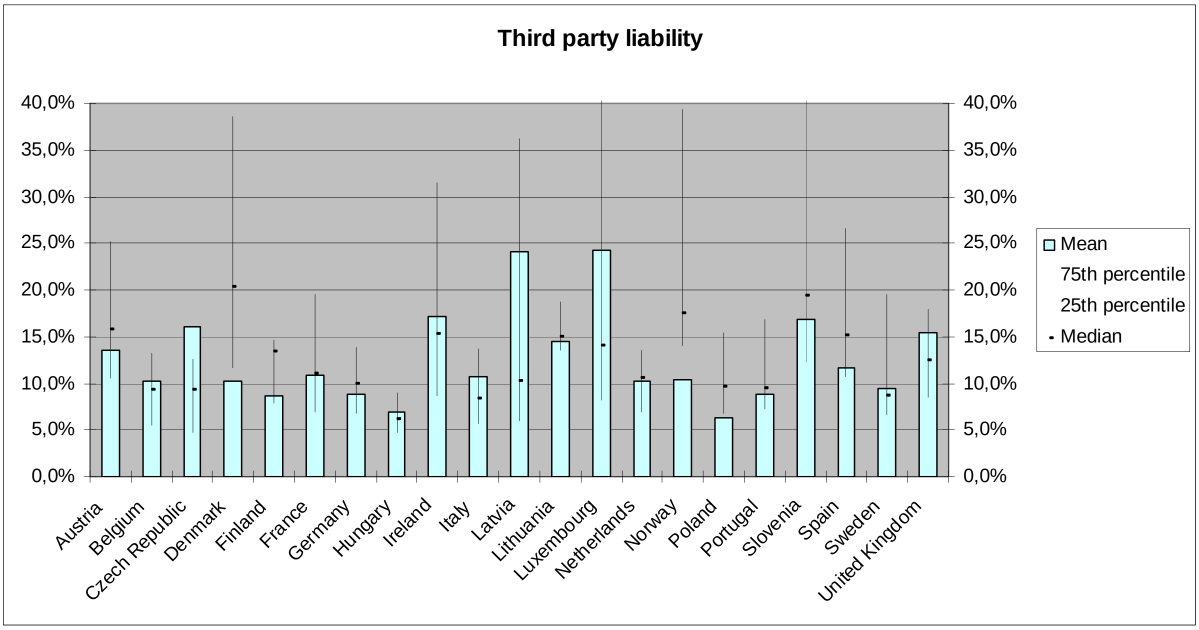

Workers' compensation
20,0%
18,0%
16,0%
14,0%
12,0%
10,0%
8,0%
6,0%
4,0%
2,0%
0,0%
Belgium
Denmark
Finland
France
Netherlands
Norway
Portugal
Mean
75th percentile
25th percentile
Median


Motor, third party liability
20,0%
18,0%
16,0%
14,0%
12,0%
10,0%
8,0%
6,0%
4,0%
2,0%
0,0%
Austria
Belgium
Czech Republic
Denmark
Finland
France
Germany
Hungary
Ireland
Italy
Lithuania
Netherlands
Norway
Poland
Portugal
Slovenia
Spain
Sweden
United Kingdom
Mean
75th percentile
25th percentile
Median


Motor, other classes
20,0%
18,0%
16,0%
14,0%
12,0%
10,0%
8,0%
6,0%
4,0%
2,0%
0,0%
Mean
75th percentile
25th percentile
Median
Austria
Belgium
Czech Republic
Denmark
Finland
France
Germany
Hungary
Ireland
Italy
Lithuania
Luxembourg
Netherlands
Poland
Portugal
Slovenia
Spain
Sweden
United Kingdom


Marine, aviation and transport
40,0%
35,0%
30,0%
25,0%
20,0%
15,0%
10,0%
5,0%
0,0%
Mean
75th percentile
25th percentile
Median
Austria
Belgium
Czech Republic
Denmark
Finland
France
Germany
Hungary
Ireland
Italy
Luxembourg
Netherlands
Norway
Poland
Portugal
Spain
United Kingdom


Fire and other damage to property
20,0%
18,0%
16,0%
14,0%
12,0%
10,0%
8,0%
6,0%
4,0%
2,0%
0,0%
Mean
75th percentile
25th percentile
Median
Austria
Belgium
Czech Republic
Denmark
Finland
France
Germany
Hungary
Ireland
Italy
Latvia
Lithuania
Luxembourg
Netherlands
Norway
Poland
Portugal
Slovenia
Spain
Sweden
United Kingdom


Third party liability
40,0%
35,0%
30,0%
25,0%
20,0%
15,0%
10,0%
5,0%
0,0%
Mean
75th percentile
25th percentile
Median
Austria
Belgium
Czech Republic
Denmark
Finland
France
Germany
Hungary
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Luxembourg
Netherlands
Norway
Poland
Portugal
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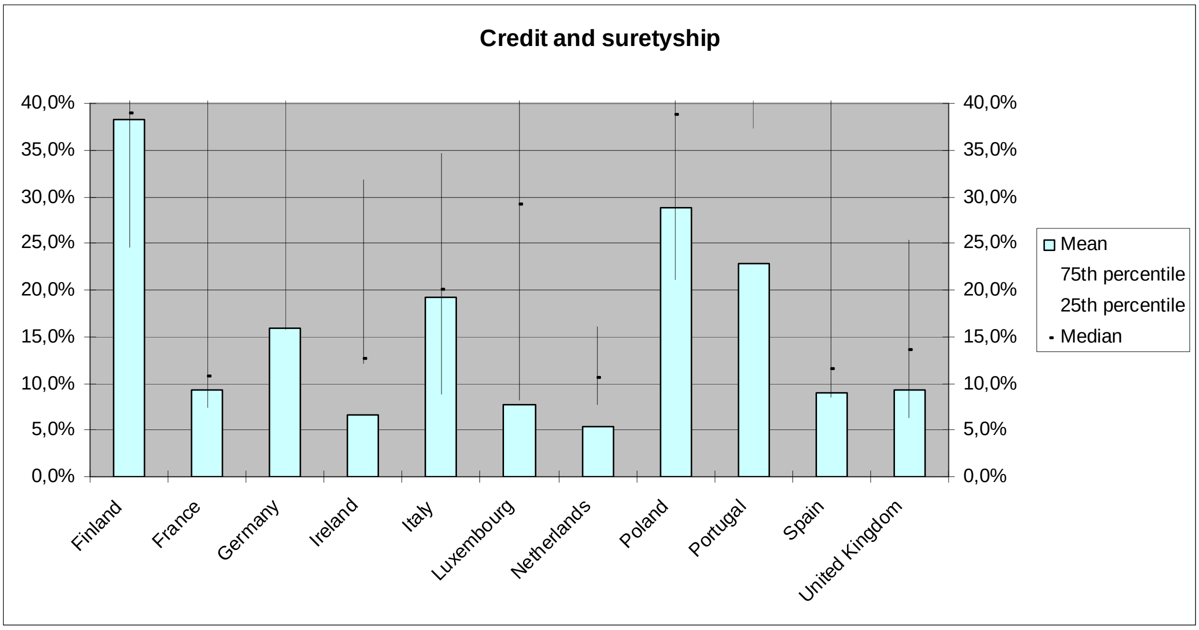
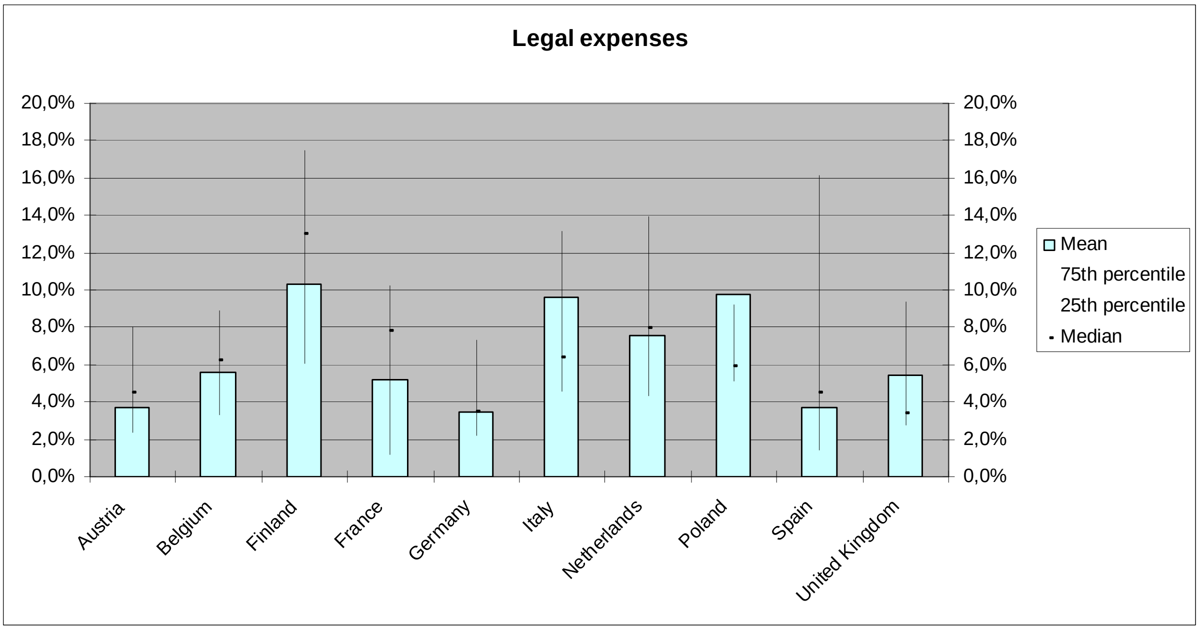
Spain
Sweden
United Kingdom


Credit and suretyship
40,0%
35,0%
30,0%
25,0%
20,0%
15,0%
10,0%
5,0%
0,0%
Mean
75th percentile
25th percentile
Median
Finland
France
Germany
Ireland
Italy
Luxembourg
Netherlands
Poland
Portugal
Spain
United Kingdom


Legal expenses
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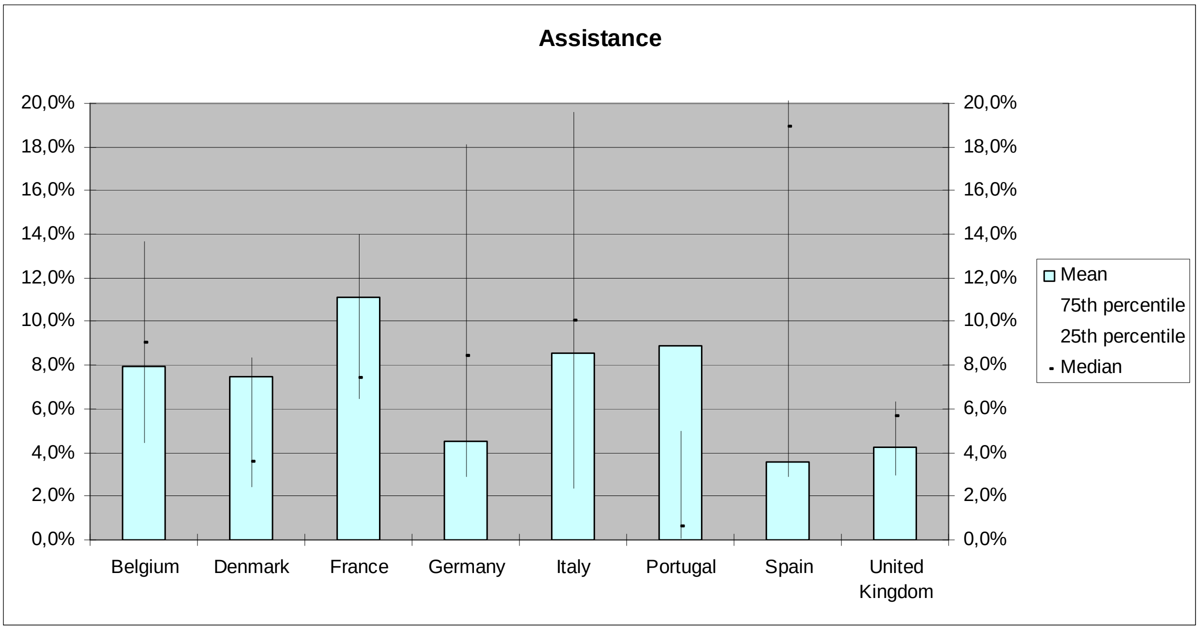
18,0%
16,0%
14,0%
12,0%
10,0%
8,0%
6,0%
4,0%
2,0%
0,0%
Mean
75th percentile
25th percentile
Median
Austria
Belgium
Finland
France
Germany
Italy
Netherlands
Poland
Spain
United Kingdom


Assistance
20,0%
18,0%
16,0%
14,0%
12,0%
10,0%
8,0%
6,0%
4,0%
2,0%
0,0%
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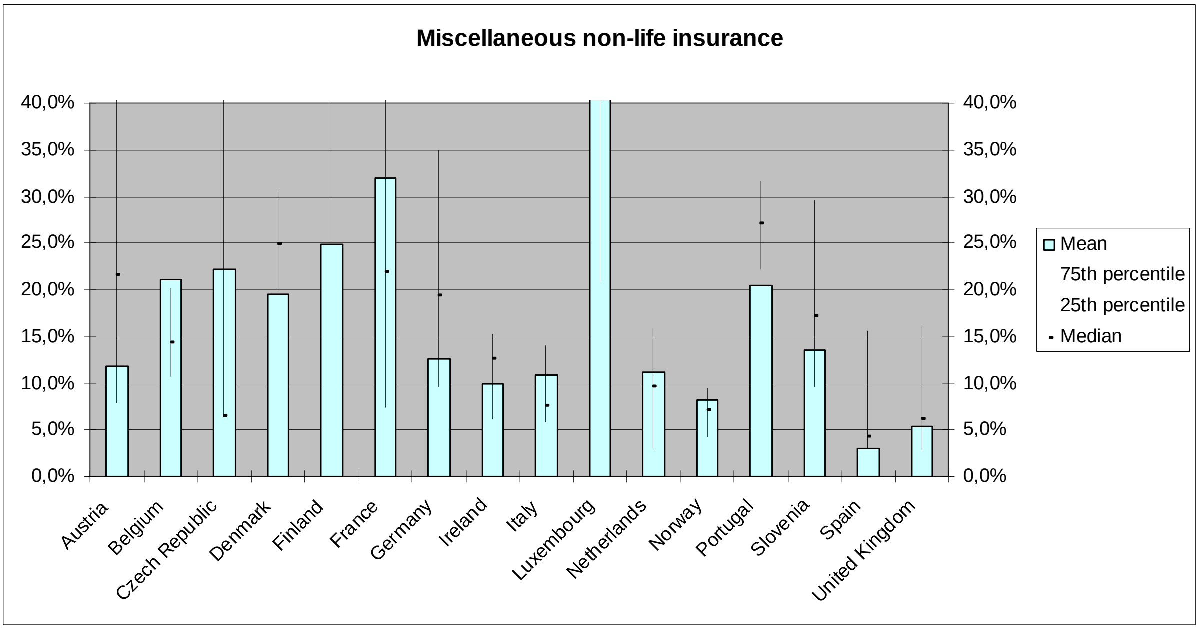
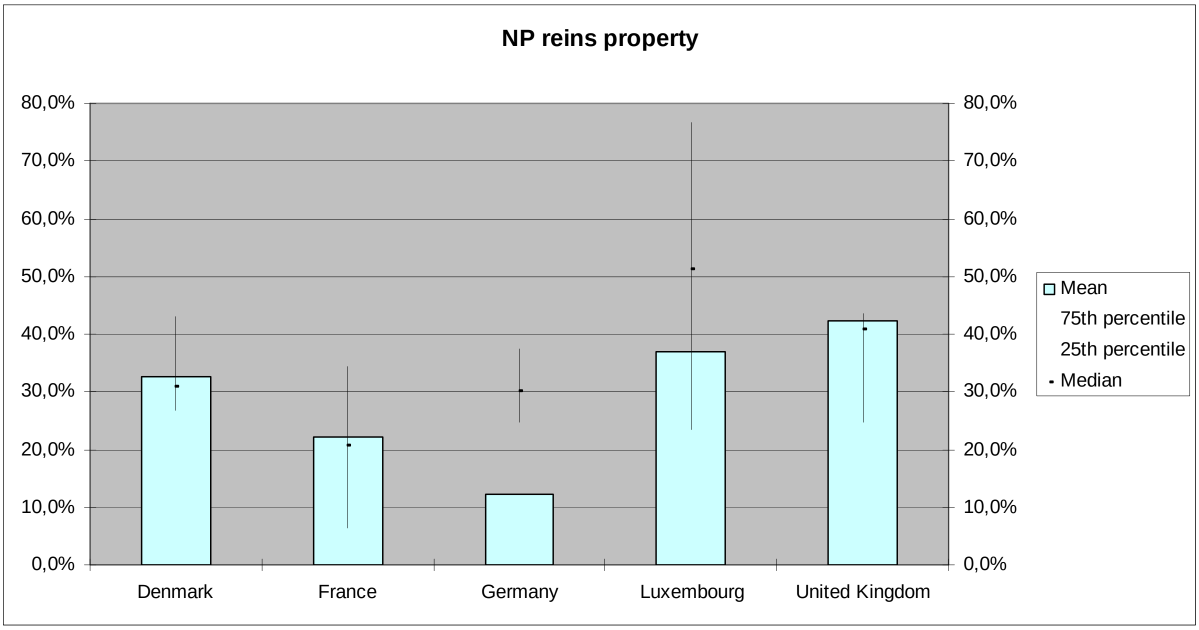
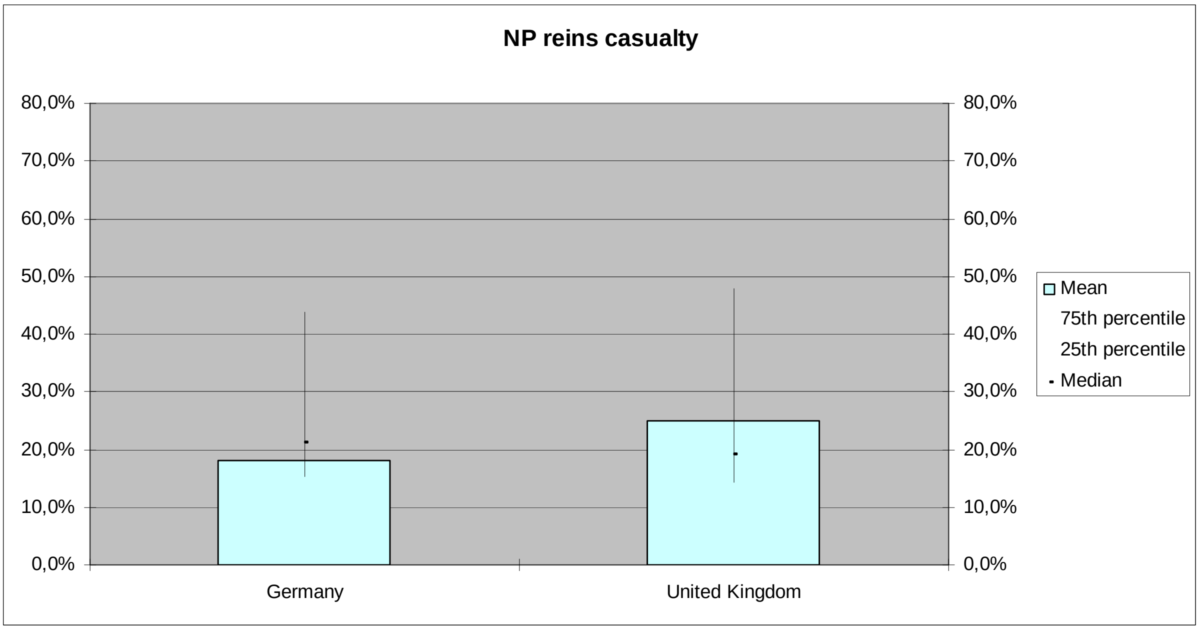
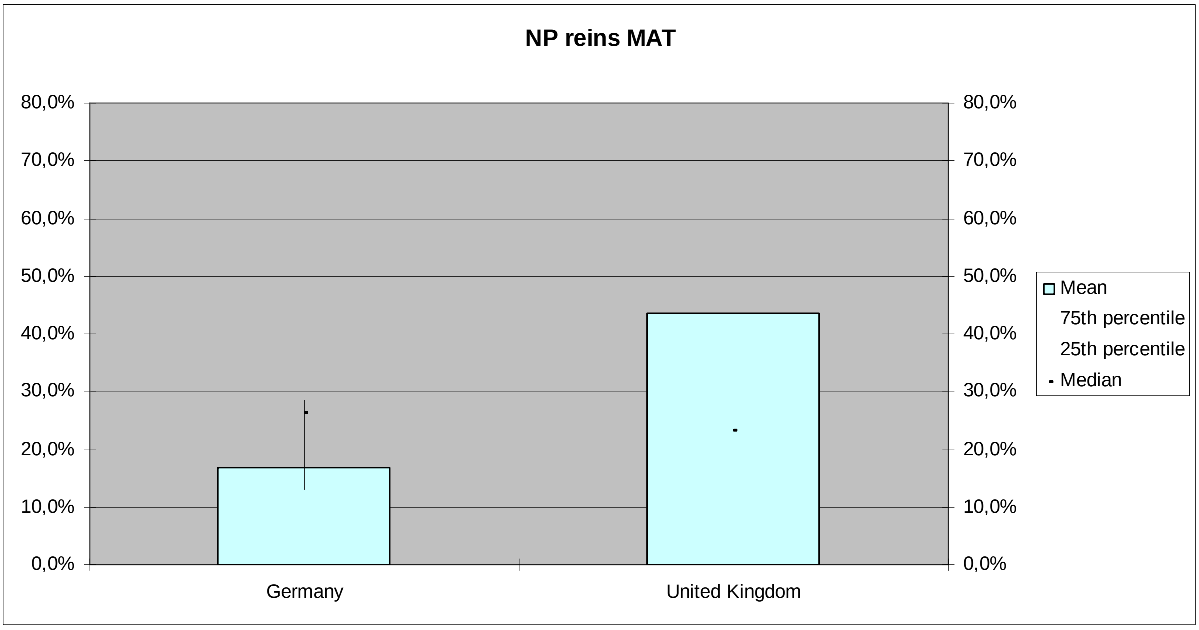
Denmark
France
Germany
Italy
Portugal
Spain
United Kingdom
Mean
75th percentile
25th percentile
Median


Miscellaneous non-life insurance
40,0%
35,0%
30,0%
25,0%
20,0%
15,0%
10,0%
5,0%
0,0%
Austria
Belgium
Czech Republic
Denmark
Finland
France
Germany
Ireland
Italy
Luxembourg
Netherlands
Norway
Portugal
Slovenia
Spain
United Kingdom
Mean
75th percentile
25th percentile
Median


NP reins property
80,0%
70,0%
60,0%
50,0%
40,0%
30,0%
20,0%
10,0%
0,0%
Denmark
France
Germany
Luxembourg
United Kingdom
Mean
75th percentile
25th percentile
Median


NP reins casualty
80,0%
70,0%
60,0%
50,0%
40,0%
30,0%
20,0%
10,0%
0,0%
Germany
United Kingdom
Mean
75th percentile
25th percentile
Median


NP reins MAT
80,0%
70,0%
60,0%
50,0%
40,0%
30,0%
20,0%
10,0%
0,0%
Germany
United Kingdom
Mean
75th percentile
25th percentile
Median

7.6 AMICE proposal for non proportional reinsurance

PROPOSAL FOR NON PROPORTIONAL REINSURANCE

This paper summarises the improvements to the QIS5 standard formula suggested by AMICE and supported by the CRO Forum concerning the non proportional reinsurance for the premium risk. It is not dealing with Cat risk which is addressed in the CEIOPS Taskforce on Cat Risk.

In order to better capture the effects of risk mitigation strategies, especially in the case of non-proportional reinsurance, the following methodology can be easily tested in the QIS5 standard formula.

Due to complexity of some non proportional reinsurance contracts, no standard formula would be able to catch all reinsurance features. We are aware that the topic is not simple if we want to keep a standard formula as operational as we can.
Nevertheless, in the standard formula framework, it is easy to improve some aspects which do not add any complexity and do not ask so much additional information.

Our proposal: a pragmatic approach

The proposal does not change the actual standard formula framework. We think the standard formula is complex enough and it does not make sense to completely change the design of the non life underwriting module.

The underlying idea is to adjust the premium factor for each line of business according to the mitigation effect due to the non proportional contract. The approach adjusts the original volatility factors for premium risk which are supposed to be calibrated gross on reinsurance. There is no change for the rest of the standard formula.

The limited scope of this approach in the standard formula is linked to its simplicity and it should be a good incentive for non life insurers to further improve risk management with partial internal models on reinsurance or undertaking specific parameters in order to capture the full reduction of volatility from the reinsurance strategy.

The adjustment ratio is based on frequency-severity approach which is intensively used in reinsurance impact studies. It is a global frequency-severity model, not only for large claims, but also for all claims for a given line of business. We suppose the independence between the frequency and the severity of the claims which is generally accepted.

The assumptions are:

- Frequency N of all claims: $N \longrightarrow Poisson(\lambda)$
- Severity X for a single claim gross of reinsurance : $X \longrightarrow Lognormal(m;\sigma)$

The choice of a Lognormal distribution for a single claim severity is rather conservative.

It is also possible to show that the frequency has no impact on volatility reduction. So there is no need to calibrate the factor λ. No assumption on the frequency is requested in the approach.

From the distribution of a claim gross of reinsurance, it is easy to estimate the average cost net of reinsurance and the volatility reduction with an Excess of Loss layer.

For a given b XoL a, the net loss is: $Y = \begin{cases} X & if\ X \le a \\ a & if\ a < X \le a+b \\ X-b & if\ X \ge a+b \end{cases}$ and the variance of the random variable S aggregate losses after reinsurance is: $Var(S^{Net}) = \lambda \cdot (Var(Y) + E^2(Y))$

We immediately have: $\frac{Var(S^{Net})}{Var(S^{Gross})} = \frac{Var(Y)+E^2(Y)}{Var(X)+E^2(X)}$ (independent from the number of claims N)

The assumptions on which this calculation is based are quite common in non life insurance. The layer can be limited or unlimited.

All the details are given in appendix 2.

Thus the adjustment ratio is based on a comparison between the volatility of a claim net of reinsurance and the volatility gross of reinsurance.

The adjusted premium factor (net of reinsurance) is: $\varphi \cdot \sqrt{\frac{vol^2(Y)+1}{vol^2(X)+1}}$

Where $vol(X) = \frac{\sqrt{Var(X)}}{E(X)}$ and $vol(Y) = \frac{\sqrt{Var(Y)}}{E(Y)}$

φ : Volatility factor for premium risk gross of reinsurance

Extra - Input data needed are also limited

In this approach, the only additional requested information is the average cost per claim for each line of business and its standard deviation. We believe that entity specific parameters are relevant.

But in a first approach for the coming quantitative impact study, country parameters could also be tested.

When the average cost of a claim and its standard deviation are given, we automatically know the value σ with the formula $\sigma = \sqrt{\ln\left(1+\frac{Var(X)}{E^2(X)}\right)}$ for a Lognormal distribution. The other parameter m is given by the formula: $m = \ln E(X) - \frac{\sigma^2}{2}$

Limitations of the proposal

All reinsurance features are not caught in this approach. The impact of an annual aggregate deductible or annual limit is not quantified. We assume in a standard framework there is only one Excess of Loss layer with unlimited reinstatements.

In some cases, the reinsurance contracts are too complicated to be considered adequately in the standard formula and would thus require partial modelling.

The capital requirement calculation is based on the usual linear assumption for lognormal random variables, closed to three times the standard deviation net of reinsurance, commonly used in the Solvency II framework.

<u>Numerical examples</u>

<u>1st example:</u>
Gross Premium Factor: 15%
The average cost is constant and equal to 3000. The coefficient of variation (standard deviation / average cost) is variable.

Coefficient of variation Gross claim	Unlimited layer XoL p				
	p = 500 000	p = 1 000 000	p = 5 000 000	p = 10 000 000	p = 15 000 000
500%	12,2%	13,3%	14,6%	14,8%	14,9%
1000%	8,3%	9,6%	12,4%	13,2%	13,7%
1500%	6,3%	7,4%	10,3%	11,5%	12,1%

<u>2nd example:</u>
Gross Premium Factor: 15%
In this example the coefficient of variation of a claim gross is constant and equal to 500%. The average cost is variable.

Average cost Gross claim	Unlimited layer XoL p				
	p = 500 000	p = 1 000 000	p = 5 000 000	p = 10 000 000	p = 15 000 000
1 000	13,7%	14,3%	14,9%	15,0%	15,0%
3 000	12,2%	13,3%	14,6%	14,8%	14,9%
5 000	11,3%	12,5%	14,3%	14,7%	14,8%

Appendix 1: The adjusted premium factor

The expected aggregate loss $S = \sum_{i=1}^{N} X_i$ for this lob is given by: $E(S) = E(N) \cdot E(X) = \lambda \cdot E(X)$

The variance is: $Var(S) = Var(X) \cdot E(N) + E^2(X) \cdot Var(N) = \lambda \cdot (Var(X) + E^2(X))$

For a given b XoL a, the net loss is: $Y = \begin{cases} X & if\ X \le a \\ a & if\ a < X \le a+b \\ X-b & if\ X \ge a+b \end{cases}$ and the variance of the random variable S after reinsurance is: $Var(S^{Net}) = \lambda \cdot (Var(Y) + E^2(Y))$

We immediately have: $\frac{Var(S^{Net})}{Var(S^{Gross})} = \frac{Var(Y)+E^2(Y)}{Var(X)+E^2(X)}$ (independent from the number of claims N)

The closed formulas to estimate $E(Y)$ and $Var(Y)$ are given in the next appendix. It only depends on (m,σ) and the layer b XoL a.

The formula used in the non life underwriting module for calculating SCR is:

$$SCR = V \cdot \left(\frac{\exp\left(2.5758 \cdot \sqrt{\ln(1+\varphi^2)}\right)}{\sqrt{1+\varphi^2}} - 1 \right) = V \cdot VaR_{99.5\%}^{Mean}(\Psi)$$

Where:

ψ : Lognormal distributed random variable with $E(\psi) = 1$ and $Var(\psi) = \varphi^2$

$VaR_{99.5\%}^{Mean}(\Psi)$: 99.5% Value at Risk of $\psi - E(\psi)$

V : Volume measure (premium)

In the standard formula, on a gross basis, it is assumed that $S^{Gross} - E(S^{Gross})$ has the same distribution as $V^{Gross} \cdot (\psi^{Gross} - 1)$

In case of non proportional reinsurance, we would like to find a random variable ψ^{Net} where $S^{Net} - E(S^{Net})$ would have the same distribution as $V^{Net} \cdot (\psi^{Net} - 1)$ (Lognormal distributed).

We have: $Var(S^{Net}) = Var(S^{Gross}) \cdot \frac{Var(Y)+E^2(Y)}{Var(X)+E^2(X)} = (V^{Gross})^2 \cdot \varphi^2 \cdot \frac{Var(Y)+E^2(Y)}{Var(X)+E^2(X)}$

With the assumption $V^{Net} = V^{Gross} \cdot \frac{E(Y)}{E(X)}$, the variance of S^{Net} becomes:

$$Var(S^{Net}) = \left(V^{Net} \cdot \frac{E(X)}{E(Y)} \right)^2 \cdot \varphi^2 \cdot \frac{Var(Y)+E^2(Y)}{Var(X)+E^2(X)} = (V^{Net})^2 \cdot \varphi^2 \cdot \frac{vol^2(Y)+1}{vol^2(X)+1}$$

where $vol(X) = \frac{\sqrt{Var(X)}}{E(X)}$ and $vol(Y) = \frac{\sqrt{Var(Y)}}{E(Y)}$

To be consistent with the standard formula, $S^{Net} - E(S^{Net})$ has the same distribution as $V^{Net} \cdot (\psi^{Net} - 1)$

where ψ^{Net} is a Lognormal distributed random variable with $E(\psi^{Net}) = 1$ and $Var(\psi^{Net}) = \varphi^2 \cdot \frac{vol^2(Y)+1}{vol^2(X)+1}$

The adjusted premium factor (net of reinsurance) is: $\varphi \cdot \sqrt{\frac{vol^2(Y)+1}{vol^2(X)+1}}$

Appendix 2: Average cost and standard deviation of a claim net of reinsurance

A claim net of reinsurance for a layer b XoL a is given by: $Y = \begin{cases} X & if\ X \le a \\ a & if\ a < X \le a+b \\ X-b & if\ X \ge a+b \end{cases}$

For a Lognormal distribution, we have the following results:

$$E(X) = e^{m+\frac{\sigma^2}{2}} \qquad Var(X) = E^2(X)\cdot\left(e^{\sigma^2}-1\right) \qquad E\left(X^2\right) = e^{2m+2\sigma^2}$$

Lemma:

$$\int_p^{+\infty} x\cdot f(x)\cdot dx = \frac{1}{\sigma\sqrt{2\pi}}\cdot\int_{\ln p}^{+\infty} e^x\cdot e^{-\frac{1}{2}\cdot\left(\frac{x-m}{\sigma}\right)^2}\cdot dx = e^{m+\frac{\sigma^2}{2}}\cdot\frac{1}{\sigma\sqrt{2\pi}}\cdot\int_{\ln p}^{+\infty} e^{-\frac{1}{2}\cdot\left(\frac{x-m-\sigma^2}{\sigma}\right)^2}\cdot dx$$

$$\int_p^{+\infty} x\cdot f(x)\cdot dx = E(X)\cdot\left[1-F_{m+\sigma^2,\sigma}(p)\right]$$

where $F_{m+\sigma^2,\sigma}$ is the distribution function of a Lognormal random variable with parameters $\left(m+\sigma^2,\sigma\right)$

$$\int_p^{+\infty} x^2\cdot f(x)\cdot dx = \frac{1}{\sigma\sqrt{2\pi}}\cdot\int_{\ln p}^{+\infty} e^{2x}\cdot e^{-\frac{1}{2}\cdot\left(\frac{x-m}{\sigma}\right)^2}\cdot dx = e^{2m+2\sigma^2}\cdot\frac{1}{\sigma\sqrt{2\pi}}\cdot\int_{\ln p}^{+\infty} e^{-\frac{1}{2}\cdot\left(\frac{x-m-2\sigma^2}{\sigma}\right)^2}\cdot dx$$

$$\int_p^{+\infty} x^2\cdot f(x)\cdot dx = E\left(X^2\right)\cdot\left[1-F_{m+2\sigma^2,\sigma}(p)\right]$$

where $F_{m+2\sigma^2,\sigma}$ is the distribution function of a Lognormal random variable with parameters $\left(m+2\sigma^2,\sigma\right)$

Average cost

$$E(Y) = \int_0^a x\cdot f(x)\cdot dx + a\cdot\int_a^{a+b} f(x)\cdot dx + \int_{a+b}^{+\infty}(x-b)\cdot f(x)\cdot dx$$

$$E(Y) = E(X) - \int_a^{+\infty} x\cdot f(x)\cdot dx + a\cdot\left[F_{m,\sigma}(a+b)-F_{m,\sigma}(a)\right] + \int_{a+b}^{+\infty} x\cdot f(x)\cdot dx - b\cdot\left[1-F_{m,\sigma}(a+b)\right]$$

$$E(Y) = E(X) - E(X)\cdot\left[1-F_{m+\sigma^2,\sigma}(a)\right] + a\cdot\left[F_{m,\sigma}(a+b)-F_{m,\sigma}(a)\right] + E(X)\cdot\left[1-F_{m+\sigma^2,\sigma}(a+b)\right] - b\cdot\left[1-F_{m,\sigma}(a+b)\right]$$

$$\boxed{E(Y) = E(X)\cdot\left[1-F_{m+\sigma^2,\sigma}(a+b)+F_{m+\sigma^2,\sigma}(a)\right] + a\cdot\left[F_{m,\sigma}(a+b)-F_{m,\sigma}(a)\right] - b\cdot\left[1-F_{m,\sigma}(a+b)\right]}$$

For an unlimited cover $b=+\infty$, the average cost net of reinsurance becomes:

$$E(Y) = E(X)\cdot F_{m+\sigma^2,\sigma}(a) + a\cdot\left[1-F_{m,\sigma}(a)\right]$$

Standard deviation

$$E(Y^2)=\int_0^a x^2\cdot f(x)\cdot dx+a^2\cdot\int_a^{a+b} f(x)\cdot dx+\int_{a+b}^{+\infty}(x-b)^2\cdot f(x)\cdot dx$$

$$E(Y^2)=E(X^2)-\int_a^{+\infty} x^2\cdot f(x)\cdot dx+a^2\cdot\left[F_{m,\sigma}(a+b)-F_{m,\sigma}(a)\right]$$
$$+\int_{a+b}^{+\infty} x^2\cdot f(x)\cdot dx-2b\cdot\int_{a+b}^{+\infty} x\cdot f(x)\cdot dx+b^2\cdot\left[1-F_{m,\sigma}(a+b)\right]$$

$$E(Y^2)=E(X^2)-E(X^2)\cdot\left[1-F_{m+2\sigma^2,\sigma}(a)\right]+a^2\cdot\left[F_{m,\sigma}(a+b)-F_{m,\sigma}(a)\right]$$
$$+E(X^2)\cdot\left[1-F_{m+2\sigma^2,\sigma}(a+b)\right]-2b\cdot E(X)\cdot\left[1-F_{m+\sigma^2,\sigma}(a+b)\right]+b^2\cdot\left[1-F_{m,\sigma}(a+b)\right]$$

$$\boxed{\begin{aligned}&E(Y^2)=E(X^2)\cdot\left[1-F_{m+2\sigma^2,\sigma}(a+b)+F_{m+2\sigma^2,\sigma}(a)\right]+a^2\cdot\left[F_{m,\sigma}(a+b)-F_{m,\sigma}(a)\right]\\&-2b\cdot E(X)\cdot\left[1-F_{m+\sigma^2,\sigma}(a+b)\right]+b^2\cdot\left[1-F_{m,\sigma}(a+b)\right]\end{aligned}}$$

For an unlimited cover $b=+\infty$, the variance net of reinsurance becomes:

$$E(Y^2)=E(X^2)\cdot F_{m+2\sigma^2,\sigma}(a)+a^2\cdot\left[1-F_{m,\sigma}(a)\right]$$